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PHILOSOPHICAL

TRANSACTIONS

OF THE

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OF

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PART I.

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ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the council-books and journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions;* which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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The PRESIDENT and COUNCIL of the ROYAL SOCIETY adjudged the ROYAL MEDALS for the year 1829 as follows:

A Royal Medal to CHARLES BELL, Esq. Fellow of the Royal Society, for his Discoveries relating to the Nervous System.

A Royal Medal to Professor EILERT MITSCHERLICH of Berlin, Foreign Member of the Royal Society, for his Discoveries relating to the Laws of Crystallization, and the Properties of Crystals.

PHILOSOPHICAL TRANSACTIONS.

I. THE BAKERIAN LECTURE.—*On the manufacture of Glass for optical purposes.*

By MICHAEL FARADAY, *Esq. F.R.S. &c.*

Read November 19, December 3 and 10, 1829.

Introduction.

PERFECT as is the manufacture of glass for all ordinary purposes, and extensive the scale upon which its production is carried on, yet there is scarcely any artificial substance in which it is so difficult to unite what is required to satisfy the wants of science. Its general transparency, hardness, unchangeable nature, and varied refractive and dispersive powers, render glass a most important agent in the hands of the philosopher engaged in investigating the nature and properties of light; but when he desires to apply it, according to the laws he has discovered, in the construction of perfect instruments, and especially of the achromatic telescope, it is found liable to certain imperfections, not essentially existing, but almost always involved during its preparation, and fatal to its use. These are so important and so difficult to avoid, that science is frequently stopped in her progress by them; a fact fully proved by the circumstance that Mr. DOLLOND, one of our first opticians, has not been able to obtain a disc of flint glass four inches and a half in diameter, fit for a telescope, within the last five years, or a similar disc of five inches in diameter within the last ten years.

It must be well known to the scientific world, that these difficulties have induced some persons to labour hard and earnestly for years together, in hopes of surmounting them. GUINAND was one of these: his means were small, but he deserves the more honour for his perseverance and his success. He commenced the investigation about the year 1784, and died engaged in it in the year 1823. **Fraunhofer** laboured hard at the solution of the same practical problem. He was a man of profound science, and had all the advantages arising from extensive means and information, both in himself and others. He laboured in the glass-house, the work-shop, and the study, pursuing without deviation the great object he had in view, until science was deprived of him also by death. Both these men, according to the best evidence we can obtain, have produced and left some perfect glass in large pieces: but whether it is that the knowledge they acquired was altogether practical and personal, a matter of minute experience, and not of a nature to be communicated; or whether other circumstances were connected with it,—it is certain that the public are not in possession of any instruction, relative to the method of making a homogeneous glass fit for optical purposes, beyond what was possessed before their time; and in this country it seems doubtful whether they ever attained a method of making such glass with certainty and at pleasure, or have left any satisfactory instructions on the subject behind them.

The philosophical deficiencies referred to above, induced the President and Council of the Royal Society in 1824, to appoint a Committee for the improvement of glass for optical purposes, consisting of Fellows of the Royal Society and Members of the then Board of Longitude. The Government on being applied to, not only removed the restrictions to experiments on glass, occasioned by the Excise laws and regulations, but undertook to bear all the expenses of furnaces, materials, and labour, as long as the investigations offered a reasonable hope of success. In consequence of these facilities, a small glass-furnace was erected in 1825, and many experiments both upon a large and small scale were made with flint and other glasses. During their continuance, Messrs. **Green** and **Pellatt** gave every instruction and assistance in their power, and evinced the most earnest desire for success. The researches, however, soon showed themselves to be a work of labour, which, to be successful, would require to be pursued unremittingly for a long period; and on May 5, 1825, a sub-committee was appointed, to whom the direct superintendence and performance of experiments were entrusted. This committee consisted of Mr. **Herschel**, Mr. **Dollond**, and myself; but in March 1829 was reduced to two, by the retirement of Mr. **Herschel**, who about that period went to the continent. From the respective pursuits of the three persons appointed upon this committee it may be easily gathered, that though all were to do what they could in every way for the general good of the cause in which they were jointly engaged, yet a distinction in the duties of each existed. It was my business to investigate particularly the chemical part of the inquiry; Mr. DOLLOND was to work and try the glass, and ascertain practically its good or bad qualities; whilst Mr. HERSCHEL was to examine its physical properties, reason respecting their influence and utility, and make his competent mind bear upon every part of the inquiry.

The experimental glass-house was erected on a part of the premises of Messrs. GREEN and PELLATT, at the Falcon Glass-works; whilst my duties as Director of the Laboratory of the Royal Institution, required my presence almost constantly at the latter place, nearly three miles from the former. As I found it impossible under these circumstances to make the numerous experiments and pay that close attention which appeared essentially necessary to produce any degree of success, the President and Council of the Royal Society applied to the President and Managers of the Royal Institution, for leave to erect on their premises an experimental room, with a furnace, for the purpose of continuing the investigation. They were guided in this by the desire which the Royal Institution has always evinced to assist in the advancement of science; and the readiness with which the application was granted, showed that no mistaken notion had been formed in this respect. As a member of both bodies, I felt much anxiety that the investigation should be successful. A room and furnaces were built at the Royal Institution in September 1827, and an assistant was engaged, Sergeant ANDERSON of the Royal Artillery, whose steady and intelligent care has been of the greatest service to me in the experiments that have been proceeding constantly from that time to the present. At first, the inquiry was pursued principally as related to flint and crown glass; but in September 1828 it was directed exclusively to the preparation and perfection of peculiar heavy and fusible glasses, from which time to the present continual progress has been made.

I have thought it right to give this brief explanatory statement of the manner in which it has happened to become my duty, on the present occasion, to give an account of what has been done in the improvement of glass for optical purposes by the Committee of the Royal Society, working at the Royal Institution. I would willingly have deferred this account until the inquiry were more complete than at present; for though glass has been made, and telescopes manufactured, yet I have no doubt that much more of improvement will be effected. It may be said that a long time has elapsed since the experiments were first instituted; and that if any thing could be done, it should have been effected in so long a period. But be it remembered, that it is not a mere analysis, or even the developement of philosophical reasoning, that is required: it is the solution of difficulties, which, as in the cases of GUINAND and FRAUNHOFER, required many years of a practical life to effect, if it was ever effected. It is the foundation and developement of a manufacturing process, not in principle only, but through all the difficulties of practice, until it is competent to give constant success: and I may be allowed to plead the acknowledged difficulty and importance of the subject as a reason, both why it may not yet have obtained perfection, and why it should still be pursued.

My wish, however, to delay the account of the researches until I could have carried the experiments further, is overcome by the conviction that much more time must be expected to elapse before I shall consider the investigation finished; by the consideration that a decided step has been made in the manufacture of glass for optical purposes; and by the feeling that the Royal Society which instituted, and the Government which defrays the expenses of the experiments, have a right to an official account of the present state of the investigation. Although much useful information has been obtained respecting flint and other glasses, yet as that train of research is very imperfect, uncertain, and will probably be resumed, I shall confine my present statement altogether to the heavy optical glass already referred to. It will be impossible for me to describe all that has been done on this subject; but I shall endeavour to give such an account of the glass, and the process by which it is obtained in a homogeneous state, as shall enable other persons to do what has been done at the Royal Institution, without incurring the laborious prefatory experiments and investigations which we have had to undertake; only introducing so much of the latter, and the principles of the process, as are necessary to make the descriptions clear to a practical man, and enable him to avoid those circumstances which might otherwise occasion failure. That the paper may appear long and tedious I am aware; but it should be remembered, that it can have no other utility than as containing efficient instructions to the few who may desire to manufacture optical glass; and that to render whatever of this character it may have, imperfect, for the sake of giving to it a more abbreviated and popular form, would have been doing injustice to the objects and motives of those who have instituted and supported the experiments.

§ 1. *Process of Manufacture, &c. &c.*

1. The general properties of transparency, hardness, and a certain degree of refractive and dispersive power, which render glass so valuable as an optical agent, are easily obtained: but there is one condition essential in all delicate cases of its application, which is not so readily fulfilled; this is, a perfectly homogeneous composition and structure. Although every part of the glass may in itself be as good as possible, yet without this condition they do not act in uniformity with each other; the rays of light are deflected from the course which they ought to pursue, and the piece of glass becomes useless. The streaks, striæ, veins or tails, which are seen within glass otherwise perfectly good, result from a want of this equality; they are visible only because they bend the rays of light which pass through them from their rectilinear course, and are constituted of a glass having either a greater or a smaller refractive power than the neighbouring parts.

2. When these irregularities are so powerful as to render their effects observable by the naked eye, it may easily be supposed to what an injurious extent their influence must extend in the construction of telescopes and other instruments of a similar nature, where these faults are not only magnified many times, but where the effect is to give an equally magnified erroneous representation of the object looked at, when the very point to be attained is to examine that object with the utmost accuracy; and it is accordingly found that these striæ are the most fatal faults of glass intended for optical purposes. Besides this, not only do the striæ themselves occasion harm, but there is every reason to believe that they rarely occur in glass otherwise homogeneous. Sometimes, it is true, a grain of sand, in passing through and at the same time dissolving in glass, will give a streak of different composition to the rest of the substance; and at others, a bubble ascending may lift a line of heavy or more refractive matter into a lighter and less refractive portion above. But very often, and especially as glass is usually manufactured and collected for use,

striæ are merely the lines or planes where two different kinds of glass approximate; and even if the striæ could be covered so as to produce no bad effect, yet the other parts, not being in every respect alike, would exert an unequal action on light, and the piece be therefore improper for the construction of a telescope. Many a disc, which upon the most careful examination has appeared perfectly free from striæ and quite uniform, has, when worked into an object-glass, been found incapable of giving a good image, on account of the existence of irregularities in the mass, which, though not sudden or strong enough to occasion striæ, still produce a confused effect; and if this happens with glass approaching so near to perfection, it happens still more frequently and to a much stronger degree with such as contain visible irregularities.

3. It must not be imagined that striæ, or those fainter differences, are, according to an expression sometimes used, due to impurity. The glass, either of the streak or of the neighbouring parts, would be equally good for optical purposes were it all alike. It is the irregularity that constitutes the fault; and hence, in this respect, a particular composition is of very little importance. As glass is always the result of a mixture of materials having different refractive and dispersive powers, it is evident that striæ must exist at one period during its preparation; and the point required is not so much to seek for a difference of composition, or for those proportions which are found by analysis to exist in specimens of tried and acknowledged good glass; as to devise and perfect a process by which the striæ period should be passed over before the glass is finished, and the formation of fresh striæ be prevented.

4. Besides these, there are other faults in glass. Sometimes it is said to be wavy, when it has the appearance of waves within its mass; but this is only a variety of that irregularity which has just been explained as constituting, when in a stronger degree, streaks and striæ. Occasionally appearances are observed in it, which seem to indicate a peculiar structure or crystallization, or an irregular tension of its parts: these, there is every reason to believe, may be avoided by careful annealing. Again: the glass sometimes includes bubbles, which, when small and numerous, render it what is called seedy. Bubbles are not usually considered as of much consequence to the performance of the glass, but objectionable only because of their appearance when the glass is looked at, rather than when looked through. They each act like a very

powerful but very small double convex lens of a rare substance in a very dense medium, or as equally deep double concave lenses of glass would do in air; they rapidly, therefore, turn the rays impingeing on them on one side, and occasion a loss of light, just as so many opaque spots would do. But as even when numerous their united area may amount to only a very small proportion of the area of the plate of glass required for a telescope, this loss of light is usually of but little consequence. In practice, it is said that no other real evil than such loss of light is dependent on them.

5. Of all these faults, that of the irregularity constituting streaks, striæ, and waves, is the most difficult to avoid, and the most injurious in its effect. It is not an improvement only beyond what is ordinarily done in this respect that is required, but absolute perfection, a homogeneity equal to that of pure water. In the two kinds of glass required to render a telescope achromatic, namely, crown or plate glass, and flint glass, it is the latter which is obtained perfect with the greatest difficulty, and to which therefore the greatest attention has been paid. The reason of this will be evident, if the general composition of the two glasses be taken into account. The required difference between them in refractive and dispersive power is found to be at command, by attention to composition; and it has been also ascertained, that crown and plate glass answer exceedingly well for the one variety, and flint glass for the other. Crown glass consists of silica, lime, oxide of iron, sometimes a little alkali, and small quantities of other matters: these substances are not very different in their refractive powers, and when fused do not produce very strong streaks, even though a little difference in the composition of different parts of the glass may exist. The glass also is not a very powerful fluxing agent upon the crucible in which it is melted; so that although it is in contact with it in a fluid and heated state for many hours, it does not dissolve much from it; and what it does dissolve having a refractive power little different from that of the glass itself, proportionately less harm is occasioned. Again: the specific gravity of the different materials used is not very different; so that the mixing agencies which affect the contents of the pot,—such as the ascent of bubbles, the ascending and descending currents from difference of temperature,—are more energetically exerted, and the whole mass approaches nearer to uniformity in a given time, or acquires it sooner than would happen were greater differences to exist.

6. With plate glass the same circumstances hold nearly in an equal degree. This substance is composed of silica and alkali essentially, other elements being only in small quantities. Its action upon the crucible is greater than crown glass, but then it has a second application of heat in such circumstances as are calculated to give a very uniform temperature to the contents of a whole pot, and it is delivered into its final form in the manner least likely to cause mixture of the different parts.

7. With flint glass many circumstances are altogether different. Oxide of lead enters into its composition to the amount of one third of its weight, or more, and by its presence gives that proportion of refractive and dispersive power, which makes the glass valuable in conjunction with crown or plate: this it does in consequence of its own powerful action on light; and it makes the glass heavy also, because of its own great specific gravity. A third property belonging to it, namely its high fluxing or dissolvent powers, it also confers upon the glass. Now these three properties are unfortunately very conducive to the formation of striæ. If the least difference in composition exists between one part and another it becomes evident, because of the great difference between the qualities of the oxide of lead and the other ingredients; and a variation in proportions which in crown or plate glass would produce no sensible effect to the naked eye, would in flint glass form strong striæ. Hence it is required that the mixture be in this case far more perfect than in the other glasses; and yet it unfortunately happens that every thing tends to make it much less so. The oxide of lead is so heavy a material, and at the same time so fusible, that it melts and sinks to the bottom, leaving the lighter materials to accumulate at the top: and so imperfect are the means of mixture, under ordinary circumstances, that glass of very different specific gravity is procured from the bottom and top of the same crucible. The following are some cases of this kind, from pots containing glass not more than six inches in depth, made from the usual materials, and retained at a full heat for twenty-four hours:—

Top......	3.38	3.30	3.28	3.21	3.15	3.73	3.85	3.81	3.31	3.30
Bottom...	4.04	3.77	3.85	3.52	3.80	4.63	4.74	4.75	3.99	3.74

These differences are great, and selected for illustration; but from appearances there is little reason to doubt that the same state of things, though not to such an extent, occurs in every pot of flint glass made in the ordinary way.

8. Another curious illustration of the predominance of oxide of lead at the bottom is shown in many of our specimens, which have been broken through vertically: they have been affected by sulphuretted vapours and tarnished; but the tarnish has occurred only at the bottom, where the lead is abundant, and is there very strong, whilst there is no appearance of it towards the top.

9. Whilst the crucible is in the condition described, it is clear that all those circumstances, as currents, bubbles, &c., which tend to mix the glass, form abundant striæ and veins of enormous strength, and do harm unless they are continued in activity until the mixture is nearly complete; a state rarely if ever acquired in the ordinary flint glass pot. But even if this could be the case, there is a constant cause of deterioration, arising from the highly fluxing and dissolving quality given to the glass by the oxide of lead. In this respect, flint glass far surpasses crown or plate glass, and it is also during one stage of its preparation more fluid: it consequently is continually exerting a solvent power upon the crucible to a considerable extent, occasioning that very irregularity in composition which produces striæ, whilst the comparative levity of the matter dissolved at the sides and bottom, and the ascending currents at the hottest parts of the crucible are constantly mixing this deteriorating portion with the general mass.

10. The difficulties which are thus introduced into the manufacture of flint glass fit for optical uses appeared to the committee, who, however, were none of them practical glass-makers, to increase, as the scale upon which the inquiries were carried on diminished: and the enormous expense of large experiments,—the time required for each,—the number necessary to give that experience which should render any one who undertook the charge of this part of the inquiry an ordinary practical workman,—and the uselessness of the resulting glass for any other purpose than the one directly contemplated,—compelled the sub-committee to consider seriously on the possibility of making other glasses than those ordinarily in use, which, at the same time that they had the high dispersive power enabling them to replace flint glass, might have also such fusibility as would allow of their being perfectly stirred and mixed, and might be

retained, without alteration, in such vessels as could be procured of any desired size.

11. The borate of lead, and the borate of lead with silica, were the substances which, after some trials, were found to offer such reasonable hopes of success as to justify a persevering series of experiments; and the metal platina was looked to as the material out of which to form the vessels intended to be used. It was soon ascertained that the borate of lead could be readily formed from dry materials, and that silica might be added with great advantage to the resulting glass; a range of proportions between the three ingredients being permissible, which gave much command over the properties of hardness, colour, weight, refractive and dispersive power, &c., and yet remained within the required range of fusibility. Platina also was ultimately found to answer perfectly the purpose of retaining the glass: for though at first it was continually liable to failure, yet it was ultimately ascertained that neither the glass nor any of the substances entering into its composition, separate or mixed, had the slightest action upon it. Finally, it was found that several kinds of glass formed of these materials, were in their physical properties fitted to replace flint glass in the construction of telescopes, in some cases apparently even with advantage; since which time the experiments have been unremittingly pursued.

12. The great proportion of oxide of lead in these glasses rendered attention to very minute points essential; for otherwise striæ were inevitably formed, and even the destruction of the apparatus involved. For this reason, after a certain number of trials upon composition had been made, one unvarying set of proportions were adopted, and the attention given altogether to the discovery and establishment of a process which should yield constantly good results. This, as far as it has been carried into effect and proved, it is now my object to describe.

13. The glass with which I have principally worked is a silicated borate of lead, consisting of single proportionals of silica, boracic acid, and oxide of lead. The materials are first purified, then mixed, fused, and made into a rough glass, which is afterwards finished and annealed in a platina tray.

14. *Purification of materials. Oxide of Lead.*—The oxide of lead at first used was litharge; but this source occasioned frequent destruction of the platina trays, in consequence of the existence of particles of metallic lead, which

alloying with the platina, rendered it fusible. When red lead was substituted for litharge, the same effect took place, due to the presence of particles of carbonaceous and reducing matter. Both these substances also contained so much iron and other impurities, as to give a deep colour to the glass, far beyond what was expected from the quantity of impurity present; this was afterwards explained. Carbonate of lead was also found to be too impure. Finally, all the oxide of lead necessary was purified, by being converted into a nitrate, and crystallized once or twice, as occasion might require.

15. For this purpose litharge is first washed, by which many black carbonaceous and ferruginous particles are separated; it is then dissolved in diluted nitric acid, so as to form a hot saturated solution, the operation being performed in clean earthenware vessels. Both the perfectly pure and the moderately pure acid have been tried without any sensible difference in the results: a little sulphuric acid does not seem injurious; and I find that sulphate of lead will dissolve perfectly in the glass; but muriatic acid has been always avoided. As the acid, water and litharge are made to act on each other by heat, either purposely applied or resulting from the chemical action going on, it will be found that when approaching towards neutrality the liquid will become very turbid. The hot saturated solution is then to be poured from the remaining litharge and undissolved nitrate of lead, and after standing a few moments, again poured from the sediment, and set aside to crystallize in a cool place. Before it is left, however, it is to be examined as to its acidity: if strongly acid to litmus paper, it is in a right state; if not, a little nitric acid should be added, for the crystals of nitrate have always been compact and pure under such circumstances, and more readily separable from insoluble matter.

16. After eighteen or twenty-four hours, the basins of crystals are to be examined; the clear mother liquor carefully poured off; the crystals broken up in the basins; and then repeatedly washed in fresh clear portions of the mother liquor, that any insoluble deposited matter may be removed. There will generally be a portion of this deposit; but if the process has been well performed, the crystals will be quite free. If they appear perfectly white or bluish white, they need not be recrystallized; but if yellow, they must be dissolved in water, a little nitric acid added, and the crystallization repeated. The nitrate in the mother liquors and washings should be purified by repeated processes.

17. The good crystals are to be washed in three or four waters, to remove the last portion of deposit and adhering soluble impurities: but to prevent excessive solution of the nitrate, the same portions of water may be used for several basins of crystals washed at the same time, by making it pass from one to another in succession. Being thus cleansed, they are to be drained, put over the sand bath, stirred and dried, and finally preserved in glass bottles. By this process much iron and sulphate of lead are excluded; and the purified nitrate is found to yield a glass very far superior in colour to that prepared with the ordinary oxides of lead, and to exert not the slightest action on the platina: its use put an end to all the accidents and failures which resulted from the presence of metallic lead in the oxide. 166 parts by weight are to be considered as equivalent to one proportional or 112 parts of protoxide of lead.

18. *Boracic acid.*—The boracic acid for these experiments was obtained pure from the manufacturer, but before being used was carefully examined. It was rejected unless it was in white or bluish white crystals, clean and entirely soluble in water. Its solution was tested for iron by the ferro-prussiate of potash and a drop of sulphuric acid, and also for other metallic impurities by a little solution of sulphuretted hydrogen. An ounce or two were heated and dissolved in a little water; and when cold, the soluble part separated and examined for sulphuric acid, by a few drops of nitrate of baryta and a little nitric acid. It was also examined for soda by dissolving three or four ounces in hot water, adding ten or fifteen drops of sulphuric acid, and allowing the whole to cool and crystallize, expressing the mother water from the crystals; concentrating it; again crystallizing, and then acting upon the mother liquor, obtained at the second time by strong alcohol, and continuing to wash with the latter fluid until all was dissolved, or an insoluble part left. If the latter circumstance occurred, the insoluble substance was examined for sulphate of soda, which if in any sensible quantity occasioned the condemnation of the boracic acid. The care respecting alkali in boracic acid was taken in consequence of observing certain bad effects produced in glasses which appeared referrible to its presence.

19. When the boracic acid was acknowledged as pure, 36 parts by weight of the crystals were considered as equivalent to 24 parts, or one proportional of the dry substance.

20. *Silica.*—This material is in its most convenient state when it forms part of a combination consisting of two proportions silica, and one oxide of lead. As yet, the silica I have used has been the flint glass-maker's sand, obtained from the coast of Norfolk, well washed and calcined. The silicate has been prepared by mixing two by weight of this sand with one of litharge, or with such quantity of nitrate of lead as is equivalent to one of litharge (16); the mixture is put into a large Hessian or Cornish crucible, which being covered over, has been put into a furnace and raised to a bright red heat for eighteen or twenty-four hours. On taking out the crucible, the charge has been found diminished somewhat in bulk, and of a porous structure and appearance like loaf-sugar. It has been freed from the crucible, the outside portions removed; and the pure parts carefully pulverized in a clean Wedgwood mortar. The powder has then been washed over in water, so as to obtain the whole in a fine state of division; after which it has been dried, and preserved in bottles. No sieve should be used in these comminuting operations, nor any reducing or metallic matter brought in contact with the substance. Every care should be taken to avoid contamination: 24 parts by weight of the silicate are equivalent to 16 parts, or one proportional of silica, and 8 parts of protoxide of lead.

21. The advantage of the silica in this combined state depends upon the known composition of the substance, its comparatively easy pulverization, and ready fusion with the other materials. That there is iron in the silica (and the litharge when used) is objectionable; and the trials for its removal have only been delayed that the investigation of a more important point, namely, a successful process, might proceed. From some brief experiments, I am led to believe that an unexceptionable source of silica will be obtained by acting upon this silicate, in a state of fine division, by nitric acid and water, or else by the use of rock crystal.

22. On some occasions I used pulverized flint glass, as the source of silica, conceiving that being already in a fusible state, it must possess an advantage over other silica, in allowing rapid mixture with the other materials. Allowance was made for the oxide of lead present, and the alkali was permitted to pass, as a substance that would probably do no harm. But a striking effect took place, which at once showed the necessity of perfectly pure materials. The glass when finished and cold was of a deep purple colour: this was immediately

referred to the manganese in the flint glass; a supposition proved by repeating the experiment with other flint glass, and then with flint glass of our own manufacture in which no manganese was used: the latter glass gave no purple colour; the former, a colour as deep as the first flint glass.

23. Thus it appears that this very heavy glass, the silicated borate of lead (and I find it to be the case with other heavy glasses), has the power of developing the colour of mineral substances far beyond what flint glass possesses; just as flint glass surpasses in the same property plate and crown glass. In the case in question, the manganese, which did not give a sensible tint to the flint glass, produced a strong colour when diluted eight or nine times by the heavy glass, for the proportion of flint glass used was only $\frac{10}{85}$ths of the whole. On making a few experiments with iron, I find that the same strong development of colour is produced with it in these heavy glasses; so that the utmost care is necessary to preserve all the materials during their preparation, and the glass in every part of the process, from metallic contamination.

24. The use of flint glass even without manganese was also objectionable, because of the alkali in it, which, as before stated, was found to produce bad effects, and rendered the glass containing it very liable to tarnish.

25. Such are the materials from which the heavy optical glass has been latterly manufactured. When the composition has been determined upon, the proper proportions and quantities of each are weighed out in a clean balance and vessels; thus, for the silicated borate of lead glass, consisting of single proportionals of each substance, 24 parts of the silicate would be taken, for they contain a proportional of silica equal to 16 parts, and in addition 8 parts of protoxide of lead: the proportional of oxide of lead has been taken as 112 parts; but there being 8 in the silicate, the quantity of nitrate of lead equivalent to 104 parts only are required, and this is 154.14 parts: the equivalent of dry boracic acid is 24, which being contained in 42 parts of the crystals, that quantity is the one required. These proportions when heated and submitted to mutual action will leave only 152 parts of glass, or thereabout, for

154.14	nitrate of lead contain	104 protoxide of lead.
24.00	silicate of lead contain	8 ditto. 16 silica.
42.00	crystallized boracic acid contain . . .	24 dry boracic acid.
		152 glass.

Hence the materials for any quantity of glass can be easily calculated; and if the above parts be ounces, about 9lbs of glass will result. The nitrate of lead is then to be broken small in a clean mortar, and the other ingredients well mixed with it in basins, the use of metal or dirty implements being carefully avoided.

26. The mixture is next melted, and made into rough glass. This preparatory operation is necessary, because from the quantity of vapourable matter which is disengaged in this part of the process, the materials, if put at once into the finishing vessel and furnace, might boil over and do injury; and the acid nature of the vapours themselves, if it did not occasion harm by acting on neighbouring iron and other parts of the furnace, would at least cause inconvenience. It is effected in a furnace, which will be particularly described in the Appendix to this paper. It will be sufficient here to state, that being a close furnace, the part immediately beyond the fire-place forms a horizontal chamber, covered above by an iron plate having large circular holes; these allow crucibles to pass through them, and to stand supported on the bottom of the chamber, whilst their edges rise above the upper iron plate. In this way the fire is applied very generally to the crucibles, whilst their mouths are altogether exterior to the furnace, so that the introduction of any reducing or colouring impurity from the fire is prevented, and the greatest facility in introducing the mixture, of watching its fusion, of stirring the glass, and finally of ladling it out, is obtained. The holes through which these crucibles are inserted are five or six in number; they are never all in use at once, and those out of use are covered by crucible covers. The heat is not given altogether by flame; but whilst coal is used in the fire-place, coke is applied between the crucibles, being introduced for that purpose, and arranged through the unoccupied holes. The iron top of the furnace is covered by a second iron plate, or, what is better, by earthenware plates, to retain the heat. The crucibles are of pure porcelain ware, and as thin as they can be obtained. The covers for them are evaporating dishes, considerably larger than the mouths of the crucibles: being turned upside down, they rest, when in their places, upon the neighbouring earthenware plate; not touching the crucibles, but preventing any thing from falling into them, and preventing the vapours from passing into the room. The latter are by the draught of the chimney drawn through by the sides of the crucible into

the furnace, and carried away up the flue, so as to occasion no annoyance to the operator. The covers are slung by a piece of platina wire, which being passed across the middle on the outside, is bent at each end round the edges, so that a rod of iron slightly curved at the extremity, easily suffices to remove them when the crucible is to be opened. Great care is always taken to put them in clean situations, and that in their removal nothing shall fall from them into the glass.

27. This furnace is found to be very effectual in its action; being connected with a high flue governed by a damper, great command of the temperature is obtained. The crucibles before being used are examined as to soundness, and then their temperature is raised gradually, and should not be above a dull red heat when the operation commences. The mixture already described (25) is then introduced, and the crucible covered; decomposition of the nitrate of lead instantly commences; the boracic acid loses its water, all the fixed elements unite; and it is remarkable that though a considerable quantity of boracic acid usually sublimes with the water when the latter is driven off from its crystals unmixed with other substances, yet scarcely a trace seems to evaporate in the present instance, in consequence of the presence of the oxide of lead.

28. The heat should not be raised too high or the operation hastened, and then the ebullition will proceed very gradually and favourably, the rough materials being by degrees converted into glass. Before the first charge is entirely melted a second is put in, and when that is fused down, sometimes a third, according to the quantity of glass present and the soundness of the crucible. When all is fused, the temperature is allowed to rise, but not too much, lest action upon the crucible to a serious extent should occur; the glass is then well agitated and mixed by a platina rake or stirrer, to be described hereafter. Finally the glass is either transferred by a platina ladle into trays roughly turned up out of old platina foil, or into a clean deep white earthenware vessel containing much distilled water. In the lattter case it is obtained in a divided state, and when drained, is dried on the sand-bath, and put up in clean bottles.

29. When a crucible has been emptied of its first portion of glass, it will serve, if carefully used, for a second, third, fourth, or for many operations; but

it should be watched for cracks and casualties, that the running of the glass into the furnace may be prevented, and, if necessary, another vessel taken.

30. The rough glass thus prepared is in the next operation to be converted into an annealed and finished plate. The size must therefore be determined upon, and we will assume it as 7 inches square, and 8 tenths of an inch thick, that being the dimension of the largest plate as yet made. For the purpose of making a competent platina vessel, a plate of that metal will be required at least 10 inches square; but if larger, it should not be cut, but either made into a tray with higher sides than is absolutely needful, or else used first in the manufacture of a larger plate of glass than the one to be described. It should be of such thickness as to weigh at least 17·5 grains to the square inch; and it is important that in its preparation a good ingot or the good part of an ingot of platina has been selected, and that it has been rolled very gradually and carefully without the formation of any holes by the adhesion of dirt or hard particles, or by the dragging of the metal in the mills. The desired perfection is, I understand, best obtained by rolling the platina between two clean plates of good copper.

31. The plate, being laid upon clean paper or a cloth on a smooth table, is to be cleansed with a cloth and a little water or alcohol, and then to be ignited at every part by a large spirit lamp. It must next be carefully examined as to its state and the occurrence of places upon its surface where holes are likely to exist. If the metal seems dragged in any place, an effect indicated by a roughness upon the surface, or by short lines parallel to each other but perpendicular to the course of rolling, such place should be noted or marked, for which purpose a dot of ink will be convenient. If a scale appears, or a small portion is apparently folded over, it should also be marked; and if a black spot is visible, (and they are sometimes formed by the adhesion of a particle of dirt or grit,) it should be examined, and removed by the point of a knife, if necessary, and its place also marked. All these places and the whole surface of the plate should then be examined for holes by a still stronger test, namely, by holding the sheet of metal before and close to a bright light, as a candle or lamp, in a dark room, and every hole observed marked. In making this examination, it must be done carefully and minutely, holding the plate in different directions to the light (for sometimes the holes are oblique),

and being careful that no reflection from illumined objects, as the hands, on to that side towards the face shall give deceptive indications. In the marking, too, the indicating spot should always be made at a certain distance from the hole, as the fourth or the third of an inch, and on the same plate constantly in the same direction or towards the same edge; the holes are then easily found again, and the mark remains during the soldering to guide the operator.

32. The holes discovered by these examinations are to be closed by little patches of platina soldered with gold; for gold, like platina, may be safely used in these experiments, when reducing matter is absent. The gold has been used in the finely divided state in which it is obtained by precipitation from its solutions by means of sulphate of iron, but it must be washed perfectly pure: the patches are formed by cutting a piece of clean new platina foil into small square or rectangular plates: a sufficient heat can usually be obtained by the use of the spirit lamp and mouth blowpipe. In the process of soldering, a little of the powdered gold is heaped upon the hole and slightly flattened by some clean instrument, the spirit lamp is applied underneath for a moment, which causes the gold to adhere slightly, a selected patch of platina is laid delicately upon the gold, and then the heat of the spirit lamp, urged by the blowpipe, is directed beneath against the place. Usually the gold will melt and run instantly, the platina patch will come into close contact with the plate, and the operation will be completed. If well done, the fused gold will appear all the way round in the minute angle formed by the edge of the patch, and also faintly at the hole on the opposite side of the plate.

33. Sometimes when the patch is large, or in the middle of a plate, the heat obtained as above is hardly sufficient to melt the gold freely and cause perfect adhesion. In such cases, a single or double piece of platina foil loosely laid over the part, prevents loss of heat from the upper surface, and frequently causes such increased elevation of temperature as to render the soldering perfect and effectual. In the few cases where this expedient has not succeeded, I have resorted to the oxyalcohol blowpipe, using a small bladder of oxygen with a little attached jet for the purpose. This has never failed to produce an effectual heat, and 15 or 20 cubical inches of oxygen are sufficient for many operations.

34. This application of patches and soldering is only secure for small holes,

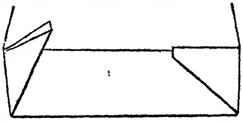
i. e. such as a pin might pass through, and smaller. The patches are always to be applied on that surface of the plate which is to constitute the outside of the tray; and therefore, before the soldering begins, the two surfaces should be examined, and the most polished and perfect selected as that intended for the inside. The patches are valuable in their use far beyond what the mere application of gold to the hole would be; for the heat afterwards applied to the tray when charged with glass, is abundantly sufficient to melt gold; in which case, if unsupported by the platina patch, the weight of glass and the action of stirring would probably force the gold out of the hole and cause the tray to run; whereas the patch of platina, although the gold holding it to the plate is liquid, still adheres by so strong a capillary action as to be sufficient to retain its place, and being outside is not disturbed by the motion of the stirrer. Besides, after a long application of heat, the gold and platina combine so perfectly as to become one piece of white alloy, infusible at the heat applied.

35. The plate is now to be folded into a tray, preparatory to which, a piece of thin board is to be provided as a guage, which in the present instance must be 7 inches square. This laid upon the plate and held tightly down, directs the foldings of the sides, and would, if placed in the middle, leave sufficient for edges one inch and a half high all round; but as the plate should serve for use several times, it is advantageous to apply the guage a little eccentric; for then, when used for a second and third operation, its place may be shifted, and the folds not occurring where they did before, there is less chance of holes being broken through the platina. The folds necessary at the corners of the tray are especially likely to render the same parts unable to bear a second and third bending; but the necessity of having them in the same place may be usefully obviated by placing the guage oblique to the sides in one direction and in another, on different occasions, and moreover gives other advantages in finishing the folding of the corners (36). These attentions, tending to the preservation of the platina for repeated service, are very needful, in consequence of the great expense of the material: the value of the plate in question is about 6*l.* 10*s.*, and when worn out, it may be sold for about half that sum. Whether it be used therefore once, twice, thrice, or four times, makes considerable difference in the expense of the resulting plates of glass.

36. When the guage is properly placed on the platina, the sides are raised

perpendicularly: this produces four projecting folded triangular corners, which being pressed close, are then turned against the sides, and a square tray is finished, which has no aperture or orifice below its upper edge. The folding of these corners is a matter of much more consequence than might be anticipated. The plate is seldom so regular that the parts of two neighbouring sides which come together at a corner are exactly of equal height; neither is it desirable that it should be so, and the unsymmetrical position of the guage to the plate, already recommended (35), is almost sure to prevent it. In that case, of the two sides of the folded corner, one will be higher than the other, and if the corner be so folded that its lower side is towards the tray and beneath its edge, a kind of syphon is formed which becomes charged with fluid by capillary action, and continues to discharge glass from the tray during the whole time of heating, notwithstanding that all the edges are much above the level of the fluid within. This in a long experiment is competent to occasion serious injury.

37. I have found, even when the edges of a corner have been of equal height, but below the edge of the side against which they are disposed, that still this capillary and syphon action has gone on, and the reason is not difficult to comprehend; the corners therefore have always been folded in such a manner, that their highest edge has been inwards, and both their edges above the level of the corresponding edge of the tray. To effect this, the line of their lateral flexure is not perpendicular to the bottom of the tray, but a little outwards above, and the proper degree of inclination is easily given by using a mould upon which to bend the corners. This should be a thick square piece of wood, having the four corners cut with different degrees of obliquity: when the corners of the tray are first imperfectly formed, it will be easy to ascertain by trial, which corner of this mould will give the obliquity and position already described as necessary, after which the folding may be easily finished upon it. The accompanying sketch represents first a good and then a bad folding.



38. All occasion for changes in the folds, especially at the corners, should be avoided. The folds should be decided upon as the work proceeds, so advantageously as to make alterations unnecessary. The closer the corners

are pressed, the smaller is the quantity of glass contained in them, and the less risk is there of the platina being broken when the finished glass is taken out; but it is proper to avoid general contact between the corners and the sides against which they are disposed, otherwise welding is likely to occur during the stirring, and the platina is injured for future experiments.

39. The tray being formed is again to be examined for holes, first by a light as before (31), and then in the following manner. Being laid upon a sheet of bibulous paper, alcohol is to be carefully poured in until the fluid is within the fourth or the sixth of an inch of the lowest edge of the tray, so as to occasion no running over at the sides or corners. If a large hole exist, it will be rendered visible immediately; but if none such appear, a large basin or some other cover is to be placed over the tray to prevent evaporation, but without touching the vessel or its contents; and the whole is to remain undisturbed for some hours. Being then examined, the wetting of the paper will indicate a hole or a badly folded corner, and will point out the faulty place: the tray may easily be shifted from one part of the paper to another for the discovery of any moistened places beneath. Sometimes holes occur so small that alcohol will not run in a sensible quantity through them. Suspected places of this kind and suspicious corners also should be examined by a clean dry point of bibulous paper, which soon shows, by its change of appearance, the transmission of any fluid: but attention is required that no false indication be produced by carelessly bringing the paper near the upper edges of the platina, especially in the folded places. These minute holes do not occasion much harm in the furnace, but no fault should be allowed to pass which care can correct.

40. When the tray is faulty, the alcohol must be removed by a small syphon, the holes soldered in the manner before described (32), and the tray again tried. When it proves good, it is, after the removal of the alcohol, to be heated red hot in every part by the flame of a large spirit lamp, and then reserved with care in a clean place until required.

41. If the platina has been used before, it should first be ascertained that none of the glass from the former experiment remain on it. If there be any portion, the plate must be returned to the weak acid or pickle out of which it has been taken. If free from glass, it should then be examined as to any

chemical injury it may have suffered. Any part which is altered in appearance, or has been attacked by the acid, or which tarnishes when heated to redness by the spirit lamp, has been thus affected; and it will depend upon the extent of the action whether the plate is unfit for further use. No chemical injury is occasioned by the proper and successful performance of an experiment.

42. An examination for holes by the candle or lamp must next be made, especially in the folds at the corners and where adhesion of the platina from welding may have occurred, and any that are discovered are to be marked as before (31). The plate should then be flattened by being put between two sheets of writing paper upon a smooth table, and the edge of a folding knife or some other smooth substance drawn over it; but if this be done whilst old glass adheres to the plate, it is almost certain to produce injury. The holes are then to be soldered and mended, the patches being applied upon the same side as before. The guage for the new tray is to be applied to the plate, shifted, if there be occasion, from its old position, as before intimated (35), and the folding of the tray, its completion and examination, to take place as before.

43. It is desirable never to cut the platina smaller than can be helped, but always to make the largest plate upon it for which it is competent. Then, when operated with a second or third time, smaller guages may be used, and the folds will not be repeated in the same place; and if injury occurs to the metal, being generally at the sides of the tray, the middle part will still be left for the preparation of smaller plates of glass.

If such large plates of platina are required for trays as can hardly be rolled at once, there is no difficulty in making a folded joint and rendering it tight by soldering with gold.

44. A kind of furnace, unlike the former, is now required for the completion of the glass, and its delivery in the state of an annealed plate. This furnace shall be described accurately in the Appendix. It may here be sufficient to state that it consists of a fire-place in which coals are burnt; of a part beyond, acting both as furnace and flue, in which coke is used; and of a chamber above, to be heated by the fire, though out of the course of both flame and smoke. It is in this chamber that the glass is made; so that, by the arrangement adopted,

at the same time the substances are fused and access for stirring allowed, the essential condition of excluding impurity or reducing matter is also fulfilled.

45. The fire-place itself is of the ordinary construction, and fed with fuel by an aperture in front in the usual way. I have found abundant reason to be satisfied that the passage of steam beneath the bars of the grate is of considerable use; for which reason an iron trough charged with water occupies the lower part of the ash-pit. The bars are by this arrangement preserved very cool and do not burn away; they are easily kept open and clear of clinkers; the free passage of air to the fire is permitted; and the action of the furnace retained at a high point for any number of hours together.

46. That part of the furnace beneath the chamber requires peculiar and careful arrangement; for at the same time that such a heat as will soften the neighbouring materials is produced there, the bottom of the chamber in its softened state and charged with several pounds of materials, has to be firmly supported for many hours together without change of position.

47. The coke necessary in this part is introduced by two or more holes in the side of the furnace, which, when necessary, are stopped by bricks. The bottom of the chamber is supported on ledges at the sides, and upon the ends of fire bricks in the middle, firmly placed at intervals so as neither to stop the passage for smoke and flame, nor the cross passages for the introduction of coke.

48. The value of the coke arrangement in this as in the other furnace is very great. The heat obtained by the united action of the coke and the flame from the fire-place, is abundantly sufficient; and whilst obtained at the necessary point does not involve that degree of mechanical action required for stoking and stirring, which is necessary with coals, and would risk the destruction of the soft thin bottom of the glass chamber. It further occasions the perfect combustion of the smoke produced in the coal fire, which at first was so considerable in quantity that, had it continued unaltered, the experiments must have been removed from the Royal Institution; in which case they would probably have been discontinued altogether.

The flue is the same as that connected with the former furnace, and has a damper for regulating the heat, especially useful during the annealing operation.

49. The chamber was at first of cast iron, that material being selected as one which would bear a sufficient temperature without melting, would conduct and transmit the heat freely to the substances within, and could be easily obtained of the requisite form. The upper aperture was closed by plate iron covers, and in the first trials all appeared to answer well; but when large experiments were made, and the heat was continued for a long time, the bottom gave way and became irregular; and upon endeavouring to rectify this, and place the tray of glass level by means of sand, the transmission of heat to the glass was prevented, the temperature of the iron rose, and the bottom melted. Besides these injurious liabilities, if the smallest portion of glass passed out of the tray, the moment it touched the iron it was reduced; the lead immediately caused fusion of the platina, and in an instant the tray was destroyed, the experiment stopped, the glass rendered black and useless, and the bottom of the chamber covered with lead and rendered unfit for another operation.

50. Finally, one very curious action of the iron was discovered, which immediately caused its rejection. Plates of glass, which seemed very good in other respects, were frequently so discoloured by dark smoky clouds as to be useless. These could not be referred to any impurity which had been left in the materials or had entered accidentally, and, as the platina was in all such cases altered and injured, was at first supposed to be occasioned by some particular action exerted between it and glass at high temperatures. But upon every fair trial to verify such chemical action, the proofs failed, however high the temperature used, or however minutely the metal was divided. At last the cause was discovered. To understand it, it must be known that the platina tray, with the glass in it, was either placed directly upon the bottom of the iron pan, or, for greater security, with only a plate of platina intervening; and that the whole was covered by an evaporating basin turned upside down, forming a sort of inner chamber within the large one. In this confined state the oxygen of the portion of air present was soon abstracted by the heated metal, an oxide of iron being formed in consequence, and at the same time also, a portion of carbonic oxide from the carbon in the cast iron. At the high temperature to which the experiment was raised, this carbonic oxide was competent to reduce a portion of the oxide of lead in the glass to the metallic state, itself becoming

carbonic acid; but as soon as the carbonic acid so produced came in contact with the heated iron, it was again converted, according to the well known condition of the chemical affinities at these temperatures, into carbonic oxide, and went back to the glass to repeat its evil operation and produce more metallic lead. In this way it was that the glass became sullied by smoky clouds consisting of metallic lead. It was the lead thus evolved also, that, by alloying with the platina, had produced the appearance of chemical action always visible in these cases; and now I knew how to account for the failure of many experiments in consequence of the formation of holes in the trays in a manner before quite inexplicable: for in the experiments purposely made to investigate this point, sometimes the glass was darkened only at the surface, the lower part being quite clear and good; and then, though the platina tray was frequently cut through as with a knife all round level with the surface of the glass, it was quite unaltered below. At other times the superficial stain was in a greater quantity, and had collected together into little drops like fat upon hot water, and upon examination each little globule was found to be soft metallic brilliant lead. At other times a much larger globule hung from the middle of the surface into the glass, barely sustained there, and ready to sink by the least agitation when in a heated state, and in some instances the bottom of the tray was alloyed and perforated by globules of lead which had thus been formed and deposited, and the glass just running out, whilst another globule was in progress of formation at the surface exactly over the place of the hole.

51. When iron was dismissed as the material of the chamber, earthenware was resorted to. The sides were built up of brick, and the bottom formed of tiles, which resting at the sides upon ledges, and at the middle upon the fire brick supports (47), could be replaced at pleasure. The same iron covers were used for the upper aperture of the chamber as before.

52. The use of earthenware as the material, made it far more difficult to apply a sufficient heat to the contents of the chamber than before, because of its inferiority to the iron as a conductor of heat; and a series of investigations were required to discover that substance, which, at the same time that it had sufficient strength and exerted no injurious influence, was also a sufficiently good conductor. Reigate fire-stone, recommended by the builders, did not answer the purpose, and moreover in thin plates was liable to fuse and slag.

Slate, however carefully heated, shivered and split, not only across, but parallel to its structure, and then, as soon as air intervened, it transmitted too little heat. It also softened, became curved, and let in air and smoke, and at last gradually fused, becoming unable to bear the weight of a large experiment. Yorkshire stone rubbed down into plates $\frac{5}{8}$ths of an inch thick, answered moderately well, if the application of heat was carefully made and gradually raised. It cracked in a few places, but did not fall to pieces ; and it was more difficult of fusion than the former substances. Fire tiles of various kinds were tried ; those made of Stourbridge clay answered the best, and when about $\frac{3}{4}$ths of an inch thick and carefully heated, might be successfully used; but that which we finally arrived at was the use of plates made of the materials from which Cornish crucibles are manufactured. These we obtained through the intervention of our President; they were purposely manufactured for us by Mr. Michell of Caleneck in Cornwall, a gentleman who has been ever willing and anxious to assist us in our inquiries, by supplying us with vessels of any size or form, or any other article which it was in his power to produce.

53. The Cornish plates have not much cohesion, and feel tender in the hand. They may be rubbed down to a flat surface, and resist any heat which can be applied to them in these or in much more powerful furnaces. They are therefore readily brought to any thickness, and when of about $\frac{5}{8}$ths of an inch, and supported in the furnace as before described (47), have strength to bear any weight required to be placed upon them. They do not crack, nor do they force themselves to pieces by expansion; but they are porous, as indeed are in a greater or smaller degree all the materials of which the chamber and its sides are now composed.

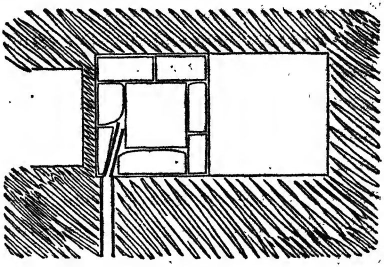
54. The porosity of these materials was of great importance; for it allowed of the passage of gaseous matter, and that even of a reducing nature, from the fire into the chamber. I have frequently had evidence that the sides and bottom might be considered as a very sieve-like partition between the fire, the flue, and the space called the chamber; for when the upper aperture has been closed, there has been a current through the chamber in the direction of the flame, the gaseous matter entering at the extremity nearest the fire, and passing out at the end towards the flue. In one or two cases, oxide of lead was actually reduced, and the glass thus rendered cloudy.

55. Hence it became necessary to use some certain means of maintaining an oxygenating atmosphere about the glass; to obtain which, and also to prevent any other injurious vapours from the fire entering the space beneath and within the earthenware covers (50), the expedient was adopted of allowing a current of fresh air to pass continually into that space and circulate about the glass. To effect this, a clean earthenware tube, glazed within, was let horizontally into the side of the furnace, in such a manner that one extremity was flush with the inside of the chamber, and of such height, that its lower edge corresponded with the level of the bottom upon which the glass in its tray was to be placed, whilst the other end of the tube reached to and was flush with the outside of the furnace. A loose piece of tube, similar in kind but smaller in diameter, being laid upon the bottom of the chamber, and applied at its end to the orifice of the larger one, served as a continuation of it until the inner extremity reached to and was under the cover of the glass experiment. When the furnace was hot, there was always a draught inwards through this tube; but the quantity of air admitted was regulated by a valve (70). The air, by first passing through the hot sides of the furnace, then through the shorter ignited tube serving for connection, was transmitted in a thoroughly heated state to the place where its presence was required, without producing any serious cooling effect; it there maintained a continually oxygenating atmosphere, and, judging from the effects, prevented the draught inwards of any vapours from the fire to the space beneath the glass covers.

56. The next point of importance, in the preparation of the glass, is the arrangement of the tray in the furnace, whose powers have just been described. To understand this, it will be necessary to say that the glass chamber is 25 inches long, 13 inches wide, and 8 inches deep, and that the fire being at one end, the flue is at the other. Plates of glass 7 inches square have been made in it; but it would probably require a larger furnace to make much larger pieces.

57. The bottom of the chamber being perfectly level and clean, the guage board, on which the tray was formed (35), should be placed on the middle of the half next the fire, and then a piece of connecting air tube taken, which being laid on the bottom of the chamber, may extend from the fixed air tube by the side of the guage as far as the middle, or even towards the other side of the chamber. After this, pieces of Cornish tile (53), or other clean

earthenware which will not fly in the fire, contains but little iron, and is free from glaze, are to be prepared, of such size that they will fit in loosely round the guage, covering the rest of that half of the chamber bottom, and serving to support the sides of the tray when in its place. This support to the tray is highly needful; for, otherwise, the weight of the glass, and the action of stirring, would be more than the thin and heated platina could support. The thickness of the pieces should be, for the plate in question, about 1 inch, and they should be all uniform in that respect. They should never rise so high as the edge of the platina, lest glass should accidentally pass from the tray to them, or impurities from them to the glass. An excellent guide to their thickness is, to make it similar to that of the intended plate. When they have been roughly arranged around the guage, the latter should be withdrawn, and the tray itself introduced, the pieces being now finally adjusted about it. They should not be so arranged as to press against its sides; but the latter should be at liberty, though only so much, that upon the least tendency of the sides outwards, they should be supported by the pieces. The assistance thus given should be directed rather to the sides than the corners, and it is better that the latter should not be in contact with these adjuncts, but be allowed to sustain themselves, for they are strong enough for the purpose, and the corners are always those places at which, from one circumstance or another, the glass is most likely to pass outwards.



58. The piece of earthenware which is fitted nearest the mouth of the air tube should have its angle taken off, or some other provision made, as by making the orifice of the tube oblique, that the passage of air may be uninterrupted; and on that side the tube itself may frequently form the support to the tray. If it does, and is glazed on the exterior, a piece of loose platina foil should be wrapped round it at the part where it touches the tray, to prevent adhesion by the glaze when cold. The general disposition of the tray, the tube, and the packings, may be seen in the sketch above.

59. When the first set of packing pieces is properly adjusted, a second series is to be arranged over them; but these are to be removed backward from the tray about the third or half of an inch all round, that accidental contact with its edges may be avoided. Their thickness should be sufficient to raise them level with, or rather above, the edges of the tray. All these adjusting pieces are to be rendered perfectly clean and free from dust before they are applied. Their use is not only to afford support and assistance to the platina tray, but also to sustain the glass covers, and likewise, by retaining the heat upon the bottom of the chamber, prevent much of the inconvenience that would otherwise occur at the times of stirring the glass.

60. The glass covers have, up to this period, consisted of inverted evaporating basins, suspended at pleasure, in the manner before described, by platina wires (26). When the platina trays used have been sufficiently small to admit of the arrangement in our present furnace, two, and even three covers have been used simultaneously, each prepared with its own platina suspension; but of such size, that the larger could be placed over and inclose the smaller, without touching it. In such cases the temperature of the glass, after being lowered by stirring, or in any other way, rose very rapidly; but with the large plate of 7 inches square, the furnace would admit of but one glass cover of sufficient size, and the only additional assistance which could be obtained was that which was given by putting a similar but smaller cover on the outside and above the principal one.

61. The first and important cover is to be selected of such dimensions, that when in its place and resting by its edges upon the packing pieces, it shall fully inclose the platina tray and its charge, not only for the purpose of accumulating heat and confining an oxygenating atmosphere within, but also sheltering the glass, and preventing any oxide of iron from the chamber covers, or dirt from other sources, falling into it. These covers, when hot, are raised and removed by means of clean iron rods, which being sufficiently thick to have abundant strength, and no injurious degree of elasticity, are made taper at one extremity, and slightly curved there. This end is easily introduced beneath the platina suspension wire, and as easily withdrawn when the cover is removed.

62. All these matters being preliminarily arranged, the final disposition of

the tray and its charge is made. The air tube is carefully wiped, and its external aperture closed by a clean loose plug of dry sponge. The tray is for the last time freed from dust by inversion and blowing upon it, and is put into its place. The quantity of rough glass necessary for the required plate, about 8lbs in the present instance (29), is carefully weighed out, and then introduced by an evaporating basin, or some other means which shall not allow of the admission of any reducing or colouring matter, or permit any portion of glass to pass beyond the edges of the tray. The glass covers are then to be arranged in their places; the iron covers of the chamber likewise adjusted, and over all are to be placed a set of thick earthenware tiles, which have been fitted together so as to constitute a general covering to the whole, well calculated to retain heat.

63. The ensuing part of the process is one in which the precise order of, and most advantageous proceedings have not yet been ascertained. Variations have been made up to the very last experiment, and it is only by still more extensive experience that the arrangement will ultimately be settled.

64. A fire being lighted in the furnace, and some coke put beneath the glass chamber, the temperature is gradually raised. In about an hour the bottom of the chamber begins to appear ignited, and in four hours the top iron covers are usually dull red hot. These appearances are useful, as indications of the progress of the operation. When the furnace has been heated for the first half hour, then every care is taken that the temperature may be fully sustained to the end of the experiment; and besides the ordinary kind of attention to the fire, particular care is taken that coke be supplied, by the lateral holes, to the part beneath the chamber; for, if the fuel there be allowed to burn out, the heat soon falls, notwithstanding the flame from the coals. Although the fire may seem quickly to have attained its best condition, yet the temperature continues to rise in the chamber long afterwards; for, from the quantity of lateral brick-work to be heated, it is usually many hours before the sides of the chamber are so hot, that the tray and its contents can attain their highest temperature. At the same time it must be understood that the heat of the glass is very much governed, especially at the early part of an experiment, by the number of covers over it, and rises far more rapidly, and much higher, with two or three covers than with one.

65. Perhaps the glass may with propriety be examined once, early in the

experiment, for the purpose of ascertaining that the tray and its contents are safe; but usually it is left for six or eight, or a greater number of hours, that the whole may fuse, the temperature rise, and the bubbles escape. When the glass is to be examined, the tile and iron covers are to be removed from over that half of the chamber containing it, by which, consequently, the glass covers are exposed; these are next to be carefully raised, one by one, using the iron instrument before described, for the purpose (61), and, as they are removed, are to be carefully put into the further part of the chamber, which still remains covered, where they will be retained in a heated state. This prevents their cracking and falling to pieces, as they would do if brought into the open air. If the experiment, and consequently the covers, are upon so large a scale that the latter cannot all be placed in this situation, then the exterior ones may be placed upon the top of the heated covers and tiles; but the particular cover, which immediately incloses the glass, being of great importance, must be put into the further safe part of the furnace, that it may be carefully preserved from injury, and ready to be replaced over the glass with the least possible disturbance.

66. The moment the last cover is removed, the glass is exposed to any falling substance from the iron plates, or tiles, or other sources, so that extreme attention is required at such times, to keep the place free from dust, and to perform every requisite operation as quietly as possible. The current of hot air which rises from the chamber, ascending and striking against the ceiling, frequently causes, by change of temperature and mechanical agitation, the separation of small particles of matter, which, descending, cause risk of injury to the glass; for which reason, it may sometimes be needful to have a temporary shelter fixed over the furnace, either of tin plate, clean boards, or some other material which shall not throw off scales or impurities of any kind.

67. If, by any unfortunate accident, a fragment of matter does fall into the glass, it should be instantly removed. It certainly will not sink, because of the great density of the glass, and may be taken out, usually with facility, by touching it, and the glass in its neighbourhood, with the platina stirrer (28), or the bottom of the platina ladle (28). In carrying it and the adhering glass away, great attention should be given, that none of the latter fall over the sides of the tray; since such portion might be a means of introducing im-

purity hereafter, or of cementing the tray and the earthenware together in a very inconvenient and injurious manner.

68. If, also, it should be observed, at this time, that there is a superabundance of glass in the tray, and not sufficient distance between its surface and the edges of the platina, the excess should be ladled out (28), an operation easily performed, but which must be done with care.

69. When the glass is ascertained to be in a proper condition, and that there is no appearance of any portion of it outside the tray, the covers are to be replaced, the chamber closed, and the heat continued. If the covers be glazed, some precaution is required in their arrangement; for on putting the second cover over the first, if they are left in contact by a portion of glazed surface, they will be found, upon their next removal, to adhere at that place. They should never be put in contact therefore with each other, or, if that cannot be avoided, a piece of old platina foil should be laid upon the place where the contact is necessary (58).

70. Whilst the glass is covered and subjected to a high temperature, there is, as before stated, an inward current of fresh air passing continually to and about it through the air tube, during the whole time of the experiment (55).

It was necessary to apply a valve to the external orifice of this tube to regulate the supply; for the draught was so considerable, that the glass was cooled by it, and much dust carried in. Finding reason to believe that even when very much diminished, the quantity of soots and dust in a London atmosphere, and especially in that portion of it taken from an experimental room in which a powerful furnace was at work, were competent to do much harm in eighteen or twenty-four hours, by giving colour and forming striæ, experiments were made on the means of cleansing the entering air. It was found easy to effect this, by the assistance of two or three Woulfe's bottles, or two or three jars, inverted one within another, using at the same time portions of diluted sulphuric acid, or such solutions of salts in the vessels as would not supply any moisture to the air, but rather take water with the dust from it. In these cases the air did not bubble through the liquid, but only passed close to its surface, and had time to deposit its dust during its passage through the inclosed spaces above the fluid: but, finally, a still simpler arrangement was used, consisting merely of a plug of clean dry sponge fitted into the end of the tube, which, at the same time that it allowed sufficient air to pass, seemed, from the appearance of the tube afterwards, to have excluded every impurity.

71. There are two conditions of the finished glass, each of great importance, which it is the object of the process to obtain in this state of the substance. One, and the most essential, is the absence of all striæ and irregularities of composition; the other, the absence of even the most minute bubbles. The first is obtained by agitation and perfect mixture of the whole; the latter, principally by a state of repose: so that the means required to be successful on both points are directly opposed to each other. Were the glass absolutely incapable of change by the long-continued action of heat, it would be easy first to render it uniform by stirring, and then to leave it in a quiescent state, until the bubbles had disappeared; but I am not yet fully assured of the fact which is necessary to this order of proceedings. That the glass as far as proportions are concerned, if changed at all, is altered only in an extremely minute and inappreciable degree, is shown by some experiments, in which, after a portion had been prepared and heated for many hours, and also stirred well, the resulting piece was divided into smaller portions, and these heated at different temperatures, in platina trays, for sixteen hours. Three portions were heated as powerfully as the furnace would admit of; three only to redness, which may be considered as a very low heat; and three to an intermediate degree: all were cooled slowly and annealed for an equal time. The specific gravities of each after the experiments were as follows:

Highest heat	5.4206	5.4211	5.4203	Mean sp. gr. 5.42066
Intermediate heat	5.4253	5.4242	5.4255	5.42500
Least heat	5.4258	5.4262	5.4235	5.42516
Original glass	5.4247	5.4261		5.42540

72. Here, notwithstanding the irregularities between the similar experiments, there seems, from the comparison of the mean specific gravities, to be a gradual though minute diminution of density, as the glasses have been more powerfully heated; and I found also, that when glass was so well stirred as to leave no doubt that it was thoroughly well mixed, yet being left in the furnace at a high temperature for eight or nine hours, it contained striæ.

73. On the other hand, first to render the glass perfectly free from bubbles and clear, and then to stir out the irregularities of composition, I have not

found to be a practicable process; because the stirring, in the manner in which I have yet performed it, tends to introduce bubbles into the glass; and though these are only small, still they are objectionable. Hence a mixed process has been adopted, which, as I have before stated, is subject to correction from future experiments. To render the process as far as it has been carried sufficiently intelligible to others, I will first describe the circumstances connected with stirring, and their influence upon striæ; and afterwards, the plans adopted for the dispersion of bubbles.

74. It is not a small degree of stirring and agitation which is sufficient to make a fluid of mixed materials homogeneous; especially when the mixture is not exceedingly fluid, but has a considerable degree of tenacity, something like tar or syrup. An idea of the extent to which it must be carried, and of the general nature of striæ in fluids, may be gained by taking a glass full of clear saturated syrup, made from white sugar, putting a few drops of water into it, and stirring the whole together. It may then be remarked how slow the striæ are in disappearing; and when they are apparently destroyed, if the whole be left for some hours, it will frequently happen that a separation will take place into a lower heavy, and superincumbent light portion, which when stirred together again produce striæ. In the glass, the stirring must be in the utmost degree perfect, for if there be the least difference in different parts, it is liable to form striæ: nor are the different portions allowed to arrange themselves by their specific gravities, in which case one part might perhaps be removed from another, after the glass was finished and cold; but the ascending and descending currents which inevitably take place in the fluid matter, are certain to arrange the irregularities in such a manner as to produce the strongest possible bad effect.

75. The instrument used for stirring has hitherto consisted of a piece of plate platina, which for the seven-inch glass, taken as illustrating the process, is 6¼ inches in length and ¾ths of an inch in breadth. It is perforated with various irregular holes, that, when drawn through the glass like a rake, it may effectually mix the parts. A piece of thick platina wire, about thirteen inches long, is riveted to it, and the extremity of this screwed into the end of a clean iron rod which answers the purpose of a handle. No small or cellular apertures should be allowed in this stirrer; for they will frequently retain air or moisture,

which may cause bubbles in the heated matter and do much harm. A little gold, therefore, should be applied to the part where the stem is attached, and fused, so that all hollows may be filled up. Stirrers of different dimensions are to be provided for different sized plates of glass. Before being used, they should be steeped in dilute nitric acid, and also heated to redness in the spirit lamp, just previous to their immersion in the glass for the first time in each experiment.

76. When a stirring is to be performed, the tiles and iron covers are removed from the first part of the chamber (44. 49. 65), the glass covers also taken off and put into the back part of the chamber (61. 65), the glass quickly examined, to give assurance that all is in good condition, and then the stirring commenced. The stirrer should be put in gently, that no air may be carried down with it, and then drawn through the glass quickly but steadily, so as to mingle effectually, but not to endanger forcing the substance over the edges of the tray or to run the risk of involving air bubbles. The chamber and its contents are cooled by the necessary exposure to the atmosphere, and therefore, when the agitation has been continued until the glass is so much lowered in temperature as to become thick, it should be discontinued, the stirrer carefully removed, the glass covers replaced, the chamber covers restored to their situation, and the temperature allowed to rise for fifteen or twenty minutes, when the operation may be renewed.

77. All the precautions against loose particles, dust, and soot, that were before spoken of (66), should be adopted in this operation. In the act of stirring, the instrument should not be struck carelessly against the bottom or sides of the tray; for the platina in this highly heated state is very soft, and a hole would readily be forced through it; nor should it be brought forcibly against the corners, for the metal is in such a favourable condition for welding, that the least blow upon a doubled part causes adhesion. By merely allowing the stirrer, when ignited, to sink upon the bottom of the tray rather more hastily than usual, it has adhered to the place; and when, for safety, an underlying plate of platina was used (50), it was always found welded to the tray at the places which the stirrer had touched a little more forcibly than the adjacent parts, and could not afterwards be separated without leaving holes in

the metal. This circumstance was the principal occasion of the advantages afforded by the use of the underlying plate being given up.

78. The heat which has to be borne during the operation of stirring, is very considerable, especially upon the hands; but at such a moment no retreat from the work, because of mere personal inconvenience, can be allowed. But the circumstance renders the use of a cover for the stirring hand very advantageous. I have found a loose linen bag, into which the hand could go freely, more convenient for this purpose than a glove; for being in contact with the skin at distant parts only, the hand is preserved at a much lower temperature. Two small holes in it, one at the front and the other at the top, allow the handle of the stirrer to pass obliquely through, by which arrangement it is easily held with firmness, and the bag itself prevented from slipping towards the glass. It should not be larger than to cover the wrist, or it will embarrass the movements; and it should be very stiffly starched, and ironed, that no fibrous particles may fly from it to the glass during the stirring.

79. The glass which, adhering to, is brought away with the stirrer, indicates, by its appearance, the general character and state of that in the tray; but during its examination, the experimenter must carefully refrain from touching it; for if the finger, or any other ordinary organic substance, come into contact with it, the next time the instrument is immersed in the ignited glass, the part touched will produce bubbles. It is therefore of importance that the stirrer be preserved perfectly clean from one stirring to another, for which purpose it may be deposited so that the platina shall be received in an evaporating basin, the mouth of which is afterwards covered over.

80. In entering upon the considerations relative to the bubbles, it will be evident, from the nature of the materials and the quantity of elastic matter originally present, that these are at first very numerous. The larger ones soon ascend to the surface, and breaking, are dissipated without inconvenience; but the smaller ones rise with far less readiness, and the smallest have so little power of elevation, that the general currents in the liquid appear sufficient to carry them downwards, or in any other direction, and thus retain them for any period within the mass. A useful idea of the length of time required for very minute bubbles to ascend through a fluid having some tena-

city, may be gained by the person who will take a glassful of clear concentrated white sugar syrup, and beat it up with a little air, until a portion of the latter is in extremely minute bubbles. If these are allowed to remain undisturbed, it will be observed, that though the larger bubbles rise quickly, and the smaller soon after, the smallest will continue for many hours under the surface, destroying the pellucidness of the fluid; and this will be the case although there are none of those descending currents, resulting from difference of temperature, which in the glass assist in retaining the bubbles beneath the surface.

81. From the great length of time which it required to liberate the bubbles even from small pieces of glass, and when no stirring was practised, I was induced to conclude that the evolution of gaseous or vaporous matter had not ceased upon the first fusion of the materials, but that the glass itself when highly heated continued to evolve small portions for some time. It occurred to me also, that in that case its formation might be hastened and the final separation advanced by mixing some extraneous and insoluble substance with the glass, to act as a nucleus, just as pieces of wood, or paper, or grains of sand, operate when introduced into soda water or sparkling champaign; in which cases they cause the gas, which has a tendency to separate from the fluid, to leave it far more quickly and perfectly than if they had not been present.

82. The substance I resorted to for this purpose was platina in the spongy state. It was chosen as being a body solid at high temperatures, uninfluenced by the glass, easily reduced to powder, and likely to retain its finely divided condition during the operation:—its preparation is described in the Appendix. In experiments made expressly to ascertain its action, it was found to assist powerfully in the evolution and separation of the bubbles, and afterwards to sink so completely to the bottom, that not a particle remained suspended in the mass. Even stirring does not render it injurious; for the particles, by that action, are welded to the bottom, and the glass ultimately equally free from mixture with them.

83. The spongy metal should be perfectly pure. It is easily reduced to powder by rubbing it with a clean finger on clean paper. No attrition with a hard substance should be allowed, as that burnishes the metal, and takes away the roughness, which is highly advantageous in assisting the evolu-

tion of the bubbles. When reduced to powder, it should be again heated upon a piece of platina foil in the flame of a spirit lamp.

84. The quantity of powdered platina which I have usually employed has been about 7 or 8 grains for every pound weight of glass. But in order to effect its more general and perfect diffusion, I have usually mixed it with ten or twelve times its bulk of pulverized glass. For this purpose, some of the rough glass, the same in composition with that to be perfected, has been crushed small in a clean agate mortar, and the finer parts separated from the coarser on an inclined and shaken sheet of paper. The former have been then mixed little by little with the platina, and rubbed slightly with the finger, to effect perfect separation of the metal, and then the coarser parts have been added, to increase the bulk. In this state it was ready for use.

85. The time of introducing this prepared platina is, like the times of stirring, as yet under investigation. It has usually been sprinkled from the platina ladle (28) over the surface of the well-fused and highly-heated glass, at the period of the first stirring. This method has the advantage of bringing the assisting substance into contact with the glass when the latter is highly disposed to throw off its adhering gaseous matter, and also allows of thorough mixture; but it also causes the addition of fresh glass after the concoction of the materials has been proceeding for many hours; and it likewise occasions the introduction of many bubbles formed by the air in the interstices of the powder.

86. On other occasions the prepared mixture of platina and glass has been introduced into the tray at the period when it was charged with the due quantity of rough glass, and before the application of fire. Particular attention was then paid to its general diffusion throughout the charge, and on these occasions its action commenced the moment the glass in contact with it was fluid. I am inclined to believe the latter will ultimately prove the better method of proceeding, both for the greater length of time during which the platina can act, and for the facility and convenience of its introduction.

87. In either mode of appliance the platina has been found highly serviceable; and in every case since its use, where stirring has not been necessary, the resulting glass has proved to be perfectly free from bubbles.

88. As already mentioned, the best periods for stirring and repose have not

been finally determined. Stirring introduces bubbles, and therefore should, if possible, be avoided towards the conclusion of the experiment. Rest, or at least that condition in which there is no other motion than what is due to the currents produced by slight differences of temperature, causes striæ even after very careful mixture (71. 72), and is therefore equally to be feared; and whatever other variations may have been adopted, I have always found it important to apply a careful concluding stirring. The following may be considered as the order of an experiment. If the spongy platina has not been introduced into the tray with the rough glass, then about the sixth hour after lighting the fire it is added in the manner already directed (85), and the glass well stirred (76). At about the twelfth hour the stirrings are recommenced, for the purpose of making the mixture perfect, and are repeated every 20 or 30 minutes, according to the fusibility of the glass and the state of the heat (60), for 8 or 9 times. The glass is then allowed to remain at rest for 6 or 8 hours, that bubbles may ascend and be dissipated, after which it is well stirred twice or thrice more with particular attention, that if possible no air may be introduced, and is finally mixed for the last time.

89. This concluding stirring is peculiar, in that it has to be continued until the glass is so cold and thick that no ascending and descending currents can be formed in it; after which its temperature is not again to be allowed to rise: hence the operation requires certain preliminary arrangements. The first point necessary is to clear out a considerable quantity of slag from the flue furnace, or that part beneath the chamber (47). This slag results from the fused ashes of all the coke which has been consumed there, with other portions that have passed on from the coal fire. It is to be drawn on to the bars of the furnace by a fire-rake which will pass into the passages beneath the chamber. If not taken out in its fused state, it would be impossible afterwards to remove it without risk of great injury to the furnace. At the same time that the slag is removed, all the coke is likewise to be withdrawn. All the fuel in the fire bars is also to be brought out of the furnace; and if the bars are embarrassed with clinkers, they are to be loosened. These things being done quickly and quietly, and the furnace apertures closed, a few moments are to be allowed for the little dust that may have been agitated to settle, and then the chamber

is to be opened and the glass stirred. The heat will have fallen very little by the preceding operations, and the glass may be well mixed, but with this precaution, that when once the stirrer is beneath the surface, it should not again be taken out until the conclusion. By opening the feed-hole, or the ash-pit, air may now be allowed freely to enter the furnace, and will rapidly lower its temperature, especially at such parts as the bottom of the pan, which are thin and at this moment exposed to the atmosphere on both surfaces. The temperature of the glass will fall in a corresponding degree, and the stirring being all this while continued, though more slowly if convenient, the substance will gradually thicken, until at last motion will endanger its being pushed out of the tray, and then the stirrer is to be carefully withdrawn. No currents in the glass need be feared, for the temperature cannot now rise higher. But a single cover being put over the tray, and the outer orifice of the air-tube closed by a good cork, the whole may be left a few minutes to cool still further for perfect security, until, the glass being supposed to have arrived at the state of a thick paste, the annealing should commence. Then the ash-pit, the fire-place, and all the other apertures to the furnace are to be closed; the second glass-cover put into its place; the chamber shut up by its iron and tile covers; a layer of bricks arranged close together over the whole upper surface of the chamber and furnace; the damper of the flue closed to prevent air passing through the fire-place, and the whole left to cool gradually for several days.

90. The interval between the common temperature and that at which the glass begins to lose solidity and acquire softness, is so much less with this variety than with flint glass, that it is probable a much shorter period of time is required for its perfect annealing than for the latter. That no failure might occur in this point, however, four days and nights have been allowed for the annealing of the large plates. If every thing were left as just described, the contents of the chamber would be warm on the sixth or even the seventh day, so gradually do the arrangements allow it to cool; but on the morning or the evening of the third day, according to circumstances, the damper in the flue is withdrawn a very little to allow the passage of a small quantity of air, and by this means the cooling facilitated and regulated.

91. When the furnace and its contents are cold, the chamber is opened: if

the experiment has been well conducted, every thing will be found loose, and unaltered in disposition from what they were when first arranged. The earthenware supports are to be removed, and the tray taken out. After examining the glass itself, the exterior of the tray should be carefully observed, whether there be any appearance of leakages either through imperceptible holes or at the corners, and such places as can be rectified by a patch should be noted in reference to the future use of the platina.

92. An operation which, to be successful, requires much care, is then to be performed; namely, the stripping off the platina from the glass. The tray should be placed on clean smooth paper upon a cloth. The corners are one by one to be opened by a blunt smooth knife, or some softer instrument, from the side towards which they were folded; and being then carefully pulled outwards by their extremities, will usually open, so that the platina becomes single again. Then proceeding from corner to corner, the platina will peel or strip easily from the sides of the glass, and will remain adhering by the bottom only. From time to time as fragments of glass are formed, they should be blown away or otherwise removed, that they may not cut the metal. If now the glass be placed a little over the edge of the table and firmly held, the platina may gradually be separated from the bottom in the same manner as from the sides, and the glass and the metal finally divided from each other without any injury to the former, and very little to the latter.

93. Immediately upon the separation of the platina, and before it can receive any mechanical injury beyond what it was impossible to avoid, it is to be put into a pickle consisting of nitric acid and water, and left there for several days. The dilute acid acts upon the adhering glass, dissolving and loosening it, and the plate is thus rendered fit for future operations (41). The stirrers also, when no longer required in an experiment, should be taken from their iron handles and put into the same pickling liquor. In this way the platina is perfectly cleaned, and being afterwards washed carefully in pure water and ignited, is again ready for use.

94. Such is the nature of the process as practised at present, by which plates of heavy optical glass seven inches square and eight pounds in weight have been prepared. I am encouraged to believe that it will admit of improvement, perhaps even to the full extent of our desires; but it will require time and patience

to effect it. As I have before said, we are in the course of our experiments only; and up to the last have seen reason to vary the arrangements, and still intend to make alterations. Every thing agrees to convince me that the size of the plate is not a circumstance involving any additional difficulty; but that, on the contrary, it will probably be safer to make a large than a small experiment. We can at pleasure obtain a glass perfectly free from striæ, unexceptionable in hardness, and with less colour than crown glass; but it is the simultaneous absence of all striæ and bubbles, with at the same time that degree of hardness and colour which will render the glass fit for optical purposes, that I am aiming at, and that I trust shortly to obtain.

95. As soon as the plates of glass are removed from the platina and briefly examined, they are sent to Mr. Dollond, who then enters upon the discharge of his particular duties in the Committee, by cutting, examining, and even working them into telescopes. It is not, however, my place to detail this gentleman's exertions (as a member of the Glass Subcommittee) in the cause of science. They will, I trust, appear in due season; and I hope that the want of perfect success on my part will not long be a cause of delay.

§ 2. *General qualities of the heavy Optical Glasses.*

96. A great variety of glasses have been formed by the use of different proportions of ingredients. They vary importantly from each other, though by no means to the extent of the difference existing between any of them and flint glass. The *specific gravity* rises very high in borate of lead, consisting of single proportions, i. e. nearly 24 by weight of boracic acid and 112 of oxide of lead; it is often as high as 6.39 or 6.4, being double that of some specimens of flint glass. In silicated borate of lead, which, in addition to the former quantities, contains 16 parts, or a proportional of silica, it is about 5.44. As the proportion of oxide of lead diminishes, so also does the specific gravity lessen, and it is in some of the specimens as low as 4.2; still permitting by the proportions present such fusibility and other qualities as consist with the process described. The specific gravity of Guinand's heavy flint glass is about 3.616; that of a specimen of ordinary flint glass 3.290; that of plate glass 2.5257; and that of crown glass 2.5448.

97. The *refractive* and *dispersive* powers of the glasses increase with their

specific gravity, as was to be expected. The powers of two of them, namely borate of lead, and silicated borate of lead, consisting always, if not otherwise expressed, of single proportionals, have been ascertained by Mr. Herschel, and are as follows:—

	Bor. Lead.	Sil. Bor. Lead.
Angle of glass prism	29° 6′	30° 26′
Refractive index for extreme red rays . . . μ =	2.0430	1.8521
———————— maximum yellow . . . — =	2.0652	1.8735
———————— extreme violet :: . . . — =	2.1223	1.9135
Dispersive index = $\frac{\delta\mu}{\mu - 1}$ = ...	0.0740	0.0703

These intense powers upon light are not accompanied by any circumstance rendering the glass optically unfit for the compensation of the dispersive powers of crown or plate glass. Three object-glasses have been constructed for the express purpose of ascertaining this point; and all of them tend to demonstrate that the compensation or correction may be effected with equal if not greater facility than with flint glass.

98. One important circumstance connected with the application of these glasses to the purposes for which they are designed, is their *colour*. The great power they have of developing strong tints from metallic impurities, has been already described and illustrated (22. 23), and creates a difficulty in the way of obtaining them unobjectionably free from colour. The usual colour is more or less of yellow, and is perhaps almost altogether, if not quite, dependent upon the presence of a little iron. Like many of those dependent upon mineral substances, it is very much heightened by elevation, and lessened by diminution of temperature. It is rapidly and permanently diminished by increasing the proportions either of the silica or the boracic acid. The silicated borate of lead has latterly been obtained of such faint tint by the precautions relative to impurities, already described, that, when 9 inches in thickness, white paper looked at through it in open daylight resembled in appearance and depth of tint the surface of a lemon. Glass consisting of 1 proportional = 112 oxide of lead, 1 proportional = 16 silica, and 1½ proportional = 36 boracic acid, when 7 inches in thickness and examined in the same manner, did not give a colour surpassing that of pale roll sulphur. The tri-borate of lead glass is almost as colourless as good flint glass, but might perhaps be found objectionable on other accounts.

99. As there is a certain quantity of light intercepted by glass which is altogether dependent upon and in proportion to its colour, it is evident that this property of the heavy glasses must be considered in relation to their use in telescopes; but there appears no reason for supposing they will ultimately prove inapplicable on this account. The colour of the glass already obtained is far less in depth than that of the crown glass constantly used in the construction of telescopes, which yet intercepts by its colour no important quantity of light; and if two plates 8 or 10 inches long, one of the yellow heavy glass and the other of crown glass, be looked through edgeways, it will be seen in a moment that the crown glass intercepts by far the most light. The colour of the glass is of no consequence, otherwise than as causing a loss of light from interception; for the tinge which is cast over objects looked at through a telescope constructed with it is scarcely perceptible to the most acute eye, and quite unimportant. When to these circumstances is added the reasonable expectation entertained of removing a large proportion of the little remaining colour by the use of purified silica (21), it will not be anticipated that experience will prove the glass faulty in this respect.

100. There is one very important action of the glass upon light, however, which may perhaps interfere more with its application, in telescopes at least, than any other, i. e. its *reflective power*. This is very strong in all the heavy glasses, far stronger than in flint, and exceedingly surpassing the similar power of crown glass. It is in proportion, as might have been expected, to the refractive power and the density of the glasses, all these properties increasing with the oxide of lead. The loss of light occasioned by the reflection from the two surfaces of a plate through which a ray is passed, appears to me to be greater than from the united action of both colour and bubbles in a piece of glass 7 inches thick.

I endeavoured to ascertain the comparative quantities of light reflected by these heavy and other glasses, in some photometrical experiments made upon the principle of similar shadows, measuring only the reflexion from the first surface of the different glasses, that from the second surface being destroyed. The ray was made incident in all the cases at an angle of 45°. It was obtained from a small single-wicked lamp *a*; and when reflected, its intensity was measured by the distance of a similar lamp *b*, whose direct light cast the comparative shadow. The uniformity of the two lights, or at least of their relation to each other, was established by trials before and after the experiments with the reflecting surfaces, and each surface was tried two or three times, at intervals, and in a mixed manner; so that no anticipation of the result could in any case bias the mind. The following Table shows the results, small decimals being neglected:

		Inches.		
Light *a* direct		10.70	1	1
——— reflected by glass	5	36.75	11.80	$\frac{1}{11.8}$
	1	40.69	14.46	$\frac{1}{14.4}$
	4	43.46	16.50	$\frac{1}{16.5}$
	9	47.31	19.56	$\frac{1}{19.5}$
	6	50.31	22.12	$\frac{1}{22.1}$
	7	51.63	23.29	$\frac{1}{23.3}$
	3	52.69	24.26	$\frac{1}{24.2}$
	8	54.33	25.80	$\frac{1}{25.8}$
	2	54.56	26.02	$\frac{1}{26.}$

The first column refers to the glasses below; the second gives the distance of the measuring flame *b*; the third, the preceding numbers squared and reduced to the direct light as unity; and the fourth, consequently, the proportion of the light *a* reflected by the first surface of each glass. No. 5 was glass consisting of 1 proportional of oxide of lead, ½ a proportional of silica, and 1½ proportional boracic acid. No. 1 was composed of 1 oxide of lead, 1 silica, and 1½ boracic acid. No. 4, of 1 oxide of lead, 1½ silica, and 1½ boracic acid. No. 9 was flint glass; No. 6, 7 and 3, different pieces of crown glass; and No. 8 and 2, different pieces of plate glass. 1, 3, 5, 6 and 7, were natural surfaces; 2, 4, 8 and 9, polished surfaces.

The deficiency of light resulting from the increased reflecting power, though considerable, may easily be compensated for by slightly increasing the area of the plate; and the power of obtaining plates of any size is professed to be given by the general process: but whether that expedient involves any other objections, it will be for the optician to determine.

101. In *hardness*, these glasses differ from each other as much as in any other

quality, and indeed more. The borate of lead is very soft; the bi-borate of lead is harder, and the tri-borate equal to flint glass in hardness. The silicated borate of lead is softer than flint glass; but the glass consisting of 1 proportional oxide of lead, 1 of silica, and 1½ proportional of boracic acid, is as hard as ordinary flint glass, at the same time that it has that degree of fusibility, colour, and other properties, which makes it a very promising variety.

102. The hardness increases with the diminution of the oxide of lead; but the *fusibility* diminishes in the same proportion; and this is a property which it is essential to preserve to a certain degree for the removal of striæ and bubbles. The borate of lead is so fusible as to soften and lose its form under the surface of boiling oil. The silicated borate, and the glass consisting of the proportions above mentioned, are quite fusible enough to allow of the processes necessary for the removal of striæ and bubbles.

103. The fusibility of these glasses, and of glass generally, must not be confounded with their relative tendency to soften by elevation of temperature. It is not that glass which softens first, that becomes most fluid at a certain given high temperature; for glasses, like other substances, vary in their readiness to pass into the fluid state. Hence it has often occurred amongst the variety of compositions tried for glasses, that when the resulting substances have been placed side by side on platina foil, and heated, that which first softened did not when heated highly become so fluid as some other specimens that longer resisted the first impression of heat. It has however always been found that those glasses which when subjected to a rising temperature, most slowly passed from the solid to the fluid state, were also those which when subjected to long annealing processes, were least liable to assume a crystalline structure; and thus very useful indications of the probable qualities of compounds under investigation were often obtained.

104. A most important consideration relative to the application of these glasses to the construction of telescopes, is their liability to change and injury by the action of substances usually occurring in an ordinary atmosphere. When the value of a good object-glass is considered, frequently amounting to many hundred pounds, this point will be thought of no little consequence; and when it is known that even flint and plate glass are frequently injured in this way,

a little anxiety for the capability of resistance in the heavy glasses may readily be allowed, since they contain so much less of the substance (silica) which confers the power of resistance, and so much more of that (oxide of lead) which is considered as the vulnerable part, than does either of the former kinds.

105. The superficial changes of glass which interfere with its optical uses are of two kinds. The one is shown by a tarnish upon the surface, which when strong is iridescent. It is quickly produced by the intentional presence of sulphuretted hydrogen, which acting upon the oxide of lead present, reduces it, and forms a sulphuret of lead. It takes place only with flint glass, and is in every case produced either by sulphuretted hydrogen or other sulphuretted vapours. In plate glass the change is of another kind, and is shown by the appearance of minute vegetations or crystallizations, which spread, obstructing the light wherever they occur. Mr. Dollond, who has shown me cases of both kinds of injury in flint and plate glass, is inclined to believe that the latter has, during his long experience, proved most injurious.

106. From the commencement of the experiments it was expected that these heavy optical glasses would tarnish more than flint glass; but as specimens of borate of lead and other dense compounds of that metal had been retained in an ordinary atmosphere, without any particular precautions, for long periods of time, yet without tarnishing, there was encouragement to continue the investigations: and though when specimens were put into atmospheres purposely contaminated with sulphuretted hydrogen, they tarnished quickly, and much more than any flint glass, yet it did not follow that they should of necessity tarnish in the telescope; especially as, being from the construction of the achromatic object-glass inclosed by the tube and the crown or plate glass lens, they would be considerably protected, and at the same time would admit of the intentional application of extraneous chemical protectors.

107. The kind of protection which occurs to the mind is the application of such substances to the interior of the tube as, having a strong attraction for sulphuretted vapours, should continually retain the atmosphere within free from their presence. Carbonate of lead, precipitated borate of lead or finely-ground litharge, mixed with the pigment which is usually applied to blacken the inside of the telescope that all extraneous light may be absorbed, will probably effect this purpose completely.

108. A very curious and important influence of alkali in facilitating the tarnish of glasses containing oxide of lead, was discovered during the course of these investigations; and when the quantity of lead in flint glass is increased but a little beyond the ordinary proportions, its effect is powerfully manifested. Ordinary flint glass consists of 33.28 oxide of lead, 51.93 silica, and 13.77 potassa; the rest of the substances present, being in very small quantity, may be disregarded. Here the oxide of lead is 33.28 hundredths of the whole; and if it be only a little increased, for the purpose of giving greater dispersive power, the glass is liable to tarnish in an ordinary town atmosphere. Such is the case with a specimen of Guinand's glass, which I have analysed, and which contains 43.05 oxide of lead, 44.3 silica, and 11.75 potassa. But provided the alkali be away, the quantity of oxide of lead may be enormously increased; and a glass containing 64 per cent of oxide of lead, in combination with 36 per cent of silica, has not tarnished by an exposure for 18 months on the same shelves with flint glasses that have tarnished. The following case will point out the effect still more strongly: A combination of equal weights of silica and oxide of lead was formed, and the compound has shown no tendency to tarnish in an ordinary atmosphere since February 1828. Eight parts of this was fused with as much pearlash as was equivalent to 1 part of potassa, and a glass was formed which has since become much tarnished. But other 8 parts being fused with 3 parts more of oxide of lead, so as almost to double the proportion of the latter, gave a glass without alkali, which does not yet exhibit the slightest trace of tarnish.

109. Hence the reason why the absence of alkali has been earnestly insisted upon in the preparation of the ingredients for the heavy optical glasses (18. 24). Hence the reason also why heavy flint glass, as already mentioned, has tarnished equally with some of the heavier glasses, though containing so much less lead, and of such inferior specific gravity. This influence of alkali is associated with, and perhaps directly referrible to, another circumstance affecting the liability of change in the glass; I mean the action of water or of aërial moisture, which is frequently considerable, and appears to be dependent upon the alkali present.

110. If a small quantity of flint glass be very finely pulverized in an agate mortar, then placed upon a piece of turmeric paper, and moistened with a drop

of pure water, strong indications of free alkali will be obtained. The same effect is produced by using plate glass; and if the pulverization be very perfect, the alkali can be detected in glasses containing far smaller quantities of that substance than either of those mentioned. This experiment, due to Mr. Griffiths, shows that in whatever state of combination the alkali may be, it can still act upon, and is subject to, the action of moisture; and that flint glass is by no means a compound resulting from very strong chemical affinities, is also shown by an experiment which I made many years ago; namely, that if flint glass be pulverised exceedingly fine, the powder will indicate the presence of sulphuretted hydrogen in air by becoming blackened, almost as readily as carbonate of lead. Glass may be considered rather as a solution of different substances one in another, than as a strong chemical compound; and it owes its power of resisting agents generally to its perfectly compact state, and the existence of an insoluble and unchangeable film of silica or highly silicated matter upon its surface.

111. The half-combined and hygrometric state of the alkali appears to be the cause of the deposited film of moisture which is well known to adhere to ordinary glass when exposed to the atmosphere at common temperatures. This film is highly calculated to condense any portion of sulphuretted vapours which may be floating in the atmosphere, and thus bring them into contact with the oxide of lead under the most favourable conditions for the production of that action which is the direct cause of tarnish. Now from this cause of action the heavy glass is free; and hence a satisfactory reason to me why the heavy glasses have suffered so little when left with common care in an usual atmosphere.

112. An extraordinary difference exists between the electrical relations of this glass and other glasses, due principally to the same absence of alkali. Ordinary glasses, either flint, plate or crown, will, from the hygrometric film of moisture upon the surface, freely conduct electricity under common circumstances. Thus if a gold-leaf electrometer be diverged, and then touched with them in their ordinary state, the electricity is instantly discharged, even though the hand be two or three feet from the part touching the instrument. If a similar experiment be made with these heavy glasses, they have no sensible power of discharging the electricity, but insulate as perfectly as sealing-wax

or gum lac. If one of these plates of glass, without any previous warming and drying, be lightly brushed or wiped with flannel or silk, it instantly becomes strongly electrical, and retains its electricity for a long time; but it would be almost impossible to develop electricity by such slight means with flint or plate or even crown glass in a similar state. Hence the glass makes as good an electrophorus as lac or resin, and may probably be found hereafter to answer many useful electrical purposes. But the great point at present in view, is the proof which such electrical properties give of the absence of that film of moisture which is so constant upon other glasses.

113. All these circumstances are favourable to the opinion that the heavy glass will not be found objectionable in the construction of telescopes, because of any undue tendency to tarnish, and especially when precautions are taken to protect it from sulphuretted vapours in the manner before described (107). No difficulty can be anticipated in preserving the air within a limited and inclosed space free from such a contamination: to preserve it dry, if that had been necessary, under the different circumstances of varying temperature and the inevitable change of the air more or less frequently, would have been a far more difficult task.

114. The other kind of superficial change, i. e. the corrosion or crystallization which takes place principally on plate glass, is doubtless also due to the alkali present. Sometimes, indeed, specimens of glass may be found where the alkali being too abundant, a similar but more extensive action has taken place over the whole of the surface, and the glass falls off in scales. Whether the alteration be due to the action of the alkali on the water only, or on the carbonic acid and other substances it finds in the air, or to its united action on all together, is of little consequence at present, as the substance on which it depends is altogether absent from the glass under consideration.

115. Among the great number of glasses made, there are several of different composition, which have been selected, because of their general characters and properties, for more extensive trial and investigation when time will permit. Of these it would be useless to speak at present, as what might be stated of them now would probably require correction from future experiments. Up to this period the attention has been devoted, as it still must be for a while, to the establishment of a process which, competent to produce with certainty a

Fig. 1.

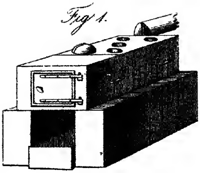


Fig. 5

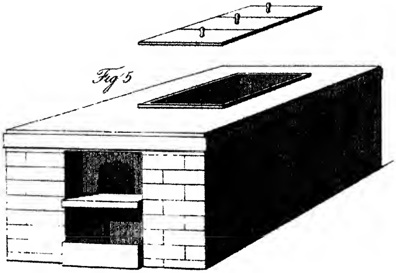


Fig. 7.

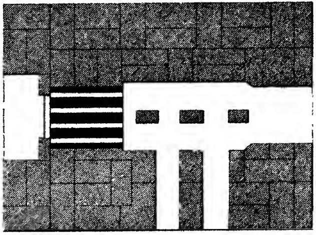


Fig. 2

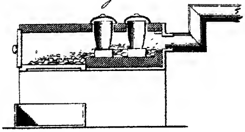


Fig. 3.

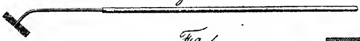


Fig. 4.

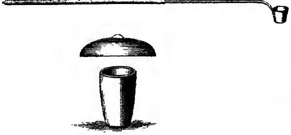
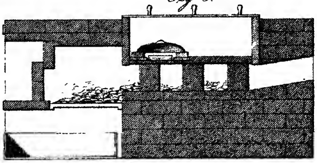


Fig. 6.



Scale

Fig. 1. 2. 5. 6. 7. half an inch to a Foot.
3. 4. one inch to a Foot.

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glass fitted for optical purposes, may have the philosophy and practice of every part so fully ascertained, as to be capable of description in a manner sufficiently clear to enable any other person, with moderate care, to obtain the same results without the labour of long and tedious investigation.

APPENDIX.

Rough glass furnace.

THE only furnace for making rough glass which has been constructed, answers its purpose exceedingly well; and though if a second were to be made, it should be upon a larger scale, yet I think it better to describe the tried one accurately, than to direct alterations which have not been experimentally approved of; especially as there seems to be nothing which, in principle, need differ in a larger furnace. An iron box (see fig. 1. & 2.) 30 inches long, 14 inches wide, and 8½ inches deep, forms the principal part of the exterior: it is open entirely at the top, and at the bottom also, in the fore part, where a fire-grate is to be placed. It has a common iron furnace door in front, the aperture of which is 8 inches wide by 6 inches high; and at the opposite end, or back of the furnace, a flanched aperture 6½ inches by 4½ for a piece of funnel pipe to connect the furnace with a powerful flue. The sides of this box, and such part of the bottom as is not appropriated for the fire-grate, are lined with fire-stone 1½ inch in thickness, except in the fire-place, where it is 2⅝. The grate is 12 inches long by 8 wide; and the part above it is closed by a fire-tile 2 inches thick and 12 inches square, which, resting on the edges of the lining, finishes the portion intended for the coal fire, leaving it 5½ inches in depth from the covering tile to the grate. The other part is covered by an iron plate 17½ inches long, 13 inches wide, and ⅝ths of an inch thick, which, resting upon the edges of the lining, incloses a space of 16 inches long, 10 inches wide, and 5 inches deep, for the reception of crucibles. This plate is formed with circular holes about 3 inches, or rather more, in diameter, arranged as in the engraving, that the crucibles inserted through them may leave plenty of room for the intervention of coke and flame. As many round crucible covers belong to the plate as there are holes, serving to close such of them as are not occupied by crucibles.

As the plate becomes very hot when in use, it is necessary to have a second above it, which may be formed of sheet-iron with corresponding holes, and when put into its place, separated from the first, a little space, by pieces of tobacco pipe, or other convenient substance, to include a layer of air. But it is much better for the retention of heat, and also for its superior cleanliness, that this second plate should consist of pieces of earthenware fitted to each other, so as to cover the surface of the iron plate, from which it should also be separated by a short interval.

The crucibles used are 5 inches high outside, 3¼ inches diameter at the top, and 2 inches diameter at the bottom. They are of pure porcelain biscuit, perfectly white and clean. They should be made as thin as possible, of the finest and most refractory kind of ware, and baked at a high temperature. We have some crucibles made about thirty years ago for Mr. Hatchett, which, though not of the size required, are precisely the right kind of ware. They have been used many times in succession without cracking or being importantly acted upon by the glass, and no sensible degree of impurity was given to it from them.

When these crucibles are arranged in the furnace, they should be supported by little stands of earthenware, formed out of brick or Cornish tile, so that their edges shall rise about the ½ or ⅓rd of an inch above the surface of the upper covering plate, that no impurity may enter them. The holes in the plate should be of such dimensions that, when hot, the crucibles may fit loosely, that they may be uninjured, and also that there may be room between for the vapours that are evolved from the mixture to pass away.

The covers to the crucibles are evaporating basins about 4½ inches in diameter. They are slung with their edges downwards by a piece of platina wire sufficiently strong for the purpose, which being first bent at the middle into an angle, is then stretched across the outside of the basin, and has its ends bent round the opposite edges of the latter. The bent extremity of an iron rod passed under the loop thus formed over the middle of the bottom, serves to raise and remove the cover from place to place. When a crucible is in use, the cover should be arranged over it in such a manner as not to touch the vessel, but rest by its edges on the earthenware plate around.

The platina stirrers in use with this furnace have been before described (28. 75) fig. 3. The platina ladle consists of a small crucible of that metal riveted to a platina wire, and that made fast by a screw to an iron rod. (fig. 4.)

The use and manner of working this furnace will be well understood from the above description, and what has before been said (26, &c.). The crucible should never be suddenly heated or cooled. The coke may be fed and arranged at such of the crucible holes as are out of use at the time. From the very valuable effects of a trough of water under the fire bars (45) experienced in the larger furnace, one is constantly used in that just described.

Finishing furnace.

This furnace on the outside is a parallelopiped, principally of brickwork, built against a wall, 64 inches in length from the fire front to the beginning of the flue, against which it is built, 45 inches wide, and 28 inches high. (fig. 5. 6. & 7.) It is the only one that has yet been built, and, for the reasons before given, shall be described exactly as it is. The fire-place is at one end, and the course of the flame and smoke is directly from that to the other end, and then immediately into the upright flue. The fire-place is 15 inches from back to front, 13 inches wide, and 11½ inches from the arched roof to the bars. Its outward side, or that from the wall, is 18½ inches in thickness of brickwork, which is intended to give stability to the structure. The mouth of the fire-place is an aperture 8 inches by 6 inches, made in a piece of fire-stone 7 inches inwards from the front of the brickwork: its lower edge is level with a fire-stone sill, which, extending forwards from the fire-place to the outer surface of the brickwork, forms a shelf, on which two bricks stand, that serve in place of a door to close the mouth of the furnace. The ash-pit is 25 inches long, 12 inches wide under the fire, and 10 inches high to the bars. A trough made of rolled iron, riveted together, and 5½ inches high on the sides, occupies its lower part. This being preserved full of water, is sustained at the boiling temperature by the radiation of heat and the hot ashes which fall into it.

From the back part of the fire-place, and 2 inches above the level of the fire-bars, the brickwork is carried on horizontally until close to the stack. The sides of this part are perpendicular, and 12 inches apart: they are continued upwards to the top of the brickwork 14 inches unbroken, except that at 5 inches from the bottom they are thrown back $\frac{1}{3}$rd of an inch so as to form a ledge there. This ledge is for the purpose of receiving the edges of certain fire-tiles, which, when put in, form the top of the flue and at the same time the bottom of the glass chamber; but the whole is so constructed, that the tiles can be put in and taken out at pleasure without disturbing the rest of the work. The side or rather end of the chamber nearest the fire is constructed of a fire-tile, which terminates and faces the brick arch over the fire-place, and extends from the surface of the brickwork downwards 9 inches to the side ledges before described: the further end of the chamber is finished in a similar way, and beyond that the flue is carried in the most convenient and direct manner, but without any unnecessary contraction, into the stack or chimney. The length of this upper aperture, afterwards constituting the chamber, is 25 inches, its breadth $12\frac{3}{4}$ inches. When the bottom tiles are in their places, they leave a depth of 5 inches for that part of the furnace or flue beneath the chamber, which is also 38 inches from the fire to the end, and, with the exception of certain supports in it, is 12 inches wide.

These supports are built in with the bottom of the flue. They are essential to the permanency and regularity of the bottom of the glass chamber, and require considerable nicety in their arrangement. They consist of fire-bricks placed up on end, so that their narrowest surfaces are towards the ends of the furnace, their sides or broadest exposed surfaces parallel with the sides of the furnace itself. They rise to the same height above the bottom of the flue as the ledges on the sides of the brickwork, or 5 inches, and with them, form the support for the bottom tiles. There are three of them in our furnace, placed in a line equidistant from the two sides of the flue; and being $2\frac{1}{2}$ inches thick, they leave spaces for the passage of flame and the reception of coke, which are $4\frac{3}{4}$ inches in width. The first of these is two inches from the back edge of the fire, and in that direction extends 4 inches; the second is 4 inches from the first; and 6 inches beyond that one is the third.

During the action of the furnace, coke is supplied to this part, and arranged through two holes level with this space, and wrought in the side of the furnace by leaving out a brick. They are made to occur nearly opposite the spaces between the supports seen when looking across the course of the flame, and are stopped by the insertion of loose bricks, and a piece of paper put before the place, which adheres from the pressure inwards of the atmosphere. These holes, being in the thickness of the walls of the furnace, are 17 inches long.

The tiles which form the bottom of the chamber and top of the flue, are of Cornish ware (52. 53), or at least the one which constitutes the half nearest the fire is of that material; but the other, which is not so highly heated, and never has to be moved, may be some other ware, and 2¼ inches in thickness. The tile nearest the fire has to transmit heat to the glass, and if of Cornish ware, and supported as described, is abundantly strong when ¾ths of an inch in thickness. It should be nicely adjusted by grinding (53), and when fitted in, the edges should be made close by a little fire lute.

There is a part of the furnace not yet mentioned, which must be arranged as the structure is raised. This is the air tube (55). It is of glazed porcelain, and passes horizontally through the side of the furnace, so that its inner aperture is 2 inches from the end of the glass chamber, and its lower edge level with the upper surface of the Cornish tile constituting the bottom, whilst the outer end is flush with the outside surface of the brickwork. Its length is 17 inches, its internal diameter ⅞ths of an inch. The short pieces of adjusting tube (55) are 6, 7, and 8 inches in length, and $\frac{7}{10}$ths of an inch internal diameter: their ends are usually finished obliquely.

All those parts of the furnace which are in contact with or near the fire, are of the best fire-bricks laid in loam; but the sides of that part of the cavity already described, which form the glass chamber, are fire-tiles, and they rise about an inch above the neighbouring brickwork, forming a raised edge all round, which, at the same time that it better excludes dirt than if level with the rest of the work, also allows the covers of the glass chamber to apply more closely. These covers are three wrought-iron plates, each ¼th of an inch in thickness and 16 inches long; but their widths vary, and are 7, 10, and 12 inches. These put side by side cover the mouth of the chamber, but varied in juxta-position, allow of more or less of the chamber being opened at once, according to whatever the experiment may require; each has a short solid handle fixed to the middle of the upper surface.

Besides the iron covers, there are a set of earthenware covers, consisting of 6 square tiles each 1⅝ inch thick(62). These are notched to receive the handles of the iron covers, and being put together over them, constitute a covering of earthenware, which very importantly assists in retaining the heat.

The tiles and brick used in the annealing process (89) are the ordinary dry varieties, with some pieces of various sizes, to allow of the close adjustment of the whole.

The earthenware supporting blocks (57) required for the arrangement and support of the platina tray should be formed out of some kind of flat unglazed ware containing as little iron as possible, and should be of various thicknesses, sizes, and forms, although parallelopipeds are the most usual. They should not be of such substance as is liable to fly or send off anything when heated; and when any portion of glass adheres to them, it should either be cleared off or the piece thrown away. The Cornish tile before described (52. 53) is excellent for this use, and may be sawn, rasped or ground into any shape required.

The glass covers (60, &c.) that have yet been used were merely inverted evaporating basins. They answer the required purpose exceedingly well, except that, when large, they are too strong, too heavy, and too deep. Some covers for the purpose are therefore in progress, and as they only have to support their own weight and hold together, they are to be thin. The covers should be of very refractory and highly baked ware; it may be desirable to have them very slightly glazed, to keep them clean, and prevent the absorption of any substance which might send off vapours injurious to the glass.

The fire tools required for this furnace will suggest themselves. Amongst the rest should be a pair of tongs which will readily lay hold either of the earthenware tile or the iron covers; a slag and coke rake (89); and a stoking iron, with its extremity bent, for the purpose of breaking the clinkers off the bars from beneath upwards.

Preparation of spongy platina.

The platina used for this preparation should be pure, and may be the refuse pieces resulting from such plate and foil as has been in use for trays in former experiments. This, after being taken out of the pickle (93), and condemned as useless for other purposes in the glass-house, should be trimmed from all alloyed parts, if any such are adhering to it, and then digested in a Florence flask, with a mixture of five measures of strong muriatic, one measure of strong nitric acid, and three measures of water. But little heat should be applied at first until the action diminishes. According to Dr. Wollaston, one ounce of platina will be dissolved by about four ounce measures of such acid, and it is advantageous to have a considerable excess of platina present. The solution obtained is to be precipitated by a strong solution of muriate of ammonia; a bright yellow pulverulent substance will fall, and a mother liquid having more or less colour remain. The precipitate being allowed to subside, the liquor is to be poured off, and the former then washed with two or three portions of water. The washing liquors and the mother water may afterwards be concentrated together; but it is better not to prepare spongy platina for this particular use from these fluids, but only from the precipitate which falls on adding the muriate of ammonia.

The yellow precipitate, when washed, is to be dried on a filter, or in a basin, and then decomposed by the application of a dull red heat. This may be done in a clean white earthenware crucible. The heat should be continued until vapours cease to arise; but this will be found a long operation, in consequence of the low temperature which is to be applied, and the exceeding bad conducting power of platina for heat when in this spongy state. The reduction may also be performed by putting the precipitate upon a piece of platina foil in a layer about $\frac{1}{8}$th of an inch in thickness, and covering it with another piece of foil; a spirit lamp will then suffice to reduce the metal, but the foil and powder must be turned occasionally, that both sides may be exposed to the flame. The platina will appear as a dull grey spongy metallic mass. It should be broken up, mingled, and then again heated to insure the dissipation of all volatile matter.

After this is done, the platina should be rubbed to powder by the clean finger, or clean paper (83), heated slightly a third time, and then preserved in a clean and well stoppered bottle.

II. *Account of Levellings carried across the Isthmus of Panamá, to ascertain the relative height of the Pacific Ocean at Panamá and of the Atlantic at the mouth of the river Chagres; accompanied by Geographical and Topographical Notices of the Isthmus. By* John Augustus Lloyd, *Esq. Communicated by* Captain Sabine, *Secretary of the Royal Society.*

Read November 26, 1829.

In November 1827 I received a special commission from General Bolivar to make a survey of the Isthmus of Panamá and Darien, in order to ascertain the best and most eligible line for a communication (whether by road or canal) between the two seas. On my arrival in Panamá in March 1828 I was joined by a brother officer of Engineers, a Swede in the Colombian service, a good mathematician and of habits of great correctness in observation.

Upon consulting together, we found that we could combine the particular object of the commission with a second object in which we both felt a deep interest, namely, the determination of the relative height of the ocean on either side of the Isthmus; and that we could best accomplish both, by taking a part of the present line of road between Porto Velo and Panamá, until we should fall in with the river Chagres about twenty miles above Cruces, which village is the usual landing-place for all articles of commerce in their transit from the North Sea to Panamá.

To avoid delay, we commenced our operations on the 5th of May, although the rainy season had for some days set in; being resolved to overlook the absence of personal comfort, the unhealthiness of the season to a European constitution, the inadequacy of our means, and various other difficulties unnecessary to enumerate, as we finally succeeded in surmounting them.

The instruments used for the levelling were,—A 20-inch spirit level of Carey's best construction, with extra telescopes, levels, shade tubes, &c., which I received from the museum at Bogota; a pair of excellent station staves made by Harris and graduated as usual, with vernier scales added by myself to read off to thousandths of a foot when required; Gunter's chains; an excellent 10-inch theodolite by Carey; and a very fine altitude and azimuth circle for the survey.

Our first level commenced at the end of the street called Callé Sal si Puede in the suburbs of Panamá and at the point of a bay called Playa Prieta, at spring tide high-water mark, observed two days after full and change, which I had subsequently the opportunity of verifying on my return to Panamá, and of ascertaining that it was 3.03 feet lower than the extreme rise of occasional tides under the influence of particular winds. From this point we followed the old road to Porto Velo, and after 732 pairs of levellings in a distance from Panamá of 1828 chains (22¾ miles), we arrived at the banks of the Chagres on the 30th of June, 633.32 feet being the greatest height we had passed over; and after building a secure station on the bank 169.84 above the level of high water mark in the Pacific, we finished our operations for that year, on account of the great inclemency of the season, the sickness and debility of the people employed, and from our own constitutions beginning to suffer by continued exposure to incessant rains, with generally no other covering than tents and ranchos or small huts built by ourselves.

On the 7th of February 1829, the dry season having for some time set in, we resumed our levelling at the station at which we had desisted the year before, having the instruments in good repair; and having descended to a station on the river fixed on for the purpose, we found the surface of the water to be 152.55 feet above high-water mark in Panamá.

We now followed the course of the river, and were enabled thereby to take longer levels than before, which were made with the greatest possible care, reducing the observations to the true levels by the most exact tables for the curvature of the earth.

After 68 pairs of levellings we arrived at Cruces in a distance of 1545 chains (19¼ miles), and found a fall in the river of 114.60 feet, leaving only 37.96 feet above the Pacific. Having nearly 50 miles more to descend, and finding so great a fall in 19 miles, we were led to expect a greater fall than 37.96 feet in the remaining distance; and consequently, at this stage of our operations, to apprehend that we should find the level of the sea at Panamá to be higher than at the mouth of the Chagres.

From Cruces to a town called Gorgona, distant 419 chains (5¼ miles), there is a fall of only 16.13 feet, and thence to a small gravel bank named by us "Playa de los Injenieros," distant altogether 1302 chains (16¾ miles) from Cruces, we found a fall from Gorgona of 21.82 feet, to a station precisely level with the high-water mark of spring tides in the Pacific, still being 34 miles from the mouth of the river. From this point we descended below the level of high-water-mark of the Pacific, until we arrived at a place called Palo Matias, distant from Cruces 2682 chains (33½ miles), and from the commencement of the levels in the river 4227 chains (52½ miles): at this point we first observed the effects (slight as they were) of the tides of the North Sea, and the height of the water was 13.65 feet below the high-water mark at the Pacific; this we therefore concluded to be the level of high-water mark in the Atlantic. We however continued our levellings 507 chains further, to a place called La Bruja, nearly 12 miles distant from the mouth of the Chagres, where the water in the dry season is very brackish, and from whence there is no perceptible current to the sea: here we found the level of the surface of the water, by several observations at the time of high water in Chagres, to be 13.55 feet below the high-water mark in the Pacific, being 0.1 of a foot less than at Palo Matias, the difference being occasioned by our not having observed the tides at the former place so correctly as at the latter, as we determined to finish the observations at La Bruja. After 935 pairs of levellings, therefore, in a distance of nearly 82 miles, we found high-water mark in the Pacific to be13.55 feet higher than high-water mark at La Bruja, which, from the circumstances above mentioned, is considered to be the high-water level of the Atlantic at Chagres.

The details of all the operations connected with the levellings are contained in a manuscript deposited in the library of the Royal Society; they are further illustrated by a section of the whole on the scale of four inches to a mile.

No proof levels were taken. I was aware, from the commencement, of the nature of the task I had entered upon, and knew too well that if I had the good fortune to be able to persevere a sufficient time to carry the levelling across the Isthmus, I should be but little capable of remaining during a third year, which such verification would have required. I therefore adopted such a scrupulous and rigid mode of proceeding as would render a verification unnecessary, and prevent the intrusion of even a trifling error; an important one I am bold to say was nearly impossible. In the whole distance overland to the river Chagres, my companion being employed with the chain, I was assisted by a Spaniard whom I had previously instructed in the management of the station staff. By means of signals I made him adjust the cross-piece correctly to the horizontal wire of the telescope; he then brought me the staff, which I read off and noted down the reading; he resumed his station, I examined the level again, adjusted the staff, recalled the Spaniard, and read off a second time.

From the Chagres to the last level my companion had the station staff himself, which when adjusted to the cross-wires was read off by him. I then re-examined the level, adjusted the staff a second time, and he again read off, writing down the two observations in distinct books, which were compared in the evening. The instrument itself I proved on most days after work, by making a set of 8 or 10 levellings in a circle; returning in the last station to the point from which I first started, and finding the sum of the differences of the levellings amount to zero.

The point from which the levellings commenced at Panamá is marked by a large stone cut for that purpose in the wall at the edge of the sea in Playa Prieta; and the concluding point at La Bruja, by a tree cut down to the exact height marked in the Observation Book above the surface of the water: the height of the section of the tree is 6.848 feet below the level of high-water spring tide at Panamá. No better means of marking this level presented itself at La Bruja; but by a reference to the manuscript detail of the observations preserved in the library, several stations are to be found in the vicinity which are not liable to decay.

By careful and continued observations, I found the rise and fall of the tide in the Pacific at Panamá as follows: between the extreme elevation and depression of the waters by occasional tides there is a difference of 27.44 feet, and the mean actual rise and fall two days after full moon is 21.22 feet.

At Chagres I observed the rise and fall of the tide at the close of the dry season in April 1829 to be 1.16 foot, and being there subsequently during the rainy season, I had an opportunity of observing that the high-water mark was the same in both seasons.

The time of high water is nearly the same at Chagres and at Panamá, namely at $3^h\ 20^m$ at full and change: hence the following interesting and curious phenomena are deducible in respect to the difference of level of the two seas.

First.—High-water mark at Panamá is 13.55 feet above the high-water mark of the Atlantic at Chagres ; half the rise and fall of spring tides is at Panamá 10.61 feet, and at Chagres 0.58 of a foot; and assuming half the rise and fall above the low water of spring tides to be the respective mean levels, the mean height of the Pacific at Panamá is 3.52 feet higher than that of the Atlantic at Chagres*.

Second.—At high water, the time of which is nearly the same on both sides the Isthmus, the Pacific is raised at mean tides 10.61 feet, and the Atlantic 0.58 of a foot, above their respective mean levels; the Pacific is therefore the highest at such times by (10.61—0.58+3.52=) 13.55 feet.

Third.—At low water, both seas are the same quantities below their respective mean levels ; therefore at such times the Pacific is lower than the Atlantic by (10.61—0.58—3.52=) 6.51 feet.

In every twelve hours therefore, and commencing with high tides, the level of the Pacific is first several feet higher than that of the Atlantic; it becomes then of the same height, and at low tide is several feet lower: again, as the tide rises the two seas are of one height, and finally at high tide the Pacific is again the same number of feet above the Atlantic as at first.

Almost every person who visits Panamá from the Atlantic side is disposed to think that the country rises from the Atlantic to the Pacific. The ascent of the river Chagres, particularly when swollen by rains and its current rendered more than usually rapid, is very toilsome; and on reaching Cruces after a four or five days tedious journey, a traveller is impressed with the persuasion that he has gained a considerable elevation above the sea that he has quitted: this impression is not diminished by the journey to Panamá, which is mostly through rough and rugged passes, continually ascending and descending ; and when, on arriving in the savannahs, a few miles from Panamá, the city is beheld for the first time with its conspicuous cathedral, the general exclamation is, "I thought Panamá had been near the level of the sea." Such is actually its situation; but as the valley from which it is first seen is several feet below the level of the sea, the first and strong impression produced, is that the city stands upon an eminence.

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* The author is aware that there are different opinions with regard to what is the mean level of the ocean; the assumption in the text is conformable to his own opinion, but as the data are given from which the conclusions are drawn, every person is furnished with the means of making his own deductions.

Topographical and Geographical Notices.

In the map which accompanies this memoir, the coast line on both sides the Isthmus is taken from the best Spanish authorities, including some very recent corrections; the interior is wholly from my own observations.

The district which extends from Panamá along the old road to Porto Velo, as far as its meeting with the river Chagres, and for three to five miles on either side, is from a survey made by my companion Captain Falmarc and myself, whilst levelling through this part of the country, in the operations already described. The angles and bearings were taken with a 10-inch theodolite by Carey, with a needle as exact as could be provided for an instrument of that size. On a separate plan (in MSS. deposited in the Society's library,) are marked the stations of the survey, and the intersection of the bearings taken from them. The principal stations are as follows:—The Cerros or Mountains of Ancon, Caledonia, Vidrio, Lirio, Algarobo, Pelado, Largo, Gordo, San Sonati, Alto, de Las Lajas, Maria Henrique, Grenadilla, &c.

From these stations, the surrounding country was sketched at once on the rough map: the same mode was continued from the point where the levellings intersected the river Chagres to the mouth of the river, including as much of the surrounding country as could be laid down by intersecting bearings, taken on either side the river.

The country west of the road from Cruces to Panamá, including a few miles of the coast by Arayjan and Chorrera, was traversed in various directions with a compass, every accessible eminence was ascended, and views taken of the country.

The country between the northern banks of the river Chagres and the North Sea was examined and sketched in an excursion in which I ascended the river Gatun, with a boat compass, and crossed from thence on foot by the gold mines of Santa Rita, (where is gained a fine view of the northern coast to La Enseñada de las Minas,) and thence to the sea beach: from this point I pursued the coast line to Porto Velo, and recrossed the Isthmus to Panamá on foot, in the rôute marked in the map, which is the old and only road from Porto Velo to Panamá, taking careful bearings from eminences over which I passed,

and gaining as many views of the surrounding country as I possibly could, by climbing the highest trees with a small tomahawk, particularly noticing the direction and figure of the Cordillera to the east and west.

The more direct line from Porto Velo to Panamá, passing through the river Chagres at a place called Calle Limon, is laid down from a manuscript furnished by a Spaniard, who with a circumferentor and a cord of 200 varas crossed from Porto Velo to Panamá as nearly north and south as possible.

It is generally supposed in Europe that the great chain of mountains which in South America forms the Andes and in North America the Mexican and Rocky Mountains, continues nearly unbroken through the Isthmus. This however is not the case: the northern Cordillera breaks into detached mountains on the eastern side of the province of Veragua. These are of considerable height, extremely abrupt and rugged, and frequently exhibit an almost perpendicular face of bare rock. To these succeed numerous conical mountains rising out of savannahs and plains, and seldom exceeding from 300 to 500 feet in height. Finally, between Chagres on the Atlantic side, and Chorrera on the Pacific side, the conical mountains are not so numerous, having plains of great extent interspersed, with occasional insulated ranges of hills of inconderable height and extent. From this description it will be seen that the spot where the continent of America is reduced to nearly its narrowest limits, is also distinguished by a break for a few miles of the great chain of mountains, which otherwise extends, with but few exceptions, to its extreme northern and southern limits.

This combination of circumstances points out the peculiar fitness of the Isthmus of Panamá for the establishment of a communication across.

On the east of the line from Panamá to the Bay of Limon, the mountains again commence, gradually thicken, and become more elevated until they connect and form Cordilleras extending from Porto Velo to the Bay of Mandinga, from whence there is another break in the province of Darien and Choco, after which the land rises into a Cordillera on a very extended scale and of very great elevation.

Two lines are marked on the map, commencing at a point near the junction of the rivers Chagres and Trinidad, and crossing the plains, the one to Chorrera and the other to Panamá. These lines indicate the directions which I consider the best for a rail-road communication. The principal difficulty in the establishment of such communication would arise from the number of rivulets to be crossed, which, though dry in summer, become considerable streams in the rainy season.

The line which crosses to Chorrera is much the shortest, but the other line has the advantage of terminating in the city and harbour of Panamá.

The country intersected by these lines is by no means so abundant in woods as in other parts, but has fine savannahs, and throughout the whole distance, as well as on each bank of the river Trinidad or Capira, presents flat and sometimes swampy country, with occasional detached sugar-loaf mountains, interspersed with streams that mostly empty themselves into the Chagres.

Should a time arrive when a project of a water communication across the Isthmus may be entertained, the river Trinidad will probably appear the most favourable route. The river is for some distance both broad and deep. Its banks are also well suited for wharfs, especially in the neighbourhood of the spot from whence the lines marked for rail-road communications commence.

As the river Chagres has been greatly dwelt on in the writings of those who have discussed the probability of communications being established between the two seas, and as considerable expectations have been formed of the facilities it might afford towards a water communication, I have given a separate plan of the river from its mouth to the point at which it was intersected by the levellings. (This plan will not admit of reduction within the compass of the plates in the Philosophical Transactions, but will remain in the Society's library, where it may be consulted. A plan of the river on a less minute scale is contained in the general map.)

The distances along the river were measured by a strong line of 10 chains in length, substituted for the usual measuring chain, properly subdivided, and its length occasionally verified; the cord was borne usually by five men, but when required in the shallow and rapid water, by as many as ten men; and when the water became too deep for the men to wade, canoes were employed to stretch the line. The soundings were taken by a man with a sounding line marked to half feet, seated behind me in the canoe; so that I could observe the line myself at every cast. The casts were generally between 30 and 40 yards

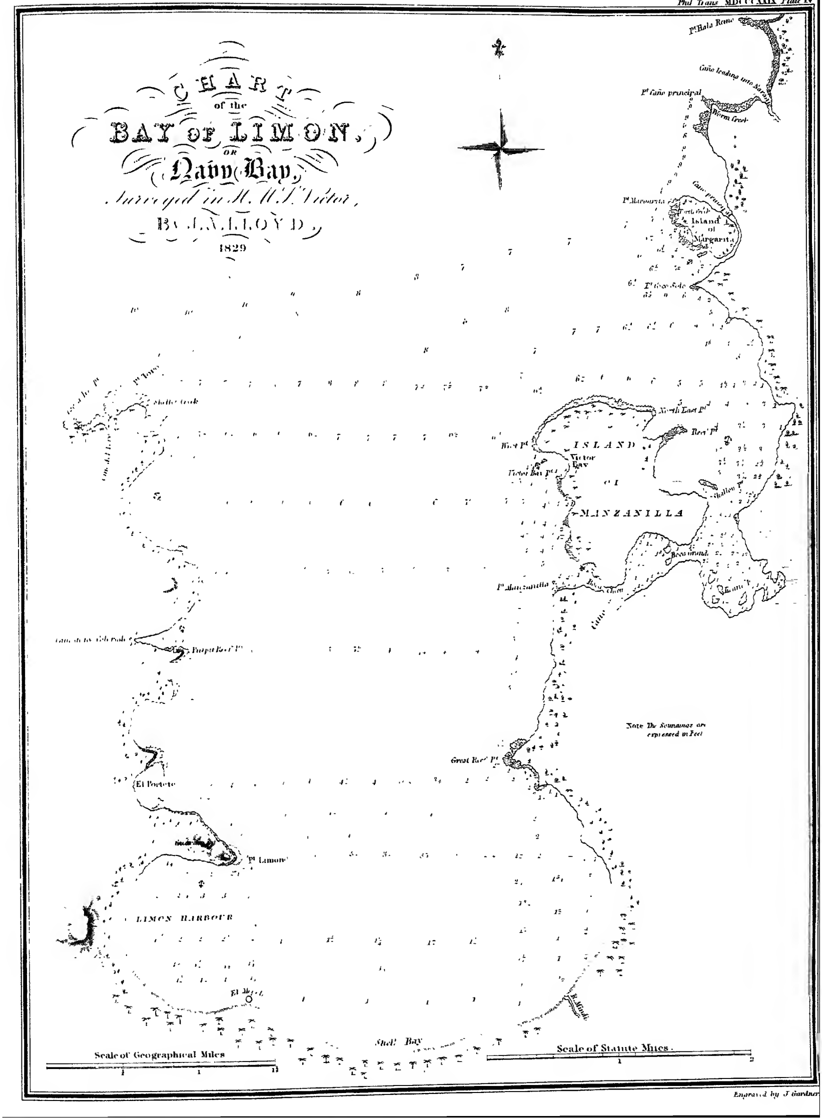
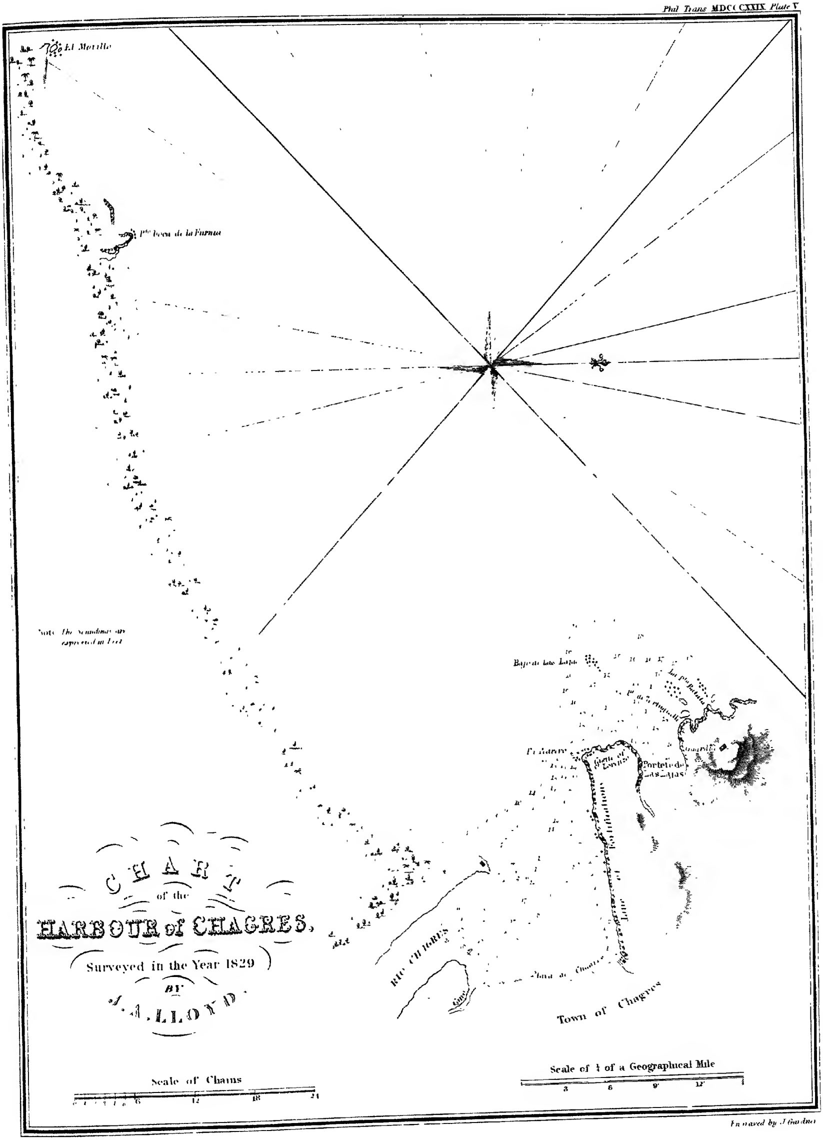


CHART
of the
BAY OF LIMON,
OR
Navy Bay,
Surveyed in H. M. S. Victor,
By J. A. LLOYD,
1829
Pt Bala Reme
Caño leading into Naranjo
Pt Caño principal
Worm Creek
Caño principal
Pt Margarita
Island of Margarita
Pt Coco Solo
North East Pt
Reef Pt
West Pt
ISLAND OF MANZANILLA
Victor Bay
Victor Bay Pt
Shallow Pt
Boca Grande
Pt Manzanilla
Boca Chica
Caño
Shelter Creek
Farpa Reef Pt
Great Reef Pt
Note The Soundings are expressed in Feet
El Portete
Pt Limone
LIMON HARBOUR
El Morro
R. Mindi
Shell Bay
Scale of Geographical Miles
½
1
1½
Scale of Statute Miles.
1
2

Engraved by J Gardner


El Morillo
Pta. boca de la Furnia
Note. The Soundings are expressed in Feet
Bajo de las Lajas
La Pta. Batata
Pta. de la Chapetilla
Pta. Arenas
Castle of Lorenzo
Portete de Las Lajas
Rio Chagres
Town of Chagres
CHART
of the
HARBOUR of CHAGRES,
Surveyed in the Year 1829
BY
J. A. LLOYD
Scale of Chains
6
12
18
24
Scale of ¼ of a Geographical Mile
3
6
9
12

Engraved by J Gardner

from one another, and the depth of water was inserted in a sketch of the river which I was at the same time engaged in making.

The river, its channel, and the banks, which in the dry season embarrass its navigation, are laid down in the manuscript plan with great care and minuteness. It is subject to one great inconvenience, that vessels drawing more than 12 feet water cannot enter the river, even in perfectly calm weather, on account of a stratum of slaty limestone, which runs, at a depth at high water of 15 feet, from a point on the main land to some rocks in the midle of the entrance of the harbour, and which are just even with the water's edge; which, together with the lee current that sets on the southern shore, particularly in the rainy season, renders the entrance extremely difficult and dangerous. The accompanying plan of the harbour will sufficiently explain the inconveniencies it is liable to.

The value of the Chagres considered as the port of entrance for all communications, whether by the river Chagres, Trinidad, or by rail-roads across the plains, is greatly limited from the above-mentioned cause. It would prove in all cases a serious disqualification, were it not one which admits of a simple and effectual remedy, arising from the proximity of the Bay of Limon, otherwise called Navy Bay, with which the river might easily be connected: the coves of this bay afford excellent and secure anchorage in its present state, and the whole harbour is capable of being rendered, by obvious and not very expensive means, one of the most commodious and safe harbours in the world.

By the good offices of H. M. consul in Panamá, and the kindness of the commander of H.M. ship Victor, I obtained the use of that ship and her boats in making the accompanying plan of this bay. The shores are laid down trigonometrically from a base of 5220 yards, the situation of which is marked; and the soundings were taken by myself with the assistance of the master. It will be seen from this plan that the distance from one of the best coves (in respect to anchorage) across the separating country from the Chagres, and in the most convenient track, is something less than three miles to a point in the river about three miles from its mouth.

I have traversed the intervening land, which is particularly level, and in all respects suitable for a canal, which, being required for so short a distance, might well be of sufficient depth to admit vessels of any reasonable draft of water, and would obviate the inconvenience of the shallow water at the entrance of the Chagres.

I have felt that I might be expected to state my own opinion of the mode that offers the greatest facility for communications across the Isthmus, since my examination and surveys were made for that specific purpose, and I have accordingly done so; but I have endeavoured at the same time to render the topographical representation of the country sufficiently detailed to enable others to draw such conclusions as may perhaps deserve to be preferred to mine, with almost as much advantage as if they had themselves visited the country.

For the opportunities that I have thus enjoyed of contributing correct topographical knowledge of a part of the world, which from its peculiar locality has attracted much philosophical and commercial interest, I am indebted to the authority and support which I received from General Bolivar; I am also indebted to his liberality for permission to make public the information I have acquired.

In a country in many respects so unsettled, it will readily be imagined that the authority and countenance, derived from a Government far from the spot, are not alone sufficient to enable a foreigner to carry through operations so extensive and long-continued as mine were. I am sensible that I could not have completed them, had it not been for the frequent assistance and constant support, which I received from the friendship of Malcolm MacGregor, Esq. H. M. consul at Panamá.

III. *On the law of the partial polarization of light by reflexion.* By DAVID BREWSTER, *LL.D. F.R.S. L. & E.*

Read February 4, 1830.

IN the year 1815 I communicated to the Royal Society a series of experiments on the polarization of light by successive reflexions, which contain the germ of the investigations, the results of which I now propose to explain.

From these experiments it appeared that a given pencil of light could be wholly polarized at any angle of incidence, provided it underwent a sufficient number of reflexions, either at angles wholly above or wholly below the maximum polarizing angle, or at angles partly above and partly below that angle; and it was scarcely possible to resist the conclusion, that the light not polarized by the first reflexion had suffered a physical change at each action of the reflecting force which brought it nearer and nearer to the state of complete polarization. This opinion, however, which I have always regarded as demonstrable, appeared in a different light to others. Guided probably by an experimental result, apparently though not really hostile to it, Dr. YOUNG and MM. BIOT, ARAGO, and FRESNEL, have adhered to the original opinion of MALUS, that the reflected and refracted pencils consist partly of light wholly polarized, and partly of light in its natural state; and more recently Mr. HERSCHEL has given the weight of his opinion to the same view of the subject.

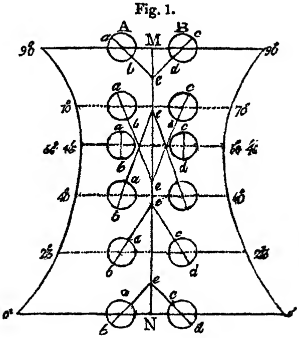
Under these circumstances I have often returned to the investigation with renewed zeal; but though the frequent repetition of my experiments has more and more convinced me of the truth of the conclusions which I drew from them, yet I have not till lately been able to place the subject in a satisfactory aspect, and to connect it with general laws, which give a mathematical form to this fundamental branch of the science of polarization.

If we consider a pencil of natural light as divided into two pencils polarized in rectangular planes by the action of a doubly refracting crystal, and conceive the light of these two pencils to return back through the crystal, it will obviously emerge in the state of natural light. When we examine the pencil thus recomposed, or when we examine a pencil consisting of two oppositely polarized pencils superposed, we shall find that they comport themselves under every analysis exactly like common light; so that we are entitled to assume such a pencil as the representative of natural light, and to consider every thing that can be established respecting the one, as true respecting the other.

In applying this principle to the analysis of the phænomena produced by reflexion, I placed the planes of polarization of the compound beam in the plane of reflexion; but though this led to some interesting conclusions, it did not develope any general law. I then conceived the idea of making the plane of reflexion bisect the right angle formed by the planes of polarization; and in this way I observed a series of symmetrical effects at different angles of incidence, which threw a broad light over the whole subject.

In order to explain these results, let AB (Fig. 1.) represent the two pencils of oppositely polarized light as separated by double refraction; let *a b*, *c d* be the directions of their planes of polarization, forming a right angle *a e c*, and let the plane of reflexion MN, of a surface of plate glass, bisect the angle *a e c*, so that the planes *a b*, *c d* form angles of + 45° and − 45° with the plane M N. Let a rhomb of calcareous spar have its principal section now placed in the plane of reflexion.

Fig. 1.



At an incidence of 90°, reckoned from the perpendicular, the reflected images of A and B suffer no change, the angle *a e c* is still a right angle, and the four pencils formed by the calcareous spar are all of equal intensity. As the incidence however diminishes, the angle *a e c* diminishes also, and the ordinary and extraordinary images of A and B differ in intensity. At an incidence of 80° for example, the angle *a e c* is reduced from 90° to 66°; at 70° it has been reduced to 40°, and at 56° 45′, the maximum polarizing angle, it has been reduced to 0°; that is, the planes of polarization *a b, c d* are now parallel. Below the polarizing angle, at 50°, the axes are again inclined to each other, and form an angle of 22°. At 40° they form an angle of 50°, and at 0°, or a perpendicular incidence, they are again brought back to their primitive inclination of 90°. Taking M N to represent the quadrant of incidence from 90° at M, to 0° at N, the curves, 90°, 0°, show the progressive change which takes place in the planes of polarization, the plane of polarization being a tangent to the curve at the incidence which corresponds to any particular point of it.

When we employ a surface of diamond in place of glass, the inclination of the axes *a b*, *c d* is reduced to 46° at an incidence of 80°, to 8° at an incidence of 70°, and at 67° 43′ the axes become parallel.

Such being the action of the reflecting forces upon A and B taken separately, let us now consider them as superposed and forming natural light. At 90° and 0° of incidence, the reflecting force produces no change in the inclination of their axes or planes of polarization; but at 56° 45′ in the case of glass, and 67° 43′ in the case of diamond, the axes of all the particles are brought into a state of parallelism with the plane of reflexion; and consequently when the image which they form is viewed by the rhomb of calcareous spar, they will all pass into the ordinary image, and thus prove that they are wholly polarized in the plane of reflexion.

All this is entirely conformable to what has been long known: but we now see that the total polarization of the reflected pencil at an angle whose tangent is the index of refraction, is effected by turning round the planes of polarization of one half of the light from right to left, and of the other half from left to right, each through an angle of 45°. Let us now see what takes place at those angles where the pencil is only partially polarized. At 80° for example, the angle of the planes *a b*, *c d* is 66°, that is, each plane of polarization has been turned round in opposite directions from an inclination of 45° to one of 33° with the plane of reflexion. The light has therefore suffered a physical change of a very marked kind, constituting now neither natural nor polarized light. It is not natural light, because its planes of polarization are not rectangular; it is not polarized light, because they are not parallel. It is a pencil of light having the physical character of one half of its rays being polarized at an angle of 66° to the other half. It will now be asked, how a pencil thus characterized can exhibit the properties of a partially polarized pencil, that is, of a pencil part of whose light is polarized in the plane of reflexion, while the rest retains its condition of natural light. This will be understood by replacing the analysing rhomb with its principal section in the plane of reflexion, and viewing through it the images A and B at 80° of incidence. As the axis of A is inclined 33° to MN or the section of the rhomb, the ordinary image of it will be much brighter than the extraordinary image, the intensity of each being in the ratio of $\cos^2 \varphi$ to $\sin^2 \varphi$, φ being the angle of inclination, or 33° in the present case. In like manner the ordinary image of B will be in the same ratio brighter than its extraordinary image, that is, by considering A and B in a state of superposition, the extraordinary image of a pencil of light reflected at 80° will be fainter than the ordinary image in the ratio of $\sin^2 33°$ to $\cos^2 33°$. But this inequality in the intensity of the two pencils is precisely what would be produced by a compound pencil, part of which is polarized in the plane of reflexion, and part of which is common light. When Malus, therefore, and his successors analysed the pencil reflected at 80°, they could not do otherwise than conclude that it was partially polarized, consisting partly of light polarized in the plane of reflexion, and partly of natural light. The action of successive reflexions, however, afforded a more precise means of analysis, in so far as it proved that the portion of what was deemed natural light had in reality suffered a physical change, which approximated it to the state of polarized light; and we now see that the portion of what was called polarized light was only what may be called apparently polarized; for though it disappears, like polarized light, from the extraordinary image of the analysing prism, yet there is not a single particle of it polarized in the plane of reflexion.

These results must be admitted to possess considerable interest in themselves; but, as we shall proceed to show, they lead to conclusions of general importance. The quantity of light which disappears from the extraordinary image, is obviously the quantity of light which is really or apparently polarized at the given angle of incidence; and if we admit the truth of the law of repartition discovered by Malus, and represented by $P_{oo} = P_o \cos^2 \varphi$, and $P_{oe} = P_o \sin^2 \varphi$, and if we can determine φ for substances of every refractive power, and for all angles of incidence, we may consider as established the mathematical law which determines the intensity of the polarized pencil, whatever be the nature of the body which reflects it,—whatever be the angle at which it is incident,—whatever be the number of reflexions which it suffers, and whether these reflexions are all made from one substance, or partly from one substance and partly from another.

The first step in this investigation is to determine the law according to which a reflecting surface changes the plane of polarization of a polarized ray. This subject was first examined by **Malus**, but not with that success which attended most of his labours. Before I was acquainted with what had been done by M. **Fresnel**, or with the experiments of M. **Arago** on glass and water, I had made a number of very careful experiments on the same subject, and had represented them by formulæ founded on the law of the tangents. These formulæ, however, I found to be defective; and I am persuaded, from a very extensive series of experiments, that the formulæ of **Fresnel** are accurate expressions of the phenomena under every variation of incidence and refractive power. If i is the angle of incidence, i' the angle of refraction, x the primitive inclination of the plane of the polarized ray to the plane of reflexion, and φ the inclination to which that plane is brought by reflexion, then, according to **Fresnel**, we have

$$\text{Tan}\,\varphi = \tan x\,\frac{\cos(i+i')}{\cos(i-i')}$$

When x is 45°, as in the preceding observations, then $\tan x = 1$, and we have

$$\text{Tan}\,\varphi = \frac{\cos(i+i')}{\cos(i-i')}.$$

In these formulæ, which are founded on the law of the tangents, $i + i'$ is the supplement of the angle which the reflected ray forms with the refracted ray; while $i - i'$ is the angle which the incident ray forms with the refracted ray, or the deviation produced by refraction.

These formulæ have been verified by M. **Arago** at ten angles of incidence upon Glass, and four upon Water; but his experiments were made only in the case where x is 45°, and where $\tan x$ disappears from the formula. As my experiments embrace a wider range of substances, and also the general case where x varies from 0° to 90°, I consider them as a necessary basis for a law of such extensive application.

The first series of experiments which I made was upon Plate Glass, in which the maximum polarizing angle was nearly 56°: hence I assume the index of refraction to be 1.4826. The following were the results:

PLATE GLASS.

Angle of Incidence.	Angle of Refraction.	Inclination of Plane of Polarization to Plane of Reflexion.		Difference.
		Observed.	Computed.	
90°	0° 0′	45° 0′	45° 0′	0° 0′
88	42 23	43 4	42 49	+0 35
86	42 17	40 43	40 36	+0 7
84	42 8	38 47	38 22	+0 25
80	41 37	33 13	33 46	−0 33
75	40 40	28 45	27 41	+1 4
70	39 20	22 6	21 3	+1 3
65	37 41	14 40	13 53	+0 47
60	35 45	6 10	6 16	−0 6
56	34 0	0 0	0 0	0 0
50	31 22	9 0	9 0	0 0
45	28 29	16 55	16 31	+0 24
40	25 42	22 37	23 1	−0 24
30	19 43	32 25	33 19	−0 54
20	13 20	39 0	40 4	−1 4
10	6 44	44 0	43 49	+0 11

These results, obtained in every part of the quadrant, completely establish the accuracy of the formula. The differences are all within the limits of the errors of observation, and amount, at an average, to 32½′ on each observation.

It is a curious circumstance, which I believe has not before been remarked, that at an incidence of 45° the deviation produced by refraction, or $i - i'$, is, in every substance, the complement of the angle of refraction i to 45°; and in the action of all substances upon polarized light at an incidence of 45°, the rotation of the plane of polarization of a pencil polarized + 45°, or − 45°, is equal to the angle of refraction; while the inclination of the plane of polarization to the plane of reflexion, or φ, is equal to the deviation $i - i$.

In order to establish the accuracy of the formula for different degrees of refractive power, I made the following experiments on Diamond, in which the index of refraction was 2.440.

Diamond.

Angle of Incidence.	Angle of Refraction.	Inclination of Plane of Polarization to Plane of Reflexion.		Difference.
		Observed.	Calculated.	
90° 0′	24° 12′	45° 0′	45° 0′	0° 0′
85 0	24 6	34 30	33 56	+0 34
80 0	23 48	24 0	23 12	+0 48
75 0	23 19	14 30	13 8	+1 22
70 0	22 39	4 30	3 54	+0 36
67 43	22 17	0 0	0 0	0 0
60 0	20 47	12 30	11 41	+0 49
50 0	18 18	24 0	23 30	+0 30

These differences, which at an average amount to 46½′, are also within the limits of the errors of observation.

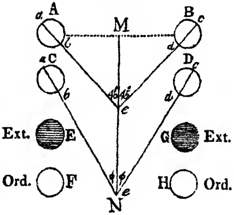
In all these experiments the value of x was 45°; but in order to determine the law of variation for φ, when x varies from 0° to 90°, I took a crystal of quartz with a fine natural surface parallel to its axis; and I found that at an angle of incidence of 75°, and when x was 45°, the inclination of the plane of polarization to the plane of reflexion was 26° 20′. I then varied x, and obtained the following results:

Values of x.	Inclination of Plane of Polarization.		Difference.
	φ Observed.	φ Calculated.	
0°	0° 0′	0° 0′	0° 0′
10	4 54	4 29	+0 25
20	10 0	10 16	−0 16
30	15 50	16 2	−0 12
35	20 0	19 12	+0 48
40	23 30	22 40	+0 50
45	26 20	26 27	−0 7
50	30 0	30 40	−0 40
55	35 30	35 23	+0 7
60	40 0	40 45	−0 45
70	53 0	53 49	−0 49
80	70 0	70 29	−0 29
90	90 0	90 0	0 0

In these experiments the average error does not exceed half a degree. The third column is computed by the formula tan φ = (tan 26° 27′) tan x.

From these experiments it appears that the formula expresses with great accuracy all the changes in the planes of polarization which are produced by a single reflexion, and we may therefore apply it in our future investigations.

Fig. 2.



Let us now suppose that a beam of common light composed of two portions A, B, (Fig. 2.) polarized + 45° and − 45° to the plane of reflexion, is incident on a plate of glass at such an angle that the reflected pencil composed of C and D has its planes of polarization inclined at an angle φ to the plane M N. When a rhomb of calcareous spar has its principal section in the plane M N, it will divide the image C into an extraordinary pencil E and an ordinary one F; and the same will take place with D, G being its extraordinary and H its ordinary image. If we represent the whole of the reflected pencil or C + D by 1, then $C = \frac{1}{2}$, $D = \frac{1}{2}$, $E + F = 1$, and $G + H = 1$. But since the planes of polarization of C and D are each inclined φ degrees to the principal section of the rhomb, the intensity of the light of the doubly refracted pencils will be as $\sin^2 \varphi : \cos^2 \varphi$; that is, the intensity of E will be $\frac{1}{2} \sin^2 \varphi$, and that of F, $\frac{1}{2} \cos^2 \varphi$. Hence it follows that the difference of these pencils, or $\frac{1}{2} \sin^2 \varphi - \frac{1}{2} \cos^2 \varphi$, will express the quantity of light which has passed from the extraordinary image E into the ordinary one F, that is, the quantity of light apparently polarized in the plane of reflexion M N. But as the same is true of the pencil D, we have $2 \left(\frac{1}{2} \sin^2 \varphi - \frac{1}{2} \cos^2 \varphi\right)$ or $\sin^2 \varphi - \cos^2 \varphi$ for the whole of the polarized light in a pencil of common light C + D. Hence, since $\sin^2 \varphi + \cos^2 \varphi = 1$ and $\cos^2 \varphi = 1 - \sin^2 \varphi$, we have for the whole quantity of polarized light

$$Q = 1 - 2 \sin^2 \varphi.$$

But $$\operatorname{Tan} \varphi = \tan x \frac{\cos (i + i')}{\cos (i - i')}$$

And as $$\operatorname{Tan}^2 \varphi = \frac{\sin^2 \varphi}{\cos^2 \varphi}, \text{ and } \sin^2 \varphi + \cos^2 \varphi = 1,$$

we have the quotient and the sum of the quantities $\sin^2\varphi$ and $\cos^2\varphi$, by which we obtain

$$\mathrm{Sin}^2\varphi = \frac{1}{\dfrac{1}{\left(\tan x\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}+1} = \frac{\left(\tan x\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}{1+\left(\tan x\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}$$

That is $$Q = 1 - 2\,\frac{\left(\tan x\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}{1+\left(\tan x\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}$$

As the quantity of reflected light is here supposed to be 1, we may obtain an expression of Q in terms of the incident light by adopting the formula of Fresnel for the intensity of a reflected ray. Thus

$$Q = \tfrac{1}{2}\left(\frac{\sin^2(i-i')}{\sin^2(i+i')}+\frac{\tan^2(i-i')}{\tan^2(i+i')}\right)\left(1-2\,\frac{\left(\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}{1+\left(\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}\right)$$

As $\tan x = 1$ in common light, it is omitted in the preceding formula.

This formula may be adapted to partially polarized rays, that is, to light reflected at any angle different from the angle of maximum polarization, provided we can obtain an expression for the quantity of reflected light.

M. Fresnel's general formula has been adapted to this species of rays, by considering them as consisting of a quantity a of light completely polarized in a plane making the angle x with that of incidence, and of another quantity $1 - a$ in the state of natural light. Upon this principle it becomes

$$I = \frac{\sin^2(i-i')}{\sin^2(i+i')}\cdot\frac{1+a\cos^2 x}{2}+\frac{\tan^2(i-i')}{\tan^2(i-i')}\cdot\frac{1-a\cos^2 x.}{2}$$

But as we have proved that partially polarized rays are rays whose planes of polarization form an angle of $2x$ with one another as already explained, x being greater or less than 45°, we obtain a simpler expression for the intensity of the reflected pencil, viz. the very same as that for polarized light.

$$I = \frac{\sin^2(i-i')}{\sin^2(i+i')}\cos^2 x + \frac{\tan^2(i-i')}{\tan^2(i+i')}\sin^2 x$$

Hence we have

$$Q = \left(\frac{\sin^2(i-i')}{\sin^2(i+i')}\cos^2 x + \frac{\tan^2(i-i')}{\tan^2(i+i')}\sin^2 x\right)\left(1 - 2\,\frac{\left(\tan x\,\frac{\cos(i+i')}{\cos(i-i')}\right)^2}{1+\left(\tan x\,\frac{\cos(i+i')}{\cos(i-i')}\right)^2}\right)$$

This formula is equally applicable to a single pencil of polarized light of the same intensity as the pencil of partially polarized light. In all these cases it expresses the quantity of light really or apparently polarized in the plane of reflexion.

In order to show the quantity of light polarized at different angles of incidence, I have computed the following table for common light, and suited to glass in which $m = 1.525$.

PLATE GLASS.

Angle of Incidence i.	Angle of Refraction i'.	Inclination of Plane of Polarization to Plane of Reflexion, ϕ.	Quantity of Light reflected out of 1000 Rays.	Quantity of Polarized Light Q.	Ratio of Polarized to Reflected Light.
0° 0′	0° 0′	45° 0′	43.23	0	0
10 0	6 32	43 51	43.39	1.74	0.04000
20 0	12 58	40 13	43.41	7.22	0.16618
25 0	16 5	37 21	43.64	11.6	0.26388
30 0	19 8½	33 40	44.78	17.25	0.3853
35 0	22 6	29 8	46.33	24.37	0.5260
40 0	24 56	23 41	49.10	33.25	0.6773
45 0	27 37½	17 22½	53.66	44.09	0.82167
50 0	30 9	10 18	61.36	57.36	0.9360
56 45	33 15	0 0	79.5	79.5	1.000
60 0	34 36	5 4½	93.31	91.6	0.9628
65 0	36 28	12 45	124.86	112.7	0.90258
70 0	38 2	18 32	162.67	129.80	0.79794
75 0	39 18	26 52	257.26	152.34	0.59154
78 0	39 54	30 44	329.95	157.67	0.47786
79 0	40 4	31 59	359.27	157.69	0.43892
80 0	40 13	33 13	391.7	156.6	0.40000
82 44	40 35	36 22	499.44	145.4	0.29112
84 0	40 42	38 2	560.32	134.93	0.2408
85 0	40 47	39 12	616.28	123.75	0.2008
86 0	40 51	40 22.7	676.26	108.67	0.16068
87 0	40 54	41 32	744.11	89.83	0.12072
88 0	40 57½	42 42	819.9	65.9	0.0804
89 0	40 58	43 51	904.81	36.32	0.04014
90 0	40 58	45 0	1000.0	0	0.0000

As the preceding formula is deduced from principles which have been either established by experiment or confirmed by it, it may be expected to harmonize with the results of observation. At all the limits where the pencil is either wholly polarized or not polarized at all, it of course corresponds with experiment: but though in so far as I know there have been no absolute measures taken of the quantity of polarized light at different incidences, yet we are fortunately in possession of a set of experiments by M. Arago, who has ascertained the angles above and below the polarizing angle at which glass and water polarize the same proportion of light. In no case has he measured the absolute quantity of the polarized rays; but the comparison of the values of Q at those angles at which he found them in equal proportions, will afford a test of the accuracy of the formula. This comparison is shown in the following table, in which col. 1. contains the angles at which the reflecting surface polarizes equal proportions of light; col. 2. the values of φ or the inclination of the planes of polarization; and col. 3. the intensities of the polarized light computed from the formula.

	Angles of Incidence i.	Inclination of Planes of Polarization to M N, or φ.	Proportion of Polarized Light or Q.
Glass: No. 1.	82° 48′	37° 33′	.2572
	24 18	37 21	.2637
No. 2.	82 5	36 47	.2828
	26 6	36 0	.3090
No. 3.	78 20	32 38	.4186
	29 42	33 1	.4064
Water: No. 4.	86 31	41 54	.1080
	16 12	41 27	.1236

The agreement of the formula with experiments made with as great accuracy as the subject will admit must be allowed to be very satisfactory. The differences are within the limits of the errors of observation, as appears from the following table:

	Deviations from Experiment.	Part of the whole Light.
Glass: No. 1.	0.0065	$\frac{1}{154}$
No. 2.	0.0262	$\frac{1}{38}$
No. 3.	0.0122	$\frac{1}{82}$
Water: No. 4.	0.0156	$\frac{1}{64}$

M. Arago has concluded, from the experiments above stated, that equal proportions of light are polarized at equal angular distances from the angle of complete polarization. Thus in Glass No. 1. the mean of 82° 48′ and 24° 18′ is 53° 33′, which does not differ widely from the maximum polarizing angle, or 55°, which M. Arago considers as the maximum polarizing angle of the glass *. In order to compare this principle with the formula, I found that in Water No. 4. the angle which polarizes almost exactly the same proportion of light as the angle of 86° 31′, is 15° 10′, the value of φ being 41° 54′ at both these angles; but the mean of these is 50° 50′ in place of 53° 11′; so that the rule of M. Arago cannot be regarded as correct, and cannot therefore be employed, as he proposes, to determine the angle of complete polarization †.

The application of the law of intensity to the phenomena of the polarization of light by successive reflexions, forms a most interesting subject of research. No person, so far as I know, has made a single experiment upon this point, and those which I have recorded in the Philosophical Transactions for 1815, have, I believe, never been repeated. All my fellow labourers, indeed, have overlooked them as insignificant, and have even pronounced the results which flow from them to be chimerical and unfounded. Those immutable truths, however, which rest on experiment, must ultimately have their triumph; and it is with no slight satisfaction, that, after fifteen years of unremitted labour, I am enabled not only to demonstrate the correctness of my former experiments, but to present them as the necessary and calculable results of a general law.

When a pencil of common light has been reflected from a transparent surface, at an angle of 61° 3′ for example, it has experienced such a physical change, that its planes of polarization form an angle of 6° 45′ each with the plane of reflexion. When it is incident on another similar surface at the same angle, it is no longer common light in which $x = 45°$, but it is partially polarized light in which $x = 6° 45'$. In computing therefore the effect of the second reflexion, we must take the general formula $\tan \varphi = \tan x \left(\frac{\cos(i + i')}{\cos(i - i')}\right)$; but, as the value of x is always in the same ratio to the value of φ, however great be the number of reflexions, we have $\tan \theta = \tan^n \varphi$ for the inclination θ to the plane of reflexion produced by any number of reflexions n,

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* Hence we have assumed $m = 1.428$, the tangent of 55°, in the preceding calculations.

† It is obvious that the rule can only be true when $m = 1.000$; so that its error increases with the refractive power.

φ being the inclination for one reflexion. Hence when θ is given by observation, we have $\tan\varphi = \sqrt[n]{\theta}$. The formula for any number n of reflexions is therefore $\tan\theta = \left(\frac{\cos(i+i')}{\cos(i+i)}\right)^n$. It is evident that θ never can become equal to 0°; that is, that the pencil cannot be so completely polarized by any number of reflexions at angles different from the polarizing angle, as it is by a single reflexion at the polarizing angle; but we shall see that the polarization is sensibly complete in consequence of the near approximation of θ to 0°.

I found, for example, that light was polarized by two reflexions from glass at an angle of 61° 3′, and 60° 28′ by another observation. Now in these cases we have

	θ after 1st Reflexion.	θ after 2nd Reflexion.	Quantity of Unpolarized Light.
Two reflexions at 61° 3′	6° 45′	0° 47′	0.00037
60 28	5 38	0 33	0.00018

The quantity of unpolarized light is here so small as to be quite inappreciable with ordinary lights.

In like manner I found that light was completely polarized by five reflexions at 70°. Hence by the formula we have

	Values of θ.	Unpolarized Light.
1 reflexion at 70°	20° 0′	0.23392
2	7 32	0.03432
3	2 45	0.00460
4	1 0	0.00060
5	0 22	0.00008

The quantity of unpolarized light is here also unappreciable after the 5th reflexion.

In another experiment I found that light was wholly polarized by the separating surface of glass and water at the following angles:

	Values of θ.	Unpolarized Light.
By 2 reflexions at 44° 51′	0° 56′	0.0005
By 3 . . . . . . 42 27	0 26	0.0001

In all these cases the successive reflexions were made at the same angle; but the formula is equally applicable to reflexions at different angles,—

1. When both the angles are greater than the polarizing angle.

	θ	Unpolarized Light.
1 reflexion at 58° 2′, and 1 at 67° 2′	0° 34′	0.0002

2. When one of the angles is above and the other below the polarizing angle.

	θ	Unpolarized Light.
1 reflexion at 53°, and 1 at 58° 2′ . .	0° 12′ . .	0.000024

This experiment requires a very intense light, for I find in my journal that the light of a candle is polarized at 53° and 78°.

In reflexions at different angles, the formula becomes $\tan\theta = \frac{\cos(i+i')}{\cos(i-i')} \times \frac{\cos(I+I')}{\cos(I+I')}$, I and i being the angles of incidence. In like manner if a, b, c, d, e, &c. are the values of φ or θ for each reflexion, or rather for each angle of incidence, we shall have the final angle or $\tan\theta = \tan a \times \tan b \times \tan c \times \tan d$, &c.

It is scarcely necessary to inform the reader that when a pencil of light reflected at 58° 2′ is said to be polarized by another reflexion at 67° 2′, it only means, that this is the angle at which complete polarization takes place in diminishing the angle gradually from 90° to 67° 2′, and that even this angle of 67° 2′ will vary with the intensity of the original pencil, with the opening of the pupil, and with the sensibility of the retina. But when it shall be determined experimentally at what value of φ, or rather at what value of Q, the light entirely disappears from the extraordinary image, we shall be able by inverting the formula to ascertain the exact number of reflexions by which a given pencil of light shall be wholly polarized.

As the value of Q depends on the relation of i and i', that is, on the index of refraction, and as this index varies for the different colours of the spectrum, it is obvious that Q will have different values for these different colours. The consequence of this must be, that in bodies of high dispersive powers, the unpolarized light which remains in the extraordinary image, and also the light which forms the ordinary image, must be coloured at all incidences; the colours being most distinct near the maximum polarizing angle. This necessary result of the formula, I found to be experimentally true in oil of cassia, and various highly dispersive bodies. In realgar for example φ is $= 0$ at an angle of 69° 0′ for blue light, at 68° 37′ for green light, and at 66° 49′ for red light. Hence there can be no angle of complete polarization for white light, which I also found to be the case by experiment; and as Q must at different angles of incidence have different values for the different rays, the unpolarized light must be composed of a certain portion of each different colour, which may be easily determined by the formula.

Such are the laws which regulate the polarization of light by reflexion from the first surfaces of bodies that are not metallic. The very same laws are applicable to their second surfaces, provided that the incident light has not suffered previous or subsequent refraction from the first surface. The sine of the angle at which φ or Q has a certain value by reflexion from the second surface, is to the sine of the angle at which they have the same value at the first surface, as unity is to the index of refraction. Hence φ and Q may be determined by the preceding formulæ after any number of reflexions, even if some of the reflexions are made from the first surface of one body and the second surface of another.

When the second surface is that of a plate with parallel or inclined faces, its action upon light presents curious phenomena, the law of which I have determined. I refer of course to the action of the second surface at angles less than that which produces total reflexion. This action has hitherto remained uninvestigated. It has been hastily inferred, however, from imperfect data; and the erroneous inference forms the basis of some optical laws, which are considered to be fully established.

Among the various results of the preceding investigation, there is one which seems to possess some theoretical importance. If we consider polarized rays as those whose planes of polarization are parallel, then it follows that light cannot be brought into such a state by any number of reflexions, or at any angle of incidence, excepting at the angle of complete polarization. At all other angles the light which seems to be polarized, by disappearing from the extraordinary image of the analysing rhomb, is distinguished from really polarized light, by the property of its planes of polarization forming an angle with each other and with the plane of reflexion. At the polarizing angle, for example, of 56° 45′ in glass, the light reflected is 79.5 rays, and it is completely polarized, because the planes of polarization of all the rays are parallel; but at an angle of incidence of 80°, where 392 rays are reflected, no fewer than 157 appear to be polarized, though their planes of polarization are inclined 66° 26′ to each other, or 33° 13′ to the plane of reflexion. This appearance of polarization, when the rays have only suffered a displacement in their planes of polarization from an angle of 90°, which approximates them to the state of polarized light, arises from the law which regulates the repartition of polarized light between the ordinary and extraordinary images produced by double refraction, and shows that the analysing crystal is not sufficient to distinguish light completely polarized from light in a state of approach to polarization. The difference, however, between these two kinds of light is marked by most distinctive characters, and will be found to show itself in some of the more complex phenomena of interference.

In my paper of 1815, already referred to, I was led by a distant view of the phenomena which I have now developed, to consider common light as composed of rays in every state of positive and negative polarization*; and upon this principle the whole of the phenomena described in this paper may be calculated with the same exactness as upon the supposition of two oppositely polarized pencils. Nothing indeed can be simpler than such a principle. The particles of light have planes, which are acted upon by the attractive and repulsive forces residing in solid bodies; and as these planes must have every possible inclination to a plane passing through the direction of their motion. one half of them will be inclined — to this plane, and the other half +. When light in such a state falls upon a reflecting surface, the — and the + particles have each their planes of polarization brought more or less into a state of parallelism with the plane of reflexion, in consequence of the action of the repulsive force upon one side or pole of the particle through which the plane passes; while in the particles which suffer refraction, the same sides or poles are by the action of the attractive force drawn downwards, so as to increase the inclination of their planes relative to the plane of incidence, and bring them more or less into a state of parallelism with a plane perpendicular to that of refraction.

The formulæ already given, and those for refracted light which are contained in another paper, represent the laws according to which the repulsive and attractive forces change the position of the planes of polarization; and as we have proved that the polarization is the necessary consequence of these planes being brought into certain positions, we may regard all the various phenomena of the polarization of light by reflexion and refraction, as brought under the dominion of laws as well determined as those which regulate the motions of the planets.

Allerly, December 25, 1829.

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* M. Biot has followed me in this opinion. See *Traité de Physique*, tom. iv. p. 304.

IV. *A Report on the stomach of the Zariffa. By Sir* EVERARD HOME, *Bart., Vice-President of the Royal Society.*

Read December 24, 1829.

HAVING submitted to the KING the following report upon the stomach of the Zariffa, I am sanctioned by HIS MAJESTY's entire approbation in laying it before the Royal Society.

The stomach consists of four cavities, like those of all quadrupeds that chew the cud. The internal surface of the paunch differs in nothing from that of the bullock, but in the projecting parts being more prominent. In the second cavity the cells met with in other ruminants are too superficial to retain water. The third and fourth cavities in every respect resemble those of the bullock. The more minute structures of these parts are distinctly shown in the annexed Plates.

As the only peculiarities in the Zariffa's stomach are in the second cavity, comparative views are given of this cavity with that in the bullock and sheep.

In considering the structure of the different parts of the stomach of ruminating animals, there can be no doubt that any peculiarity met with in particular tribes is to serve the purpose of enabling them more readily to subsist upon the food which is intended by nature for their use. Of this the reservoirs for water in the stomach of the camel are a remarkable instance. In the bullock and sheep the cud formed of grass in its return after the second mastication into the stomach is rendered dry, and when it drops into the second cavity requires being re-moistened before it can be spread between the membranous folds of the third cavity; this is effected by the water contained in the cells with which this cavity is furnished. In the Zariffa the cud formed from the leaves and twigs of the acacia is so succulent as not to require being again moistened in passing through the second stomach, and therefore that cavity in this animal is not furnished with cells of the same depth as in the other animals that chew the cud.

Explanation of the PLATES.

PLATE VI.

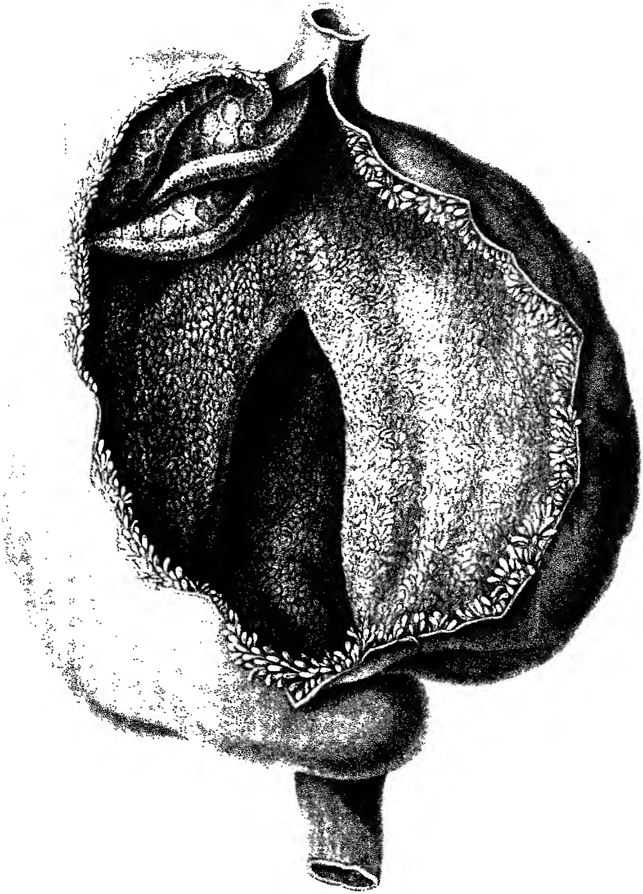
A front view of the stomach of the zariffa, on a reduced scale of 4 inches for a foot.

PLATE VII.

A back view of the same.

PLATE VIII.

Fig. 1.—A portion of the first cavity of the zariffa's stomach; natural size.
Fig. 2.—A very small portion of the same; magnified 4 diameters.
Fig. 3.—A portion of the second cavity of the zariffa's stomach; natural size.
Fig. 4.—A very small portion of the same; magnified 4 diameters.
Fig. 5.—A portion of the third cavity of the zariffa's stomach; natural size.
Fig. 6.—A very small portion of the same; magnified 4 diameters.
Fig. 7.—A portion of the first cavity of the bullock's stomach; natural size.
Fig. 8.—A very small portion of the same; magnified 4 diameters.
Fig. 9.—A portion of the second cavity of the bullock's stomach; natural size.
Fig. 10.—A portion of the first cavity of the sheep's stomach; natural size.
Fig. 11.—A very small portion of the same; magnified 4 diameters.
Fig. 12.—A portion of the second cavity of the sheep's stomach, in which the cells are deeper than in the zariffa's stomach, but less so than in the bullock's stomach; natural size.



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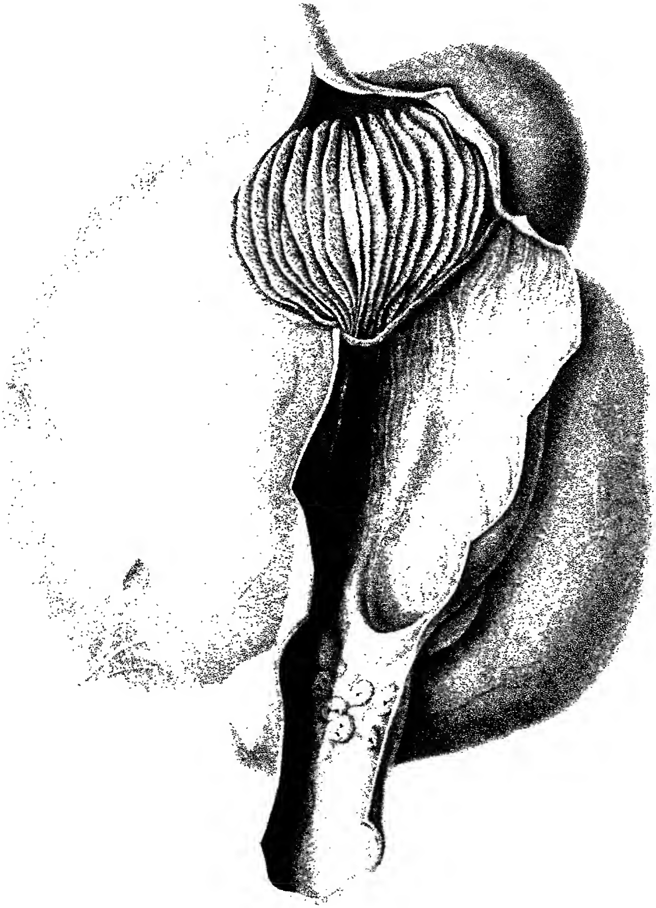

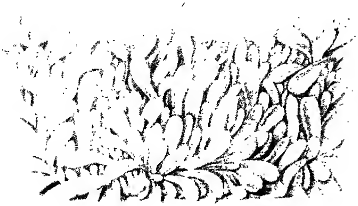

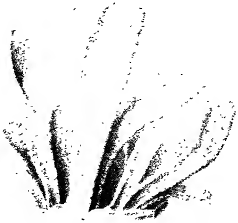

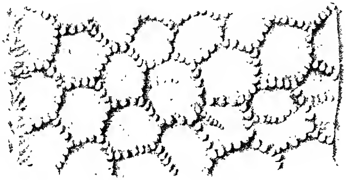

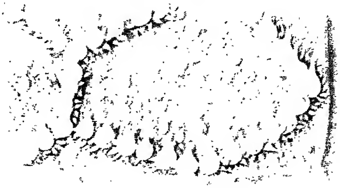

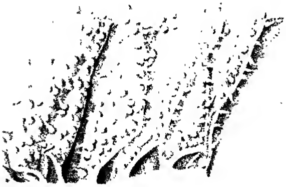

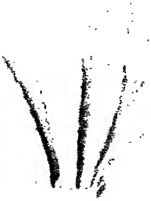

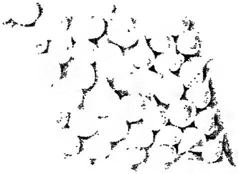

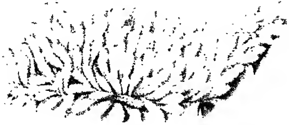

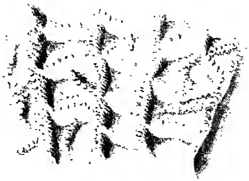

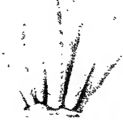

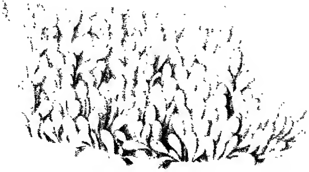

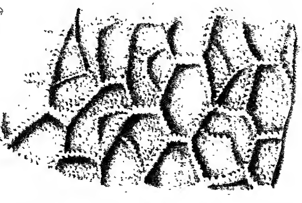

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V. *On the production of regular double refraction in the molecules of bodies by simple pressure; with observations on the origin of the doubly refracting structure.* *By* David Brewster, *LL.D. F.R.S. L. & E.*

Read February 11, 1830.

In various papers already printed in the Philosophical Transactions, I have had occasion to show that the phenomena of double refraction may be produced artificially by certain changes in the mechanical condition of hard and soft solids*. In all these cases the phenomena are related to the form of the mass in which the change is induced; and in the case of hard and elastic solids, they vary with any variation of form which alters the mechanical state of the particles. In isinglass and other bodies to which double refraction has been communicated by induration, the particles take a permanent position, which is not altered by any change of shape; but still the phenomena exhibited by a given portion of the mass are related to the surfaces where the indurating cause operated, and also to those by which the isinglass was bounded; and they depend on the position which that portion occupies in the general mass.

In all these cases the phenomena are entirely different from those of regular crystals, and in none of them is the doubly refracting force a function of the angle which the incident ray forms with one or more axes given in position.

As long ago as 1814 I communicated to the Royal Society the following experiment on the depolarizing structure of white wax and resin:

"When resin is mixed with an equal part of white wax, and is pressed between two plates of glass by the heat of the hand, the film is almost perfectly transparent by transmitted light, though of a milky white appearance by reflected light. It has not the property of depolarization when the light is incident vertically; but it possesses it in a very perfect manner at an oblique incidence, and exhibits the segments of coloured rings†".

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* Phil. Trans. 1814; 1815, pp. 1, 30, 60; 1816, pp. 46, 56.

† Ibid. 1815, pp. 31, 32.

The subject of double refraction was then so little developed that this experiment excited no notice; and it was only brought to my own recollection by the accidental appearance of the specimen itself. This depolarizing film has suffered no change by remaining fifteen years between the plates of glass. The vertical line along which it is destitute of the property of depolarization is a single axis of double refraction; and the coloured rings at oblique incidences are produced by the inclination of the refracted ray to the axis of double refraction. In order to examine this remarkable effect under a more general aspect, I made a considerable number of such plates with different kinds of wax, and with various proportions of resin, and I was led to results which seem to possess considerable interest.

When the white wax is melted alone and cooled between two plates of glass, it consists of a number of minute particles, each possessing double refraction, but having their axes turned in all possible directions. If the film of wax is made extremely thin, the particles are not sufficiently numerous to exhibit any action upon polarized light.

When resin alone is melted and cooled in a similar manner, it exhibits no doubly refracting structure, whether it indurates slowly or under the influence of pressure.

If resin and white wax are mixed in nearly equal proportions, the compound possesses considerable tenacity. When a proportion of it is melted and cooled between two plates of glass, it shows the quaquaversus polarization of bees'-wax, the axes of the elementary particles being turned in every direction. It possesses a considerable degree of opalescence, and a luminous body seen through it is surrounded with nebulous light. This imperfect transparency evidently arises from the reflexion and refraction of the rays in passing from one molecule to another, occasioned by a difference in the refractive power of the ingredients, or by the imperfect contact of the particles, or by both these causes combined.

In order to observe the modifications which these phenomena received from pressure, I took a few drops of the melted compound and placed them in succession on a plate of thick glass, so as to form a large drop. Before it was cold, I laid above the drop a circular piece of glass about two thirds of an inch in diameter, and by a strong vertical pressure on the centre of the piece of glass, I squeezed out the drop into a thin plate. This plate was now almost perfectly transparent, as if the pressure had brought the particles of the substance into optical contact.

If we expose this plate to polarized light, we shall find that it possesses one axis of positive double refraction, and exhibits the polarized tints as perfectly as many crystals of the mineral kingdom. The structure thus communicated to the soft film by pressure does not belong to it as a whole, nor has it only one axis passing through its centre like a circular piece of unannealed glass. In every point of it there is an axis of double refraction perpendicular to the film, and the doubly refracting force varies with the inclination of the incident ray to this axis, as in all regular uniaxal crystals. When the two plates of glass are drawn asunder, we can remove one or more portions of the compressed film, and these portions act upon light exactly like films of uniaxal mica or hydrate of magnesia, and develope a doubly refracting force of equal intensity.

This remarkable experiment presents an interesting subject of inquiry. That the regular double refraction of the film is developed by the agency of pressure cannot be doubted; but it does not at first sight appear whether it is the immediate effect of the pressure, or is the same doubly refracting force which produces the quaquaversus polarization that takes place when the resinous film indurates without constraint. In this state of the film the axes of double refraction are clearly turned in every conceivable direction; and it is impossible to suppose that a pressure in one direction could suddenly arrange all these axes in parallel positions. The double refraction of each particle of the film has therefore been developed by the compressing force similarly applied to them: and in producing this effect, it must have deprived each particle of the doubly refracting structure which it previously possessed. The substitution of one doubly refracting structure for another may be easily effected in many bodies. Even in regular crystals we can by heat or pressure modify or remove their double refraction. Nay, we can take away one axis from a biaxal crystal, and communicate a second axis to an uniaxal one. When the doubly refracting structure is produced by induration, we can remove it wholly by pressure, and replace it with another even of an opposite character; and when it is generated by the living principle, as in the case of the crystalline lenses of animals, we can take it away entirely, and substitute a new and more powerful doubly refracting structure by induration.

We may therefore consider it as clearly established that the uniaxal double refraction of the resinous mass has been communicated to the individual molecules by simple pressure; the increased transparency arising from the molecules being brought into closer contact, and the regular double refraction from the variable density impressed upon each elastic molecule, and symmetrically related to the axis of pressure. The effect thus produced on the resinous mass is precisely the same as what would take place by subjecting elastic spheres to a regular compressing force. The axis of pressure becomes an axis of positive double refraction, and the double refraction increases with the inclination of the ray to the axis, and becomes a maximum in the equator of the molecules.

By this view of the preceding facts, we are led to a very simple explanation of the origin and general phenomena of double refraction in regular crystals. That this property is not inherent in the molecules themselves may be easily proved. The particles of silex, for example, do not possess it in their separate state. In tabasheer, in many opals, and in melted quartz, there is not the slightest trace of the doubly refracting structure: but when the particles of silex in solution are allowed to combine, in virtue of their polarities or mutual affinities, they then instantly acquire, at the moment of their combination, the property of double refraction, and they retain it while they continue in this state of aggregation. The manner in which this takes place may be easily conceived: a number of elastic molecules existing in a state of solution, or in a state of fusion, are kept at such a distance by the fluid in the one case, and by the heat in the other, as to preclude the operation of their mutual affinities; but when, in the process of evaporation or cooling, any two molecules are brought together by the forces or polarities which produce a crystalline arrangement, and strongly adhere, they will mutually compress one another, and each will have an axis of double refraction in the directions of the line joining their centres, in the same manner as if they had been compressed by an external force.

From the phenomena of crystallization and cleavage, it is obvious that the molecules of crystals have several axes of attraction, or lines along which they are most powerfully attracted, and in the direction of which they cohere with different degrees of force. Guided by the indications of hemitrope forms, and supposing the molecules to be spherical or spheroidal, we infer that their axes are three in number and at right angles to each other, and are related in position to the geometrical axis of the primitive form. In like manner the phenomena of double refraction are related to the same axis of the primitive form, and may be all rigorously calculated by a reference to three rectangular axes. In uniaxal crystals, the three axes A, B, C must be such that two of them are equal and of the same name; while the third, corresponding with the apparent axis, may be of the same or of a different name. In biaxal crystals, the three axes A, B, C are unequal, and in crystals with no double refraction the axes are equal and destroy each other*.

This approximation of these two classes of facts is too remarkable to be accidental, and would go far to establish their dependence, even if it were not indicated by other arguments which I shall proceed to illustrate.

Among those crystals which have the obtuse rhomboid for their primitive form, there are many with one axis of negative double refraction, and only one or two with one axis of positive double refraction. In the former, the negative doubly refracting structure will be produced round the axis of the rhombohedron by the compression arising from attractions in the direction of two equal rectangular axes A, B, which will dilate the molecules in the direction of the third axis C, and make it a negative axis of double refraction, equal in intensity to either of the other two. Here we require the combination only of two axes; but if we suppose that there is in the direction of C a third axis of attraction either more or less powerful than the other two, then if it is less powerful, the compression of the molecules produced by it will diminish the dilatation arising from the united action of A and B, but will still leave an unbalanced dilatation, or a single negative axis of double refraction in the axis of the rhomb.

If C, on the contrary, is an axis in which the attractive force of the molecules is greater than along A and B, the compression which it produces will exceed the dilatation arising from A and B, and we shall have an axis of compression along C, or an axis of positive double refraction as in quartz and dioptase*. The same observations are applicable to minerals that crystallize in the pyramidal form.

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* In uniaxal crystals, the resultant of the two equal axes A, B may have any relation to C but that of equality; excepting when C is of a different name from A and B.

In biaxal crystals, any two axes A, B, may be converted into three A + C, B ± C, ± C. See Phil. Trans. 1818.

When the three axes A, B, C are all equal, the three rectangular compressions, produced by the aggregation of the molecules, will destroy one another at every point of the molecule, and the body which they compose will have no double refraction, and cleavages of equal facility. Hence all crystals in which it is known by cleavage that the particles cohere with equal force in three rectangular directions have actually no double refraction.

If the three attractive axes A, B, C are all unequal, the difference of density which they produce in the molecules will be related to two axes of double refraction, the strongest of which will be negative or positive according as the compression along C is less or greater than the dilatation produced along C by the united compressions of A and B. Hence all crystals belonging to the prismatic system, in which we are informed by cleavage that the particles cohere with unequal forces in three directions, have invariably two, or, as we have already explained, three unequal axes of double refraction, of which the strongest is sometimes positive and sometimes negative.

We have supposed the elementary molecules of bodies to be spherical when existing singly, or beyond the sphere of their mutual action; but although their form must, in the case of doubly refracting crystals, be changed into oblate, prolate or compound spheroids, yet the deviation of these spheroids from the sphere may be so small, that the forms of the bodies which they compose may be regarded as arising from the union of spherical molecules. It is more probable, however, that the form of the molecules suffers a considerable change, and we may consider that change as determining the exact primitive form of the crystal and the inclination of its planes.

The circumstance of almost all rhombohedral crystals having negative double refraction, which can only be produced by axes of compression in the equator of a prolate spheroid, excludes the supposition, that the ultimate molecules are spherical particles converted by the forces which unite them into those oblate and prolate spheroids, by means of which, according to the views of HUYGENS, all the varieties of rhombohedrons may be formed*; for if this were the case, the obtuse rhombohedrons should possess one positive axis, and the acute ones one negative axis of double refraction. We are constrained therefore to suppose that in rhombohedral crystals the molecules have the form of an oblate spheroid, with its axes so related, that the change superinduced upon it by the forces of aggregation determines the exact form of the combination. In carbonate of lime for example, where the precise inclination of the faces of the rhombohedron can be produced only by oblate spheroids whose polar is to their equatorial axis as 1 to 2.8204, we may suppose that the spheroids were originally more oblate, and that the forces by which they receive the doubly refracting structure dilated them in the direction of the smaller axis, so as to produce a spheroid having its axis as 1 to 2.8204. Hence if we could suppose the molecules placed together without any forces which would alter their form, they would compose a rhombohedron with a greater angle and having no double refraction. But when they are combined by the attractive forces of crystallization, they compose a rhombohedron of 105°, possessing negative double refraction.

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* Since this paper was written, I have seen the very valuable researches of M. Savart on the structure of crystallized bodies as developed by sonorous vibrations. The curious result of his experiments, that the axis of calcareous spar, a negative axis of double refraction, is the axis of least elasticity, while the axis of quartz, an axis of positive double refraction, is the axis of greatest elasticity, harmonizes in a remarkable manner with the above views.

In this view of the subject, the form of the ultimate molecules of crystals existing separately, may be regarded as determining within certain limits the primitive form to which they belong; while the doubly refracting structure and the precise form of the crystal are simultaneously produced by the action of the forces of aggregation.

These views receive a remarkable illustration from a new doubly refracting structure, which I discovered many years ago in chabasie, and which will form the subject of a separate communication. In certain specimens of this mineral, the molecules compose a regular central crystal, developing the phenomena of regular double refraction; but in consequence of some change in the state of the solution, the molecules not only begin to form a hemitrope crystal on all the sides of the central nucleus, but each successive stratum has an inferior doubly refracting force till it wholly disappears. Beyond this limit it reappears with an opposite character, and gradually increases till the crystal is complete. In this case the relative intensities of the axes or poles from which the forces of aggregation emanate, have been gradually changed, probably by the introduction of some minute matter, which chemical analysis may be unable to detect. If we suppose these axes to be three, and the foreign particles to be introduced, so as to weaken the force of aggregation of the greater axis, then the doubly refracting force will gradually diminish with the intensity of this axis, till it disappears, when the three axes are reduced to equality. By continuing to diminish the force of the third axis, the doubly refracting force will reappear with an opposite character, exactly as it does in the chabasie under consideration.

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* See HUYGENS's Traité de la Lumière, chap. v. and the Edinburgh Journal of Science, No. xviii. pp. 311, 314.

From the mutual dependence of the forces of aggregation and double refraction, it is easy to understand the influence which heat produces on the doubly refracting structure, as exhibited in the phenomena discovered by M. MITSCHERLICH in sulphate of lime and calcareous spar, and in those which I detected in glauberite*. This eminent philosopher has found, by direct experiment, that heat expands a rhomb of calcareous spar in the direction of its axis, and contracts it in directions at right angles to that axis†; that the rhomb thus becomes less obtuse, approaching to the cubical forms which have three equal axes, and that its double refraction diminishes. All these effects are the necessary consequences of the preceding views. The expansion in the direction of the axis, and the contraction of all the equatorial diameters diminish the compression of the axes of the oblate spheroidal molecules, and must therefore diminish its double refraction, as well as the inclination of the faces of the rhomb. In like manner it will be found that in sulphate of lime and glauberite the expansions and contractions will be so related to the three axes, as to explain the conversion of the biaxal into the uniaxal structure, and the subsequent reappearance of the biaxal structure in a plane at right angles to that in which the axes are found at ordinary temperatures.

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* See Edinburgh Transactions, vol. xi.

† It follows from this fact, that massive carbonate of lime, in which the axes of the molecules have every possible direction, should neither expand nor contract by heat, and would therefore form an invariable pendulum. As there must be, in any given length of massive carbonate of lime, as many expanding as there are contracting axes, then, if the contractions and expansions in each individual crystal are equal, they will destroy one another; but if they are proportional to their lengths, the contractions will exceed the dilatations. In this case, we have only to combine the marble with an ordinary expanding substance, to have an invariable pendulum. The balances of chronometers might be thus made of mineral bodies.

The phenomena exhibited by fluids under the influence of heat and pressure, and those of doubly refracting crystals, exposed to compressing or dilating forces, are in perfect conformity with the above views; so that even without the fundamental experiment described in this paper, we might have been entitled to conclude that the forces of double refraction are not resident in the molecules themselves, but are the immediate result of those mechanical forces by which these molecules constitute solid bodies.

Allerly, October 5th, 1829.

VI. *Experiments on the influence of the Aurora Borealis on the magnetic needle. By the Reverend* James Farquharson, *F.R.S. Minister of Alford, Aberdeenshire. In letters addressed to Captain* Edward Sabine, *Secretary of the Royal Society.*

Read January 28, March 4, and April 1, 1830.

Alford, December 15*th*, 1829.—THE apparatus, belonging to the Royal Society, with which these experiments were made, consists of a horizontal brass circle, about one foot in diameter, graduated to divisions of 10 minutes, and capable of adjustment to a perfect level by means of spirit levels and screwed feet. Concentrically within this divided circle moves a circular horizontal brass plate, its edge touching the divisions, and having at opposite points two verniers, which, by means of attached microscopes, indicate the movements which it makes to 60th parts of 10 minutes, or 10″. The movement of the plate within the circle is effected by means of a screw. A circular brass needle-box is attached to the surface of the inner plate, and a vertical pointed steel wire for supporting the needle forms the centre. At opposite points in the needle-box are fixed two micrometers with cross wires in the foci, for adjusting the needle to a level, and observing any change in its direction. The top of the needle-box is a circular plate of ground glass in a brass ring, made to slip easily off and on, and having screwed into its centre a vertical brass tube about 8 inches long, for the purpose of suspending the needle with fibres of silk, for measuring the time of its oscillations. A horizontal brass pin, with a minute perforation for the silk near its middle, passes through the vertical tube near its top, and being contrived with several motions, serves to adjust the suspended needle, and bring it correctly over the steel point, where its levelling can be completely ascertained.

The magnetic needlec itself is a rectangular plate about 5 inches long, half an inch broad, and $\frac{1}{40}$th of an inch thick. An agate cup set in brass admits of being screwed in either at the narrow or flat side of the needle; and a little fixt ring of brass, with a minute perforation in its top, rising over the cup, admits of the ready attachment of the silk; so that the needle can be placed on the steel point or suspended with the silk, with its flat face either vertical or horizontal.

This apparatus measures, with great accuracy, very minute changes in the declination of the needle. A change as small as 10″ is quite sensible by it. It was placed on a firm-set table, in a room of my house, on the 21st of September last; and by shifting the whole apparatus, the cross wires of the micrometers were brought into a line with the needle, when at rest at 8 o'clock in the evening; the index of the vernier being at the same time brought to zero on the divided circle. The readings of the variations of the needle are therefore reckoned from its position at that hour, which was made choice of as being that when the Aurora borealis was most likely to appear, and thus the diurnal variation would be brought to interfere as little as possible with any variation induced by that meteor.

The needle, where it is stationed, is subjected to some influence from several fixt pieces of iron, but not to any from iron that is moved from place to place. There was a necessity for removing the apparatus temporarily from its station on the 24th of November; but it was carefully replaced on the 2nd of December; marks having been made in the floor for the feet of the table, and on the table for the feet of the apparatus; and the cross wires brought to the line of the needle at 8 P. M. as before.

In making observations on the intensity, the time occupied by a certain number of horizontal vibrations of the suspended needle is measured by a stop-watch, the character of which it is necessary to describe. It is a time-keeper, on which, indeed, not much reliance ought to be placed, if it were necessary to have the intervals estimated in absolute mean time; as, although it is adjusted to return nearly to mean time at the end of every twenty-four hours, when only wound up once during that period, yet it goes very unsteadily at many of the intermediate hours. I have found, however, that it keeps time nearly with a well-regulated pendulum clock, from 12 minutes after it is wound up till about an hour after that time. It is therefore always prepared for the observations by being stopped 12 minutes after it is wound up; and thus, although the intensities measured by it could not very safely be compared with those measured by a more correct watch, yet considerable reliance may be placed on the comparison of measures thus made by itself at different times. I have in fact already discovered, by using these precautions, such an increase of intensity as was to be anticipated during the cold season now come on. My observations have been too desultory, owing to numerous other avocations, to permit me to look out for a diurnal change of intensity.

The balance of the watch makes 290 vibrations in a minute, so that the vibrations are not commensurate to the seconds; recourse therefore must be had to reading off the time by approximation on the seconds dial-plate. The time is estimated to the nearest quarter of a second by means of a microscope, and the watch relieved from the point where it stops for a new observation; thus neutralizing any incorrectness in the reading off, and any inequality in the divisions of the dial-plate on the principle of the repeating circle. The stop is on the balance of the watch, and is therefore instantaneous.

After trials of various arcs, I have found it most convenient, for insuring that the different series of observations shall commence in equal arcs, to make the needle vibrate by its own breadth at the extremity, that is, in arcs of nearly 12°, which is correctly determined when the corners of the needle come alternately to the cross wires of the micrometer. The watch is relieved the instant the centre of the needle comes to the cross wires the third time after the extent of the arc has been noted by the corners; 50 oscillations are then reckoned, and the watch stopped for reading off the time. A very small piece of iron is employed for moving the needle, which is instantly deposited at some distance at right angles to it; and for night observations my candlestick is of glass.

I had no leisure for making observations on the intensity for several days previous to the 14th of December. The last I made were on the 2nd, 3rd, and 4th of December,—4 sets of 50 oscillations each day at 8 P. M.; average of each set, and of the whole, in 225″.25: Therm. from 35° to 42°.

I shall now copy from my journal the observations of the 14th of December. They were set down at the time, at intervals of a quarter of an hour.

$5\frac{3}{4}$ P. M.—Aurora seen in various quarters of the sky.

6 P. M.— Arch of nebulous light in N. about 25° high; another S. about 30° high; vertex of each about the mag. mer.: S. arch just over a continued line of clouds. Many detached clouds in region of N. arch.

Sky between these quite clear of clouds, with many brilliant streamers of Aurora, forming evidently parts of two fringes coming towards the zenith from N. Bar. 30°, Therm. 39° ; nearly calm.

6¼ P. M.—Many groups of very brilliant short streamers now passing into the zenith. Needle on steel point 2′ W. Intensity, 1st set of 50 oscill. in 225″.25 ; 2nd in 225″.25 ; 3rd in 225″.

6½.—Narrow arch near the zenith, very faint, and small clouds appearing there. Arch in the S. descending to the horizon, and clouds descending with it. That in the N. now about 35° high, accompanied by many clouds. Needle on steel point shifted to 5′ E.

6¾.—Needle on steel point now 23′ E. S. arch now extinct. N. one about 45° high ; nebulous lights under it, and clouds rising with it.

7 P. M.—Needle on steel point slowly returning westward.

7¼.—Needle at zero.

7½.—Needle 4′ W. Aurora still forming an arch at N. ; but not advancing higher than 45°.

7¾.—Needle still 4′ W. Several arches of nebulous light in N. under 45°, with many clouds ; gentle gale a point or two S. of W.

8 P. M.—Arch of nebulous light again formed at S. about 45° high. W. and middle parts of N. arches expiring as they reach 45° ; but their E. ends going now much further S. and passing the prime vertical to the mag. meridian ; not fully seen, however, for clouds, which are following the Aurora in all its flittings ; the rest of the heavens being quite clear. Needle on steel point now 7′ 50″ E. Intensity, 1st in 225″.25 ; 2nd in 225″.25. Therm. 38°.

8¼.—Needle on steel point 13′ 40″ E.

8½.—Needle on steel point 8′ 50″ E. Intensity, 1st in 225″.25 ; 2nd in 225″.25. Nebulous light at N.W. and very brilliant groups of streamers at N.E. Aurora in the S. now extinct.

8¾.—Aurora fading ; needle on steel point 4′ 20″ E.

9 P. M.—Intensity, 1st in 225″.25 ; 2nd in 225″.25. Needle on steel point 2′ E.

9¼.—Three arches of Aurora of unusual brilliancy at mag. N. coming up rapidly with most splendid streamers. Fragments of the uppermost soon passed the zenith, and needle shifted slowly to 19′ W.

9½ P. M.—Very obscure narrow nebulous arches across the zenith, and a little S. of it at right angles to mag. mer.; being the remains of some of the arches seen at 9¼. Needle shifted to 9′ 50″ E.

9¾.—Only a few streamers near N. mag. mer. 25° high. Needle now 12′ E.

10 P. M.—Very brilliant arch of streamers in N. about 25° high; and narrow obscure arch across mag. mer. a little S. of zenith. Needle 14′ E. Intensity, 1st in 225″.25; 2nd in 225″.25. Therm. 37°. No clouds.

10¼.—Arch of streamers in N. about 25° high, not very brilliant. Many groups of faint streamers higher up; and nebulous narrow belts of pale light across the zenith and southward of it a little, at right angles to mag. mer. These are all expiring as they go S. in succession. Needle shifted as I was watching it from 14′ E. to 3′ W. in the space of about 6 minutes.

10½.—Many groups of very pale streamers over all the northern half of the sky, and the western half of a very pale zenith arch has passed to about 20° S. of the zenith. Needle now 21′ 30″ W. Intensity, 1st in 225″.5; 2nd in 225″.75. Therm. 35°.

10¾.—Aurora nearly faded every where, and heavy clouds forming in N.W. Needle returned to 3′ W.

11 P. M.—Aurora extinct. Needle at 5′ W.

12 P. M. Nebulous light at the horizon in mag. N. has continued for nearly an hour, fading at 10° high, and rising again from the hills. Needle 1′ E. and has remained so for three quarters of an hour. Many falling stars whose paths are parallel to the lines of the streamers, or at right angles to them in the planes of the fringes. Intensity, 1st in 225″.5; 2nd in 225″.25. Breeze a point or two S. of W.

December 15, 1 P. M.—Intensity, 1st in 225″.25; 2nd in 225″.5. Therm. 34°.

It ought to have been stated, that from noon on the 10th of December till the evening of the 14th, there was a continued series of the hardest gales at S.W. and W. that have occurred this autumn.

It is obvious from the above detail, that the disturbance of the magnetic declination by the northern lights on the evening of the 14th of December was so great, and so frequently reversed from E. to W., and the contrary, as to leave no shadow of doubt regarding the reality of the phenomenon. It is to be observed, however, that no disturbance took place till the fringes of the Aurora had gone so far south as to place the needle in their planes, or in the line of prolongation of the middle or most elevated streamers of the fringes, which is that of the dipping needle. The numerous observations formerly made by me this autumn, of which I have already put you in possession*, and those also now detailed, afford a proof,—a negative one indeed, and therefore imperfect, but now very extensive,—that the needle is not affected by the Aurora, till it comes into the plane of the dip: and we have thus, I trust, an explanation of the discordant results obtained and announced regarding this matter by different former observers. When the disturbance has been observed, there have been no doubt zones of Aurora in the plane of the dip, though perhaps obscure and liable to be overlooked. Most of those I observed in that position on the 14th were so faint, that I might not have noticed some of them, had I not traced their progress southward from stations where they were much more brilliant.

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* 10th of October.—11 A. M. Series of 50 oscillations: 1st in 227″.75; 2nd in 227″.5; 3rd in 227″.2; 4th in 227″.5; 5th in 227″.75; 6th in 227″.5.

2 P. M.—1st in 227″.5; 2nd in 227″.5.

4 P. M.—1st in 227″.75; 2nd in 227″.5.

11th of October.—6½ P. M.—Very brilliant streamers seen, at an opening among clouds at N. W. reaching upwards to about 35°. Series of 50 oscillations: 1st in 227″.75; 2nd in 227″.5; 3rd in 227″.5; 4th in 227″.75.

7½ P. M.—Column of very brilliant light near the horizon at the W. extremity of the prime vertical to the mag. mer. and arches of Aurora in the N. Series of 50 oscillations: 1st in 227″.5; 2nd in 227″.75; 3rd in 227″.5; 4th in 227″.25.

It is evident that the differences between the averages of these series at the several periods, are within the limits of the probable errors of observation; so that the intensity was not sensibly affected by the Aurora.

At 8 P. M. the needle on the steel point remained steady at its usual position at that hour.

At 20 minutes past 8, needle on the steel point was still steady, with many brilliant groups of streamers in the N. and an arch of light below them, and another bright column at mag. W.

On the 25th of October the Aurora again appeared; but neither at this time could I find that either the intensity or direction of the needle were altered.

17th of November.—3 P. M. 50 oscillations: 1st in 225″.75; 2nd in 225″.5; 3rd in 226″.25: 4th in 226″.

6 ¼ P. M.—A complete arch of pale nebulous light, with its vertex about 20° high at mag. mer. Intensity, 1st in 225″.25; 2nd in 225″.25. Needle on steel point steady at its usual place at 8 P. M.: and intensity, 1st in 225″.5; 2nd in 225″.5.

A succession of arches rose one below another, and successively expired at about an elevation of 20 till 11 P. M. when the western end of one of them became unusually brilliant, and dense. Streamers. Intensity at this hour, 1st in 225″.5; 2nd in 225″.75; 3rd in 225″.5. Needle has now 3′ E. var. from its position at 8 P. M. At 12 P. M. Intensity 1st in 225″.25; 2nd in 225″ 25. Var. 3′ E.

On the 18th of November, Aurora first seen at 6 P. M. At 8 P. M. with a very brilliant arch about 20° high, and a few streamers, the remains of a preceding one, about 35° high. Intensity, 1st in 225″.25; 2nd in 225″; 3rd in 225′.5; 4th in 225″.5. Variation at 8 P. M. 2′ E.—At 9½ P. M.; Intensity 1st in 225″.25; 2nd in 225″.—At 10 P. M. Var. 4′ E.—At 11½, Var. 3′ 20″ E.

19th of November.—At 2 P. M.; Intensity observed, 1st in 225″; 2nd in 225″; 3rd in 225″; 4th in 225″.25.

At 8 P. M. Aurora first seen. A pale light through many clouds about 15° high. Variation not altered. Intensity, 1st in 225″; 2nd in 225″.25.

[The variations at 11 and 12 P. M. of the 17th of November, and at 8, 10, and 11½ P. M. of the 18th of November, were within the limits of the irregularities of the diurnal variation.]

It would be as yet premature to infer any connection between the easting and westing of the declination, and the vividness of the fringes in their eastern or western ends. What occurred at 8 P. M. and ½ past 10 P. M. might lead us to suspect that the needle declines towards the most vivid end.

In regard to the intensity, I cannot venture to say, that the slight decrease indicated at ½ past 10 is not an error of observation; being conscious that the limits of errors of this kind ought not to be taken at less than the change then noted. I could not take the intensity at ¾ past 6, when the declination was greatest, from a desire to watch the extent of the change in declination produced by the Aurora, of which I was for the first time fully assured. When I receive the separate apparatus for the intensity, with which I am to be favoured, I shall expect more certain results on that point.

Alford, December 26th, 1829.—Since I addressed you on the 15th current, I have made some additional observations on the Aurora borealis, of which I proceed to put you in possession in continuation.

From Journal of observations.

Frost during the 16th, 17th, and 18th Dec. Therm. at night 19°.

19th Dec.—10 P. M. Bar. 29°.7. Therm. 30°.—At ½ past 11 P. M. a heavy low fog cleared up, and exposed a very brilliant Aurora. An arch of very brilliant streamers about 25° high over dense clouds in the N. A broad lane of clear sky above these from W. to N. E. Over head many broken clouds giving partial views of a narrow arch of Aurora from E. to W. in the plane of the mag. dip, and many streamers approaching the zenith from N. To the S.

of these another clear lane of sky E. and W. and nebulous Aurora over dense clouds at S. horizon. Needle on steel point 36′ 20″ E. Before the watch was prepared for the intensity, needle shifted to only 33′ E. Intensity then found 50 oscillations in 226″.25. Needle at conclusion of the observation 14′ E. —12 P. M. Another arch getting into plane of the dip, the rest of the Aurora presenting nearly the same appearances as before. Needle shifted to 22′ E. Intensity 50 oscil. 1st in 225″.5; 2nd in 225″.5. Therm. still 30°. Needle after observation still 22′ E. Steady slight wind near S. W.

20th Dec.—From ½ past 8 P. M. till 11 P. M., a very splendid Aurora continued, with periodical obscurations and revivals, over a dense cloud resting on, and capping the summits of the North hills. Aurora never getting higher than 20°. Rest of the sky entirely clear of clouds, and slight steady wind at W. Therm. 25°. Needle not affected.

21st Dec.—8 P. M. Intensity 225″. Therm. 27°. Ground covered with snow.

22nd Dec.—I have been informed, this day, by the Rev. JAMES PAULL, minister of Tullynessle, that on the 20th of Dec. about ¼ past 9 P. M. he saw the Aurora remarkably bright, near the zenith, at his house, about 2 miles N. of this place, and I am permitted to use his name for the fact.—8 P. M. Intensity 224″.75; 225″. Therm. 28°.

23rd Dec.—8 P. M. Intensity 225″. Therm. 30°.

24th Dec.—8 P. M. Intensity 225″. Therm. 30°.

25th Dec.—8 P. M. Intensity 224″.5; 224″.75; 224″.75. Therm. 29°.

I hope the Royal Society will find in the above observations, and in those I transmitted to you on the 15th, a settlement of the much agitated question regarding the disturbance of the magnetic needle by the Aurora borealis. Mr. DALTON was the first, as far as I am acquainted, who observed that the streamers direct their upper extremities to that point in the heavens to which the dipping needle is directed, and that the arches they form are at right angles to the magnetic meridian; but the definite order in the southward progress of the fringes of streamers not having been then ascertained, those circumstances under which alone it now appears that the magnetic needle is disturbed, were for a time overlooked; and results apparently the most opposite were announced by different observers of distinguished skill and reputation.

A proper value will now be put on the results obtained by Captain Franklin and Lieutenant Hood, chiefly at Fort Enterprise. They observed a remarkable disturbance of the needle when the Aurora passed the zenith; that is, when the fringes came into the plane of the dip, which is there 86° 59′. But the full admission of this fact does not invalidate the results of an apparently opposite nature obtained by others, or their importance. The observations I made on the 21st, 22nd, and 26th of September, 1st, 3rd, 11th, and 25th of October, and the 17th, 18th, and 19th of November last, and now of the 20th December, show, that with very brilliant Aurora, there is no disturbance of the needle, if the fringes do not come into the plane of the dip.

It is evident that as the needle is affected in those places only where the fringes are in that plane, observers in different latitudes may obtain very discordant results on the same evening. The numerous observations collected by Mr. Dalton, of the appearances of Aurora on the 29th March, 1826, (Phil. Trans. 1828,) prove that many fringes of streamers may be parallel to each other at remote distances; and the observation, by the President of the Royal Society, of a luminous arch in Cornwall, 29th September, 1828, simultaneously with a remarkable Aurora of many arches over the whole of Aberdeenshire, proves that the meteor is sometimes active over a space nearly coincident with the extent of this kingdom; and we have no reason to suppose it may not extend often much further. There might therefore be an extensive succession of observations of disturbance and non-disturbance of the needle, at the same instant, from N. to S., over many degrees of latitude. Next to the discovery of the truth, it will give me the highest satisfaction if the Royal Society shall be of opinion, that the result of the observations they have enabled me to make, is to reconcile the conflicting statements made regarding this matter, leaving unimpeached the accuracy of all the observers.

The observations of the intensity, on the 19th December, lead now more plainly to the inference, that it is decreased under the influence of an arch of Aurora in the plane of the dip; but the indications of decrease are too small to be fully confided in, until after a long series of similar results.

I consider Mr. Paull's observation on the evening of the 20th December, viewed in conjunction with mine, of great importance in various respects. The same Aurora that I saw here under an angle not exceeding 20° high, he saw at the Manse of Tullynessle very near the zenith. Now the Manse of Tullynessle is about one point E. of due N., distant from this place little more than two miles in a direct line. It is in a narrow side valley that enters northward into a ridge of hills, the nearest summits of which are nearly two miles from here, about the N. point of the magnetic meridian. These are the summits that were capped by the clouds, to the region of which the Aurora was confined the whole evening. A line drawn over them at mag. E. and W. would pass very nearly over Mr. Paull's station. The height therefore of this particular Aurora at its upper extremities, did not exceed 4000 feet above the level of this place.

In my paper on the Aurora borealis, which the Royal Society honoured with a place in their Transactions of 1829, I stated "that I have here seen the meteor much more frequently in the form of a light near the northern horizon, than in any other form." Mr. Paull's observation has convinced me, that the true place of this light near the horizon is often no other than the nearest mass of hills to the north.

If any doubt had yet remained regarding the height of the region occupied by the meteor, this observation would have determined the point. The merit of first accurately ascertaining that region is due to Lieutenant Hood and Dr. Richardson, by their observations at Cumberland House and Basquiaw Hill. It is the region immediately above that of the clouds, and of course varies much in height with different states of the atmosphere. Although this region was very low on the 20th December, it is, we know, at times several miles high, agreeing with the observations of those distinguished travellers. I have seen the Aurora here when the height of the clouds could not be estimated at less than two or three miles; and I state this again more precisely, because I understand what I had formerly said, has, by some persons, been misunderstood to imply, that its height never exceeds six or eight thousand feet.

Captain Franklin and Dr. Richardson first observed also the connection of the Aurora with the formation of clouds. The latter even says, "I am inclined to infer that the Aurora borealis is constantly accompanied by, or immediately precedes, the formation of one or other of the various forms of cirro-stratus." He could not however determine whether the Aurora was dependent on the formation of the cloud, or the formation of the cloud on it. I conceive the observations of the 20th December throw a satisfactory light on this subject, and shall now briefly state the conditions of the phenomena of that evening, and their localities, which lead me to this conclusion.

The mass of hills, named Coreen, to which both the clouds and the Aurora of that evening were entirely confined, is eight or ten miles long from E. to W., and about four miles broad in some places from N. to S. It is nearly bisected by the magnetic meridian of this place, which cuts off the broadest and highest part of it to the W. About twelve or fifteen square miles rise above the limits of cultivation; that is, about six or seven hundred feet above the bottoms of the conterminous valleys; and some of the summits attain an elevation of nine hundred and one thousand feet.

On the W., a little by S., of this mass of hills, there extends for eighteen or twenty miles a succession of valleys of considerable width, all bounded on the S. and N. by high lands of great extent, and separated from each other by comparatively low eminences. Through these the river Don flows from W. to E. On descending this succession of valleys with the course of the river, the Coreen hills are seen directly in the line of them, and apparently shutting them nearly up at the E., the extension of the low ground turning there, for some distance, suddenly N. and afterwards E., by the N. side of these hills; and the river, on the contrary, when it reaches these hills, turning several points S. of its former course, and entering this valley of Alford at its N. W. corner, by a very narrow defile between Coreen and another hill on the S., of nearly equal elevation but comparatively little extent, which is placed more in the direction of the general outline of the high lands bounding the valleys on the south.

We have then, in the situation of the Coreen hills, a most satisfactory explanation of the formation of clouds over them on the 20th December, in well established principles independently of the Aurora. The wind coming steadily the whole evening from W. carried the air out of the upper Don valleys, (where, at a comparatively low elevation, it was nearly saturated with aqueous vapour, to which the river in its long course would much contribute,) over the surface of Coreen; that is, lifted the whole mass several hundred feet perpendicularly, and the diminished pressure and consequent expansion lowered its temperature to the point of saturation.

As the Aurora was entirely confined to the region of the cloud, the cause of the formation of which we can thus otherwise clearly trace, we are surely justified in considering it as an effect of the condensation of vapour; and as the evaporation and condensation of the aqueous fluid are ascertained agents in developing electricity, the meteor must be considered as a peculiar manifestation of the electric fluid *.

The parallelism of the streamers with the dipping needle; the position of the fringes at right angles to the magnetic meridian; the movement of these fringes away from the N. magnetic pole, and their effect on the needle when they come into the plane of the dip,—all prove it to be equally a magnetic phenomenon; so that we here find another relation between the electric and magnetic influences.

I conceive the result of these observations coincides with many discoveries of a very recent period, to show that we are on the point of being compelled to resolve the long received theory of the magnetic action of the nucleus of the earth, into a peculiar influence of its atmosphere and superficial electricity.

Alford, February 11*th*, 1830.—Between January 10th and 20th, several series of observations of the intensity of the needle were taken at 8 P. M. Therm. uniformly near 30°; and 50 vibr. very uniformly in 225″.25. [It thus appears that the annual increase of the intensity was greatest in the end of December.]

January 25th.—No Aurora was seen since 20th December till this evening, when for about 3 hours from 7 P. M. there were seen, over many detached clouds resting on the Coreen hills in the N., several low arches and parts of arches of brilliant streamers, rising in succession below one another, and expiring about 20° high. Rest of the sky quite clear. Therm. 30°. Needle not affected. Many falling stars in paths parallel to streamers. None of these meteors were seen during the absence of the Aurora.

January 26th.—A steady gale all day a point or two S. of W. Therm. 34° and 35°. Gale became hard after dark, and much snow melted before next morning. The ground has been covered with snow, of which there have been many falls since the 20th December, with a nearly steady frost, generally from 30° to 35° Fahr., but once or twice as low as 12° at sunrise. Wind since that time till now in all points, except W. and S. W.

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* This is in accordance with Mr. DALTON's conclusions, that electricity appears to be a consequent rather than an agent in the formation and decomposition of clouds. (Observations on Meteorology, &c. Manchester Memoirs, Second Series, vol. iv.)

January 27th.—Again frost and calm. Therm. 30°. Intensity $225''.25$.

January 28th, 8 P. M.—Several low arches of Aurora of great brilliancy, with many streamers over clouds resting on the Coreen hills. Rest of the sky quite clear. Needle steady. Therm. 25°.

½ past 8.—An arch has come slowly forward and passed the zenith into the plane of the dip; its west part fading as it reaches that position; but the east part from the mag. merid. continuing moderately bright. Needle shifted to $21' 30''$ E. or towards the bright part. Intensity $225''.25$. After this observation, needle on steel point returned slowly to $6'$ E.

5 min. before 9.—Another very bright arch, about 3° broad at the vertex, and extending nearly from the E. to the W. horizon, but narrowing much at its extremities, came forward into the plane of the dip. The light of this very beautiful arch was nearly uniform from end to end. Sky quite clear, when it was in the plane of the dip, excepting over the Coreen hills, where there were still many clouds. Needle on steel point moved incessantly at this time, but irregularly and somewhat fitfully, alternately to E. and W. within the limits of about $30'$ in all. Intensity $226''$. Arch faded slowly about 30° S. of the zenith; and needle on steel point returned to rest nearly at zero.

Many falling stars in paths parallel to the streamers, i. e. to pencils of rays of the Aurora, were they at the same places.

Many low arches rose in succession after this to about 25° above the Coreen hills; some of them with very brilliant streamers. Needle steady. A slight steady breeze a point or two S. of W. set in after 9 P. M. At 11 P. M. therm. 24°.

January 29th.—Therm. 36°. A steady gale all day, a point or two S. of W. till sunset, when it shifted to N., and therm. fell to 30°. 8 P. M. Intensity $225''.25$.

February 6th and 7th.—Steady moderate frost till now, when there occurred a terrible storm at S.S.E. with a heavy fall of snow. At 8 P. M. of 6th, therm. 27°. Intensity $225''.5$.

It will be seen that the observations now detailed confirm the result of those I formerly sent, viz., that the magnetic needle is disturbed by the Aurora only when the fringes come into the plane of the dip. I employ the term fringe, which I first used in my paper on the definite arrangement and order of progress of the meteor, as the term arch can have reference properly to two only of its dimensions, while fringe may include the idea of all the three.

The observations of the 28th January lead to the conclusion, that the N. pole of the needle (meaning the pole that is directed towards the N.) shifts towards the most brilliant end of the fringe. The successive phenomena of that evening were remarkably distinct, giving great facility to accuracy of observation; and I remember nothing of the kind surpassing them in splendour. The first fringe of that evening, which crossed the zenith, and faded in its W. end as it got into the plane of the dip, exhibited, in the most distinct way, the manner of the progress southward, by the extinction of streamers at its northern face, and the sudden formation of new ones along the whole of its southern face.

After making the attempt, I have found it impracticable to keep any journal in detail, of the falling stars, on account of the great numbers that often occur in the same evening; but I have not failed to observe them since my attention was directed to them in the remarkable manner I mentioned to you in a former letter*. I have now no doubt whatever that they are a branch or modification of the Aurora borealis. In a vast majority of cases their paths are parallel to the streamers of the aurora, that is, they are directed away from that part of the heavens to which the upper end of the dipping needle points; and in those cases, comparatively few in number, where this parallelism

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* *Date Nov.* 11.—During the course of these observations, my attention has been incidentally directed to another meteor, that of falling stars; and the conclusions to which my observations of these phenomena (as yet I must acknowledge very limited) have uniformly led, I have no doubt you will acknowledge to be interesting. I would have introduced this subject to you in my last, had I not felt it proper to wait till the views regarding them, which had at that time opened to me, should be ascertained to be correct or not, by some continuance of observation.

On the evening of the 21st of September at 10¼ P. M. there occurred an Aurora, rising above the northern horizon in detached groups of bright streamers and nebulous patches of light. Many flashes of sheet lightning were seen, during and after its continuance, in the western part of its space; and in the eastern part several falling stars descending among, and having paths quite parallel to, the streamers. I could not quite satisfy myself whether the course of the sheet lightning was vertical in a downward direction, on account of its suddenness; but that was the impression to which my observations chiefly led. But the correct parallelism of the paths of the falling stars to the streamers led is departed from, the paths can generally be referred to the planes of the fringes of the Aurora, and, within these, to lines at right angles to the streamers. I have even seen a succession of three or four or more falling stars, at momentary intervals, whose paths were all wholly within the plane of a fringe, or such a narrow zone as an elevated fringe would occupy; and again, after a considerable interval, have seen a similar succession in a similar zone, further S., as if the planes in which they are active moved southward like the fringes of the Aurora. But the circumstance which perhaps most convincingly connects them with the Aurora, is their occurring in great numbers on those evenings when it is visible, and their rareness at any other time, a fair example of both which cases is to be found in the foregoing journal.

It will be perceived from this and my former communications, that the Aurora precedes and accompanies W. and S. W. gales, which are generally the hardest we experience. In the absence of more direct observation, I may be permitted at present, although it is not strictly legitimate, to refer to the popular belief, that the falling stars also are indicatory of hard gales.

I think there is now no doubt that the horizontal intensity is affected by the Aurora borealis, as the indications are uniformly on one side, and longer practice has given me more confidence in the correctness with which I am enabled to measure the times.

me to examine the direction of those I have seen since. They are now very numerous, and, without any exception, their paths have been directed from a point near the meridian, within the limit of from 10° to 20° south of the zenith. These meteors, too, have been most frequent on those evenings when the Aurora borealis appeared, with the exception of the evening of the 16th October, when there was no Aurora, but a great number of falling stars. The state of the weather and wind that night was, however, such as led me to anticipate and make preparation for a display of the former. The paths of those falling stars I have seen near the zenith have been all short in comparison with those seen on the same night lower down.

These observations of mine have been too few, and limited to too short a period of time, to give confidence in the inferences which are to be deduced from them; but being now put into your possession, they will either soon be confirmed or their partial character shown: but it may surely be worthy of inquiry, whether the falling stars of southern latitudes are the equivalents of our Aurora borealis.

March 24*th*, 1830. (Journal continued.)—Therm. 45°, observed a remarkable Aurora. A bright fringe having a little deficiency near its vertex, but reaching low towards each horizon, and about 4° broad, is near the plane of the dip. To the northward of this, and crowding up to it with intervals only of about 3°, 4°, and 5°, many fragments of another fringe, some of them 10° or 20° long from E. to W., others much shorter ; the general breadth of these varying from about 4° to 6° or 7°. N. of these again other fragments of a fringe or fringes, having their most lengthened dimensions very perceptibly in an E. and W. direction; and the whole northern part of the sky, down to a long dense cloud stretching the length of the Coreen hills and elevated only 8° or 10° above them, is filled with brilliant streamers, at times presenting apparently inextricable confusion, and at other times partial appearances of arches. No clouds in the region of the Aurora, nor southward of it, with the exception of one on the S.W. horizon. A bright nebulous light about 15° high along the whole S. horizon.

The whole lights in the N. part of the sky made a rapid progress southward ; and the manner of this progress was repeatedly finely exhibited in the fringes and fragments that had reached or passed the zenith, by the extinction of streamers at their northern faces and the formation of new ones at their southern faces. The advanced southern fringe expired when it had reached about 25° S. of the zenith ; and all did so, either when they attained a similar angle S., or before they had gone so far. The confused mass of streamers in the N. as they came forward in succession to the zenith, and passed that point, unfolded themselves into narrow zones of light at right angles to the magnetic meridian, or very nearly so ; for there was occasionally a small deviation from parallelism among themselves. These zones were more numerous than I have seen on any evening before, and were separated from each other by less intervals, sometimes not exceeeding 3° or 4°, sometimes however 15° or 20°. There were among them two complete and very lengthened fringes, besides the one first described ; but the larger proportion consisted of only fragments of similar fringes, cut short more or less in their E. and W. dimensions.

The needle on the steel point, from the first moment of observation was very unsteady, shifting sometimes more slowly, and sometimes more rapidly and fitfully, within the limits of about 50°, in the most extreme case, on each side of the ordinary magnetic line.

5 min. past 9 P.M.—With a very lengthened fringe in the plane of the dip, varying much in intensity of light from end to end; the needle on the steel point shifted to 32° W.; I then made trial of the intensity with the needle, and observed the arc of oscillation, at the conclusion of 50 oscillations, much less than I have uniformly before observed it. Time of 50 oscillations 224″. Dropt the needle on steel point, and found it now (a few seconds only after concluding the observation) 25′ E. Repeated the trial of the intensity immediately, and again found the arc of oscillation at the conclusion greatly diminished. 50 oscillations in 224″.25. Needle instantly dropt on the steel point now returned to 34′ W.

Having examined the needle on the steel point for some considerable time, and the extremely irregular nature of its movements, under the influence of succeeding fringes coming into the plane of the dip, I conceive the true explanation of the irregularity in the extent of the arcs occurred to me. The arc of oscillation being bisected by the line of the magnetic force, when this line changes its direction suddenly, as is indicated by the needle on the steel point, the arc is either enlarged or diminished suddenly at the same time, according as the line shifts from or towards that of the needle, at the moment when it has completed an oscillation, to return in another; and the time of the oscillations is thus sensibly changed.

This conclusion might indeed have been drawn from the previously observed phenomena, but did not occur to me till the experiments of this evening brought it into view.

Besides changes in declination, the needle on the steel point often exhibited a slight vertical movement and tremor, indicative of a change of the dip, and showing thus another impediment to an accurate determination of the intensity.

At a quarter to 10 P.M. the region occupied by the Aurora was extensively obscured by a haze, preventing further good observation.

Several circumstances occurred this evening confirming the views that the observations of the 20th December last had led to; viz. "That the Aurora is dependent on the condensation of vapour." It is familiar to every one, resident in a mountainous country, that clouds, even during hard gales, continue apparently stationary over the higher summits, and present themselves in these situations when the rest of the sky is clear. The explanation is quite simple. The lower saturated strata of air being carried over the summits, the diminished pressure and consequent expansion lower the temperature to the point of condensation; and as the air in its progress again descends, a reverse effect occurs, and the vapour is again dissolved. A cloud of this kind appeared to rest the whole time of the appearance of the Aurora over the line of the Coreen hills, little elevated above them, and appearing to furnish, as it were, the base of the bright Aurora in the northern part of the sky.

It has been stated that a detached cloud rested on the S. W. horizon; and the description of the circumstances attending it has been deferred till now. It continued apparently stationary all the time of the observations from 20 minutes to 9 till a quarter to 10; and I had no difficulty in referring it with certainty to the summit of a large hill about six miles from this place, having considerably more elevation than the Alford hills, and standing several miles apart from any summit of nearly equal height. Now over the top of this cloud, which was quite free of the northern Aurora, and also of the nebulous Aurora at the S. horizon, there continued the whole time a very bright nebulous light. The large hill referred to is not itself visible from this place, on account of a high ground about a mile distant.

I have little doubt also that the nebulous Aurora at the S. horizon, which likewise continued the whole time, ought to be referred to a range of hills extending E. and W., about seven miles distant, and nearly as high as the Coreen hills. A high ground about three quarters of a mile distant conceals these hills from view, and would also conceal clouds over them, as high as those above the Coreen, on this evening.

It would thus appear that the pencils or bundles of parallel rays of the Aurora, denominated streamers, and whose longitudinal dimensions always show themselves parallel to the dipping needle, become indistinct at the distance of a few miles, being blended and softened down into a nebulous light by the refraction and haze of the atmosphere.

The wind the whole time was a rather strong gale a point or two S. of W. There have been very severe gales from the same quarter for three days past.

I have this evening again seen several falling stars during the continuance of the Aurora, having seen none since I saw it last. Four were over the Coreen hills, and the paths of all were nearly but not quite parallel to the streamers. As if in correspondence with the great brilliancy of the Aurora, they were very large and bright, fully equal at least to Jupiter.

My attention having been sometimes directed to the diurnal variation of the needle, for the purpose of ascertaining whether any interference might be expected between it and the influence of the Aurora, I have been led by many observations (of the whole of which indeed I have no regular notes, but only of a few,) to conclude that the quantity of the variation depends on the brightness or gloominess of the weather. Thus, on the brightest day that has occurred since I received the apparatus, the 2nd of October 1829, therm. 52°, the diurnal variation at noon was 26′ 20″ W. On the two gloomiest days that have occurred, the 3rd and 4th of December, 1829, therm. about 42°, the diurnal variation at noon was 3′ 20″ and 3′ 40″ respectively. Again, some observations in the month of January last, on bright days, when yet the earth derived little or no heat from the sun, owing to a close covering of snow and a hard frosty wind,—and others, on bright days likewise, when the earth, much cleared of snow, imbibed much heat,—would lead me to believe, that the direct radiating heat of the sun has more effect than the light in causing the increased variation, when the state of the earth's surface is such as to imbibe that heat.

VII. *Remarks on several icebergs which have been met with in unusually low latitudes in the southern hemisphere. By Captain* James Horsburgh, *Hydrographer to the East India Company, F.R.S.*

Read Feb. 4, 1830.

An apology might probably be necessary, in offering the following remarks to the notice of the Royal Society, if they did not pertain to a phenomenon of a novel nature, which may be interesting to those members of the Society who are anxious for the elucidation of cosmographical science.

It appears that icebergs, until lately, have seldom been seen by navigators in their passage near the Cape of Good Hope and the coast of South Africa ; for the journals of the ships belonging to the East India Company, during the whole of the last century, do not specify that any icebergs had been seen in the route of their navigation in the southern hemisphere, although several of these ships proceeded into the parallels of latitude 40°, 41°, and 42° south.

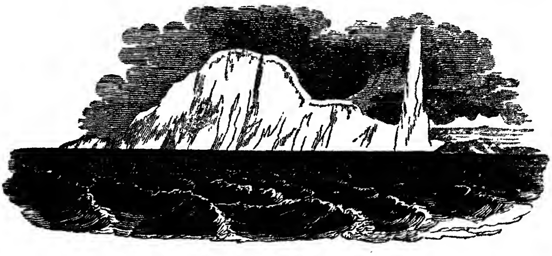
On April the 7th, 1828, the Harmonie, French ship from Calcutta, bound homeward, in latitude 35° 50′ south, longitude 18° 5′ east of Greenwich, saw several icebergs, some of them appearing to be 100 feet elevated above the sea, and passed between two of them, upon which the sea broke violently. When amongst these icebergs, the Harmonie fell in with the Spanish ship Constancia from Manilla, bound to Cadiz, the pilot of which describes them as follows :

April 7th, 1828, at 10¾ A.M. saw a small island, appearing like a white cloud, and soon afterward some shadowy lines were observed in it, as is usual in land : upon a nearer approach, it appeared to be a large island of considerable height divided into two summits. Soon after, three other small islands were discovered at a short distance from the former. At 11½ A.M., perceived that they were white, and that the light of the sun was reflected from their surface as from a mirror. We were perplexed with this phenomenon till noon, being then in latitude 35° 56′ south, longitude 16° 59′ east of Greenwich, by chronometer, corresponding with lunar observations. Sounded, but got no bottom at 135 fathoms; and the sea continuing of a green colour, we concluded these were icebergs, which had drifted to latitude 35° 54¾′ south, longitude 17° 59′ east. Steered W.S.W. till 2 P.M. and spoke the French vessel L'Harmonie from Calcutta. At 3½ P.M. discovered two more icebergs, which we passed at 4½ P.M., the most southerly of these presenting a square of 25 or 30 toises of elevation, but without an apex like the other near it. At the distance of three miles to the north of these, another iceberg of large size appeared.

On the 28th of April, 1828, the brig Eliza from Antwerp, bound to Batavia, fell in with five icebergs in latitude 37° 31′ south, longitude 18° 17′ east of Greenwich, having the appearance of church steeples, and apparently from 250 to 300 feet high, which were passed within the distance of a quarter of a mile; and the sea broke so violently against these enormous masses of ice, that at first they were thought to be fixed on some unknown shoal, until on sounding, no bottom could be obtained.

These icebergs, seen by the Eliza three weeks after the former were discovered by the ships Harmonie and Constancia, although nearly on the same meridian, were 32 leagues to the south of those first seen, and therefore must have been different masses following nearly in the track of the former, and carried along from a high southern latitude by the current and waves towards the coast of South Africa.

The East India Company's ship Farquharson, April 20th, 1829, in latitude 39° 13′ south, longitude 48°46′ east, fell in with a large iceberg, and when near it the measured altitude gave 150 feet for its height above the surface of the sea, and it appeared to be about two miles in circumference. If the height measured from an altitude of this iceberg be correct, the whole height from its base to the apex must have been 1000 feet or upward, by allowing for the difference of gravity between ice and water, compared with the form of the iceberg above the sea. When near it, the annexed view was taken.



Antecedent to these icebergs, discovered in April 1828, and April 1829, none appear to have ever been seen to the northward of the 42nd or 43rd degree of south latitude, in the Southern Ocean; but His Majesty's store-ship Guardian struck upon one in latitude 44° 10′ south, longitude 44° 25′ east, on the 24th December 1789.

In the Encyclopedia by the late Dr. Rees, it is stated that "floating ice has occasionally been found in both hemispheres as far as 40° from the poles, and sometimes, as has been said, even in latitudes 41° and 42°." But it is now ascertained that icebergs are carried to a greater distance from the poles before they are dissolved.

As the icebergs in the southern hemisphere have been found further from the pole in April than at any other time, it might be expected that in the corresponding month of October they would in the northern hemisphere be found at the greatest distance from the Arctic pole; it however appears that in the same month of April or May*, icebergs have been seen at the greatest distance from the latitude of their formation, in the northern as well as in the southern hemisphere; in accordance with which the following examples may be stated:

April 14th, 1817, the Minerva from New York, bound to Liverpool, fell in with four large icebergs in latitude 42° 47′ north, longitude 47° west.

April 3rd, 1823, the Mountstone sailed from Plymouth for St. Johns Newfoundland, and on the 7th of May struck against a mass of ice during a thick fog, and shortly afterward filled with water:—the latitude not given.

May 14th, 1814, the fleet bound to Quebec, in latitude 44° 18′ north, longitude 50° 50′ west, fell in with upwards of twenty large icebergs, some of which were 80 feet above water; and in the afternoon of the same day the convoy passed a field of ice about 20 miles in extent, and about 30 feet elevated above the sea, some parts considerably higher.

From the foregoing observations, the following remarks naturally arise:

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* There are, however, some exceptions to this remark; as icebergs (it is said) have been seen in July or August not far distant from the Azores; but perhaps this seldom occurs, for in these months southerly winds prevail, with a warm atmosphere, and frequently a current setting to the northward: all, together, forming a barrier of resistance to the advancement of icebergs to a low altitude in the North Atlantic Ocean.

First,—That in April, or early in May, both in the northern and southern hemisphere, icebergs, or large masses of ice, have been found in lower latitudes than at other times, which appears to be anomalous, as a difference of six months in time might be expected between the nearest approach to the equator of the northern and southern icebergs.

Secondly,—That the existence of a large tract of land near the Antarctic circle seems to be necessary for the origin and accretion of the southern icebergs, and probably situated somewhere between the meridian of London and longitude 20° east; from whence these icebergs have been carried in a N. and N.N.easterly direction by the united forces of current, winds, and waves, prevailing from S.S.W. and S.W.; for Sandwich Land, in latitude about 60° south, longitude 27° west, seems to be too far west from the prevailing line of direction of the currents, winds, and waves, to be the place of formation of the icebergs found near the Cape of Good Hope, and of that seen by the Farquharson. Bouvet's Island, and Thompson's Island, in latitude about 54° south, longitude 5½° east, are not of magnitude sufficient to have been the basis of these icebergs; and Kerguelen's Island, in latitude 49° south, longitude 70° east, is too far to the eastward to have been their original base.

Thirdly,—That from the unprecedented appearance of icebergs in the vicinity of the south coast of Africa in April 1828, and in April 1829 further to the eastward, some unknown cause probably produced a disruption of these icebergs from the place of their formation, such as an earthquake, or volcanic forces, which seldom or never had before happened in those regions; but more particularly during the last century, a period when icebergs have not been seen in the Southern Ocean near the coast of South Africa.

If, however, these icebergs were dislocated from the land by some unknown cause, which seldom or never had before happened, the anomaly adverted to above, of the northern and southern icebergs appearing in the lowest latitudes in April or May, would in such case only apply to these icebergs having been carried from the polar regions by similar currents in the same months, instead of a difference of six months, in reference to the set of the North Atlantic current and that of the Southern Ocean, as might have been expected.

VIII. *On the progressive improvements made in the efficiency of steam engines in Cornwall, with investigations of the methods best adapted for imparting great angular velocities.* *By* DAVIES GILBERT, *Esq. President.*

Read March 4, 1830.

IN the year 1827, some observations I had made on steam engines were honoured by a place in the Philosophical Transactions. I am therefore induced to lay before the Society further particulars illustrative of the progress by which that most important machine has reached its actual high state of improvement. On a subject of less magnitude I should not have presented to the Society a mere collection of matter in detail, unconnected by any general arrangement of the facts: but every thing appears to me of great interest that bears on the history of an invention that has continually advanced towards perfection by the aid of chemical, mechanical, and mathematical sciences; an invention that has already altered and improved the condition of mankind; and seems destined to produce consequences the most beneficial to civilized society, by extending the dominion of intellect over muscular power and brute force. I am moreover desirous of preserving information derived from documents which have never yet passed out of private hands, and are consequently liable to be lost or destroyed.

For all practical purposes the steam engine must be considered as originating with Mr. NEWCOMEN; the introduction of a moveable diaphragm between the active power and the vacuum or less elastic medium, being essential to the very principle of the machine as a moving power.

Mr. NEWCOMEN's engines were brought into Cornwall very early in the last century, where they immediately superseded the laborious method of drawing water by human exertions, applied through the simple medium of a chain pump, similar in construction to those at present used on board large ships. So inartificial, indeed, was the machinery in mines at this comparatively recent period, that I well remember an individual who used to boast of his having assisted in constructing the first Whim, or contrivance for applying the strength of horses to the lifting of weights, that was seen in the peninsula west of Hayle and of St. Michael's Mount.

The only material improvement ever made to Mr. NEWCOMEN's engine, had, I apprehend, been effected previously to its introduction into Cornwall, which consisted in the automatic opening and shutting of the valves. It consequently remained in nearly the same state up to the time when a new æra commenced from the appearance of Messrs. BOULTON and WATT.

The use of Mr. WATT's engine, secured to him by a patent in 1769, and extended by Parliament to the year 1800, was offered on terms reasonable in themselves, and fairly growing out of the subject. These were, payments of one third part of all the savings in fuel, to be estimated by a comparison of the new engines with those on the former construction executing equal quantities of work.

Experiments on a large scale were accordingly instituted in the summer of 1778 on two atmospheric engines, then supposed to be the most perfect of any at work in Cornwall; conducted by Messrs. BOULTON and WATT themselves on one part, and by the engineers and managers of mines on the other, assisted by some of the most respectable gentlemen of the county, who were interested either as proprietors of the land, or as adventurers in the mines.

The following is a copy of their Report:

"Poldice Mines, October 30, 1778.

"We the under subscribers having carefully examined the books of the mine touching the consumption of coals of the two eastern engines during the months of August and September of this present year, and attended to the working of the engines during those months, do hereby certify, that the said two eastern engines did consume in 61 days of those two months 220 weys of coal, each wey being 64 Winchester coal bushels, which amount to 14'080 bushels in 61 days.

"We do also certify, that the said two engines together do work pumps in four lifts which are 17 inches diameter in the working barrels, and the whole depth from whence these pumps drew water to the adit is 58 fathoms.

"We further certify, that the said engines did during those months of August and September last, work the said pumps at the rate of 6 strokes of 5½ feet long each in every minute, which amounts to 8640 strokes per 24 hours.

"We have also made an accurate calculation, by which it appears that when the new fire-engine to be erected by Messrs. Boulton and Watt is completed, and actually works a pump of the same depth of 58 fathoms and 17 inches in diameter at the rate of 6 strokes of 5½ feet long each in a minute, and consequently making 8640 strokes per 24 hours, it will draw a quantity of water equal to that now drawn by both the present engines, and consequently whatever smaller quantity of coals it uses than 14'080 bushels for 61 days when going at the rate of 6 strokes per minute, will be the real savings in fuel occasioned by the said new engine at that rate of going.

(Signed) "James Watt.
"Matthew Boulton.
"H. Hawkins Tremayne.
"Richard Williams.
"John Williams.
"Thomas Brown."

A pound avoirdupoise lifted through one foot had not at that period been established as the dynamic unit.

The product of pounds raised, and of the number of feet through which they are lifted in a given time, divided by the number of bushels of coal (supposed to weigh 84 pounds) burnt in the same interval, give what is now termed the duty of the engine, and afford a perfect criterion of its comparative merit.

The most convenient method of forming this estimate is by multiplying together, the diameter in inches squared, of the lifting box, or of the plunger piston, the height of the lifts in fathoms, and 2.04 (log. 3.3095101) the weight in pounds avoirdupoise of a cylinder of water one inch in diameter and six feet long, the product gives the weight.

This, multiplied by the length of the stroke in feet, and by the number of strokes in a given time, and finally divided by the number of bushels of coal consumed, will give the duty.

In the certified case,

Inches in the diameter of the lifting box 17, sq. 289	Log. 2.4608978
Length of the column in fathoms 58	1.7634280
Constant multiplier 2.04	0.3095101
Weight in pounds 34185	4.5338359
Length of lifts 6 feet × $5\frac{1}{2}$ the number in a minute = 33, 33 multiplied by 1440 = 47520, and this number × by 61 = 2'898720	6.4622063
Pounds into feet during the whole period of 61 days 99092'800000	10.9960422
Bushels of coal consumed in the 61 days 14.080	4.1486027
The duty 7'037800	6.8474395

If the quantity of water lifted at each stroke is required in imperial gallons: square as before the diameter of the lifting box or plunger in inches, multiply by the length of the stroke in feet, and by 0.034 (the fractional part of an imperial gallon in a cylinder one inch in diameter and a foot long—the logarithm 8.5313588): the product will be the gallons raised.

Diameter in inches squared as before	Log. 2.4608978
6 feet	0.7781513
Constant multiplier 0.034	8.5313588
Number of gallons 58.94	1.7704079
$5\frac{1}{2}$ strokes in a minute, × minutes in 61 days 87'840 = 483.120	5.6840550
Imperial gallons in 61 days = 28'475000	7.4544629

Each separate engine on the new construction underwent a constant comparison, during the whole time of its continuing in action, with the duty of the two standard atmospheric engines as reported by the committee.

The diameter of the various lifting boxes or plungers, the length of the lifts or columns of water, and the lengths of the strokes, were matters of common notoriety; and the number of times moved by each engine in a given period was ascertained by a contrivance denominated a Counter, placed on the great beam; this apparatus involves a series of wheels and pinions set in motion by a weight rolling in each direction, and acting through the medium of an escapement similar to that of a clock.

In the year 1793, that is fifteen years afterwards, an account was taken of the work performed by seventeen engines on Mr. WATT's construction, then working in Cornwall. This statement is not verified in the authentic manner of the former; but there do not appear to be any reasons for doubting of its correctness. The average duty was 19'569000 pounds of water raised one foot high, by the consumption of one bushel of coal; exceeding the standard experiment on the two atmospheric engines in the proportion of 2.78 to 1, or nearly as 2¾ to 1. So that work requiring by the atmospheric engines 278 bushels of coal, would be performed by Mr. WATT's steam engine by 100 bushels, consequently 178 would be saved. One third part of these must have been paid to Messrs. BOULTON and WATT as patentees, leaving a clear gain to the mine of 118 bushels, being more than the quantity consumed.

Some years later, disputes took place as to the real performance of Mr. WATT's engines, and a reference was agreed on between the parties to five individuals, of whom I had the honour to be one; and in May 1798 returns were made by the agents in various mines, of all the particulars respecting twenty-three engines, from which I then deduced their respective duties for the information of the referees.

It will not be necessary to trouble the Society with more of these particulars than the diameter of the cylinder, whether it worked single or condensed, both above and below, and the final result or duty in millions of pounds lifted one foot high by the consumption of one bushel of coals.

	Diameter of Cylinder in inches.	Duty.	Observations.
1	20	10’015000	
2	21	16’385000	
3	45	29’668000	It was believed at the time that some inaccuracy must have occurred in the communications respecting these two engines.
4	36	28’212000	
Double 5	42	18’193000	On the same mine. The length of strokes in all but one, six feet; in that, eight feet. Average duty of the whole, 15’985000.
6	63	15’190000	
Double 7	45	15’180000	
8	45	15’571000	
9	45	15’090000	
10	45	14’384000	
Double 11	42	18’740000	
12	42	15’532000	
13	36	18’465000	
14	—	12’226000	The diameter of the cylinder not returned.
15	30	14’050000	
16	20	12’366000	
Double 17	14¾	6’097000	
18	30	13’931000	
19	28	19’739000	
20	36	24’514000	
21	21	13’215000	
22	20	15’034000	
23	48	27’503000	Supposed to be the best engine.
		17’671000	The general Average in 1798.

It may be observed that the average duty was here somewhat less, than it had been found in 1793, confirmatory of an opinion generally entertained that the steam engines had deteriorated from the time of Mr. Watt's quitting his residence near Redruth.

The principles and even the mechanism of Mr. Watt's engines have remained unaltered since their first introduction, unless a change in the precise periods of opening and closing the valves could be considered a variation. But to such an extent has the economy of fuel been carried by the use of steam at a high degree of temperature and consequently of pressure, usually from fifty to sixty inches of mercury above the atmosphere, by extending the expansive action to two thirds or even to three quarters of the whole descent of the piston, by making small fire-places, with sharper drafts, in iron tubes surrounded by the water of the boiler, by more effectually preventing the escape of heat, by enlarging the engines themselves, and perhaps by executing the work with superior accuracy, that in the monthly return of duty performed in Cornwall by the steam engines in December 1829,—the best engine with a cylinder of 80 inches did 75'628000, exceeding the duty performed in 1795 in the proportion of 3.865 to 1, or as 27 : 7 nearly: and exceeding the standard atmospheric engines of 1778 by 10.75 : 1. But subject as in former times to a great variation between different engines apparently similar in all respects; the average being about forty-one million and a half.

Greatly as we are indebted to Mr. Watt for his improvement of the steam engine used in exhausting water from mines, our obligations to him are still greater for originating and carrying almost to a state of perfection, the application of steam as a moving power to machinery, in all the complicated and varied uses of mechanical inventions in this country.

In effecting this most important object, the double engine was first brought into use, the extremely ingenious contrivance for producing parallel motion was invented, and the principle of centrifugal force enabled an apparatus called a Governor to regulate a supply of steam inversely proportionate to the velocity which might at any instant be acquired; and the use of fly-wheels, perfectly understood in theory, became subservient to the regularity of motion, and to the gigantic efforts of our most ponderous machinery. We owe further to Mr. Watt the introduction, at least into general use, of what is termed bevelled geer.

When wheels acting on each other by means of teeth have their axes parallel, the teeth however curved on themselves, must obviously be parallel also on their line of contact. But when the axes are inclined, the line of contact between the teeth should then be directed to the point where the two axes would intersect, thus assimilating their action to that of a cone revolving round a centre on a circular plane.

Having on various occasions had my attention drawn to the consideration of machines forced into rapid action by great powers moving at a comparatively slow rate, I have been induced to make endeavours for ascertaining the amount of friction in several instances.

The mode adopted has been to impart equal velocities to the machine performing the work for which it was destined, and in a state disincumbered from all extraneous impediments, and then to compare the forces applied under these different circumstances. The media of communication being in all cases axes with wheels, impelled by teeth, or cogs.

In some instances the friction was found to equal two-thirds of the whole resistance, in others a half, and in some few instances one-third: evidently becoming less as the wheels were increased in size; and always greatly diminished by the introduction of an additional axis for multiplying the angular velocity. In similar machines recently constructed on the best principles, friction is said not to exceed a tenth.

With the hope of reducing the amount of this great impediment to the useful application of motive force, I have been led to consider what would be the most proper form of teeth or cogs, and by how many intermediate steps a given increase of angular velocity might be most advantageously effected.

It is quite clear, with respect to cogs or teeth, that to give them theoretical perfection, they should be so formed as to communicate an equable velocity from the driving to the driven wheel; and at the same time to roll upon themselves free from any sliding, the cause of friction and of abrasion.

Either of these properties may be separately obtained; but the two are utterly incompatible, except at the limit where teeth disappear and the wheels themselves are in contact.

For producing equable velocities.

The involute from a circle, receives continually an accession of length to its radius of curvature equal to the development of the generative arc. Consequently, if involutes are formed in opposite directions from two circles of magnitudes inversely proportionate to the angular velocities of their respective wheels, the extremities of the two radii of curvature will always remain in contact, forming together a tangent to the evolute circles crossing the line of the centres. The angular velocities must therefore be uniform, while the surfaces will slide on each other, and create a friction proportionate to the difference of length between the radii of curvature.

For avoiding friction between the cogs or teeth.

As logarithmic spirals preserve always a constant angle between the ordinate and the curve, if two similar logarithmic spirals act against each other, they must continue to roll without friction, the ordinates remaining in contact on the line of the centres, but causing angular velocities in the inverse proportion of those ordinates.

It is plain therefore that the two properties are incompatible, since the loci of contact between the curves producing them are in different lines. But since the involutes may be generated from circles indefinitely small, without reference to the size of the wheels, and the logarithmic spirals may be inclined at any angle: at their ultimate limit the lines of the centres and of the tangents will coincide, producing equable velocities and avoiding friction; but at this point the teeth or cogs disappear, and the wheels revolve with a contact of their peripheries.

As cogs become oblique, their contact produces an increase of pressure with augmented friction on the axes: but I am induced to believe, from the results of such trials as I have been enabled to make, as well as from theory, that an uniform angular motion is the great object to be sought, especially in ponderous machinery, and above all where fly-wheels are applied. The expedient usually resorted to, of making the teeth small and consequently many in number, renders attention to any precise curvature less necessary; but this subdivision is limited by want of strength; and I would venture to recommend the involute form, producing an equability of angular motion.

When angular velocities are to be multiplied in large machines, the effect can scarcely be produced in any other way than by the usual one, of wheels unequal in diameter and consequently in the numbers of their teeth.

The question then to be resolved is this:

By the interposition of how many such pairs of wheels can the ultimate angular velocity be most advantageously produced?

Wishing to obtain a solution accurate to no further degree than what may be sufficient for the guidance of practical experiments, I omit to take any separate account of the friction caused by the teeth of the wheels as distinct from that of their axes, and I shall consider the wheels and axes throughout the train in a constant ratio to each other, although as the actual pressure becomes less, the axes may be reduced in size, and thereby friction diminished.

When two forces act on an axis from the peripheries of two wheels attached to it; one for receiving motion, and the other for conveying it on through the train, the pressure on the axis may be the sum of the two forces, their difference, or any intermediate quantity according to the positions of the points communicating with the preceding and with the subsequent wheels; the pressure is most usually their sum, as convenience generally requires that the power should be received and carried forwards at opposite points.

Let the diameters of the two wheels be as $1 : d$, then will the amount of pressure on the axis be $d + 1$. Assuming the axis and wheels next in succession to be similar in all respects, the pressure on them will be less than on the former in proportion to the increased angular velocity, but the prime mover having the disadvantage of leverage in the same proportion, the retarding effect of friction will be precisely equal in both; whence it is obvious, that the same ratio should continue throughout all the series, or that the multiplication of angular velocity should proceed in a geometrical proportion.

Let then the whole increase of angular velocity be represented by a, and let the number of axes employed be x. Then in the usual mode of applying the wheels the friction on each axis will be $a^{\frac{1}{x}} + 1$. And the friction of the whole of the axes will be $(a^{\frac{1}{x}} + 1)\,x$. To find when this is a maximum, let $A =$ the natural logarithms of a. Then will the fluxion of $(a^{\frac{1}{x}} + 1)\,x$ be

$$- A \, . \, a^{\frac{1}{x}} . x^{-1} \, \dot{x} + a^{\frac{1}{x}} \, \dot{x} + \dot{x} \text{ when this is put } = 0$$

$$x = \frac{A}{1 + a^{-\frac{1}{x}}}$$

By approximating it will be found that when

$a = 120$ x or the number of axes $= 3.745$, and $(a^{\frac{1}{x}} + 1).x$ or the friction 17.9 instead of 121 if x were unity and the whole angular velocity communicated at once, about one seventh part.

$a = 100$ $x = 3.6$ friction 15.64 about 10 parts in 64 of the whole, as above.

$a = 40$ $x = 2.88$ friction 13.25, about one third part of 41.

When $a = 3.59$ the minimum of friction falls on a single axis.

$a = 12.85$ the minimum of friction is on two axes.

$a = 46.3$ the minimum of friction is on three axes.

$a = 166.4$ the minimum falls on four axes.

In practice it is obvious that x must represent a whole number.

Let $a = 64$ x may be either two or three.

If $x = 2$ the friction will be 18.
 3 the friction will be 15.

If the wheels be so arranged that the resultant of the two forces on the axes equal the larger in magnitude, x becomes = to A, and the amount of friction $a^{\frac{1}{A}} \times A$ but $a^{\frac{1}{A}}$ = the radix of the natural system of logarithms; consequently the friction in this case will be $e \times A$.

If the points of the two wheels receiving and communicating motion are placed in the same right line parallel to the axes, the general expression becomes $x = \frac{A}{1 - a^{-\frac{1}{x}}}$ evidently giving to x an infinite magnitude: but this can indicate no more in practice than the advantage of so arranging the communications of motion when it can possibly be done.

As fly-wheels are of essential use in preserving uniform velocities, or for accumulating power in almost all rotatory movements produced from those that alternate, and as the power of the centrifugal force has sometimes exceeded the cohesion even of iron, I shall conclude this paper with an adaptation of a well-known theorem to common use.

Let r = the radius of a wheel expressed in feet.
v = the velocity of the rim where all the weight is supposed to be accumulated, expressed in feet in a second.
s = the space in feet through which a body descends in one second 16.0899; log. 1.2065541.
F = the centrifugal force.

$$\text{Then } F = \frac{v^3}{2\,r\,.\,s}$$

Let n = the number of revolutions in a second.
c = the periphery of the circle to diameter unity = 3.14159. log. 0.4971499.

$$\text{Then } F = r\,.\,n^2 \times 1.2268 \text{ (log. } 0.0887757\text{)}.$$

Consequently for an approximate value,

The centrifugal force = the radius × number of revolutions in a second squared × 1.2.

If N = the number of revolutions in a minute, the product found as above must be divided by $60 \times 60 = 3600$; consequently $F = r N^2 \times 0.00034078$ (log. 6.5324732.)

For an approximate value,

The centrifugal force = the radius × number of revolutions in a minute squared × 34 and cutting off five places of figures.

IX. *On the laws of the polarization of light by refraction. By* David Brewster, *LL.D. F.R.S. L. & E.*

Read Feb. 25, 1830.

In the autumn of 1813 I announced to the Royal Society the discovery which I had then made of the polarization of light by refraction*; and in the November following I communicated an extensive series of experiments which established the general law of the phænomena. During the sixteen years which have since elapsed, the subject does not seem to have made any progress. From experiments indeed stated to have been performed at all angles of incidence with plates of glass, M. Arago announced that the quantity of light which the plate polarized by reflexion at any given angle was equal to the quantity polarized by transmission; but this result, founded upon incorrect observation, led to false views, and thus contributed to stop the progress of this branch of optics.

I had shown in 1813, from incontrovertible experiments, that the action of each refracting surface in polarizing light, produced a physical change on the refracted pencil, and brought it into a state approaching more and more to that of complete polarization. But this result, which will be presently demonstrated, was opposed as hypothetical by Dr. Young and the French philosophers; and Mr. Herschel has more recently given it as his decision, that of the two contending opinions, that which was first asserted by Malus, and subsequently maintained by Biot, Arago, and Fresnel, is the most probable,—namely, that the unpolarized part of the pencil, in place of having suffered any physical change, retains the condition of common light.

I shall now proceed to apply to this subject the same principles which I have already applied to the polarization of light by reflexion, and to establish on the basis of actual experiment the true laws of the phenomena.

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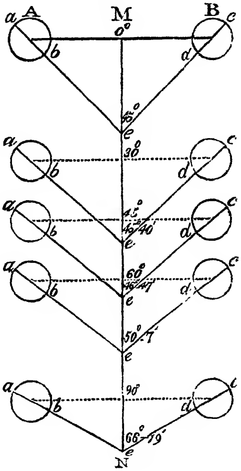
* In this discovery I was anticipated by Malus.

The first step in this inquiry is to ascertain the law according to which the polarizing force of the refracting surface changes the position of the planes of polarized light,—a subject which, in as far as I know, has not occupied the attention of any other person.

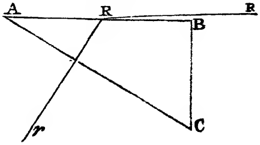
If we take a plate of glass deviating so slightly from parallelism as to throw off from the principal image the images formed by reflexion from its inner surfaces, we shall be able to see, even at great obliquities, the transmitted light free from all admixture of reflected light. Let this plate be placed upon a divided circle, so that we can observe through it two luminous discs of polarized light A, B (Fig. 1.) formed by double refraction, and having their planes of polarization inclined + 45° and — 45° to the plane of refraction. At an angle of incidence of 0°, when the light passes perpendicularly, the inclination of the planes of polarization will suffer no change; but at an incidence of 30° they will be turned round 40′; so that their inclination to M N or the angle *aec* will be 45° 40′. At 45° their inclination will be 46° 47′. At 60° it will be 50° 7′; and it will increase gradually to 90°, where it becomes 66° 19′. Hence the maximum change produced by a single plate of glass upon the planes of polarization is 66° 19′ — 45° = 21° 19′, an effect exactly equal to what is produced by reflexion at angles of 39 or 70°. It is remarkable, however, that this change is made in the opposite direction, the planes of polarization now approaching to coincidence in a plane at right angles to that of reflexion. This difference is exactly what might have been expected from the opposite character of the resulting polarization, the poles of the particles of light which were formerly repelled by the force of reflexion, being now attracted by the refracting force.

Fig. 1.



In this experiment the action of the two surfaces is developed in succession, so that we cannot deduce from the maximum rotation of 21° 19′, the real action of the first, or of a single surface, which must be obviously more than half of the action of the two surfaces, because the planes of polarization have been widened before they undergo the action of the second surface.

Fig. 2.



In order to obtain the rotation due to a single surface, I took a prism of glass A B C (Fig. 2.) having such an angle B A C, that a ray RR, incident as obliquely as possible, should emerge in a direction R r perpendicular to the surface A C. I took care that this prism was well annealed, and I caused the refraction to be performed as near as possible to the vertex A, where the glass was thinnest and consequently most free from the influence of any polarizing structure. In this way I obtained the following measures.

Glass.

Angles of Incidence.	Inclination of Planes *ab*, *cd*, (Fig. 1.) to the Plane of Reflexion.	Rotation.
87° 38′	54° 15′	9° 15′
54 50	47 25	2 25
32 20	45 22	0 22

I next made the following experiments with two kinds of glass,—the one a piece of parallel plate glass, and the other a piece of very thin crown. The latter had the advantage of separating the reflected from the transmitted light.

	Plate Glass.		Crown Glass.	
Incidence.	Inclination.	Rotation.	Inclination.	Rotation.
0°	45° 0′	0° 0′	45° 0′	0° 0′
40	47 28	2 28	47 18	2 18
55	49 35	4 35	49 19	4 19
67	52 53	7 53	52 16	7 16
80	58 53	13 53	58 42	13 42
86½	61 16	16 16	61 0	16 0

I was now desirous of ascertaining the influence of refractive power, although I had already determined in 1813, that a greater quantity of light was polarized, at the same angle of incidence, by plates of a high than by plates of a low refractive power. I experienced great difficulty in this part of the inquiry, from the necessity of having plates without any crystalline structure. I tried gold leaf in a variety of ways; but I found it almost impossible to obtain correct results, on account of the light which was transmitted unchanged through its pores.

By stretching a film of soapy water across a rectangular frame of copper wire I obtained the following measure.

WATER.

Incidence.	Inclination.	Rotation.
85°	54° 17′	9° 17′

I next tried a thin plate of metalline glass of a very high refractive power.

METALLINE GLASS.

Incidence.	Inclination.	Rotation.
0°	45° 0′	0° 0′
20	45 42	0 42
30	46 50	1 50
40	48 0	3 0
55	51 12	6 12
80	62 32	17 32

From a comparison of these results it is manifest that the rotation increases with the refractive power.

In examining the effects produced at different angles of incidence, it becomes obvious that the rotation varies with the deviation of the refracted ray; that is, with $i - i'$ the difference of the angles of incidence and refraction. Hence from a consideration of the circumstances of the phænomena I have been led to express the inclination φ of the planes of polarization to the plane of refraction by the formula,

$$\operatorname{Cot} \varphi = \cos (i - i'),$$

the rotation being $= \varphi - 45°$.

This formula obviously gives a minimum at 0°, and a maximum at 90°; and at intermediate points it represents the experiments so accurately, that when the rhomb of calcareous spar is set to the calculated angle of inclination, the extraordinary image is completely invisible,—a striking test of the correctness of the principle on which it is founded.

The above expression is of course suited only to the case where the inclination x of the planes of polarization *ab*, *cd*, (Fig. 1,) is 45°; but when this is not the case, the general expression is

$$\text{Cot } \varphi = \cot x \cos (i - i').$$

When the light passes through a second surface, as in a single plate of glass, the value of x for the second surface is evidently the value of φ after the 1st refraction, or in general, calling θ the inclination after any number n of refractions, and φ the inclination after one refraction,

$$\text{Cot } \theta = (\cot \varphi)^n$$

When θ is given by observation we have

$$\text{Cot } \varphi = \sqrt[n]{\cot \theta}.$$

The general formula for any inclination x and any number n of refractions is

$$\text{Cot } \theta = \big(\cot x \cos (i - i')\big)^n, \text{ and}$$

$$\text{Cot } \varphi = \sqrt[n]{\cot x \cos (i - i')}.$$

And when $x = 45$ and $\cot x = 1$ as in common light,

$$\text{Cot } \theta = \big(\cos (i - i')\big)^n.$$

$$\text{Cot } \varphi = \sqrt[n]{\cos (i - i')}.$$

As the term $\big(\cos (i - i')\big)^n$ can never become equal to 0, the planes of polarization can never be brought into a state of coincidence in a plane perpendicular to that of reflexion, either at the polarizing angle, or at any other angle.

In order to compare the formula with experiment, I took a plate of well annealed glass, which at all incidences separates the reflected from the transmitted rays, and in which m was nearly 1.510, and I obtained the following results.

Angles of Incidence.	Angles of Refraction.	Rotation observed.	Inclination observed.	Inclination calculated.	Difference.
0°	0° 0′	0° 0′	45° 0′	45° 0′	
10	6 36½	0 13	45 13	45 6	+0° 7′
20	13 5	0 27	45 27	45 25	+0 2
25	16 15	0 32	45 32	45 40	−0 8
30	19 20	0 40	45 40	46 0	−0 20
35	22 19	1 12	46 12	46 25	−0 13
40	25 10	1 30	46 30	46 56	−0 26
45	27 55	1 42	46 47	47 34	+0 47
50	30 29	2 48	47 42	48 24	−0 42
55	33 52	3 54	48 54	48 59	−0 5
60	35 0	5 7	50 7	50 36	−0 29
65	36 53	6 48	51 48	52 7	−0 19
70	38 29	8 7	53 7	53 59	−0 52
75	39 45	9 55	54 55	56 18	−1 23
80	40 42	12 10	57 10	59 5	−1 55
85	41 17	15 45	60 45	62 24	−1 39
86	41 21	16 39	61 39	63 9	−1 30
90	41 28			66 19	

The last column but one of the Table was calculated by the formula,

$$\text{Cot}\ \theta = \left(\cos\ (i - i')\right)^2$$

n being in this case 2. The conformity of the observed with the calculated results is sufficiently great, the average difference being only 41′. The errors however being almost all negative, I suspected that there was an error of adjustment in the apparatus; and upon repeating the experiment at 80°, the point of maximum error, I found that the inclination was fully 58° 40′, giving a difference only of 25′ in place of 1° 55′. I did not think it necessary to repeat all the observations; but I found, by placing the analysing rhomb at the calculated inclinations, that the extraordinary image invariably disappeared, the best of all proofs of the correctness of the formula.

In these experiments $x = 45°$ and cot $x = 1$; but in order to try the formula when x varied from 0° to 90°, I took the case where the angle of incidence was 80° and $\varphi = 58° 40'$ when $x = 45°$. The following were the results.

Values of x.	Inclination observed.	Inclination calculated.	Difference.
0°	0° 0′	0° 0′	0° 0′
2½	7 10	7 20	−0 10
5	9 40	8 19	+1 21
10	17 10	16 25	+0 45
15	24 42	24 6	+0 36
20	32 30	31 19	+1 11
25	39 15	37 54	+1 21
30	44 10	43 57	+0 13
35	49 38	49 28	+0 10
40	54 36	54 31	+0 5
45	58 40	59 5	−0 25
50	63 10	63 19	−0 9
55	66 58	67 15	−0 17
60	70 18	70 56	−0 38
65	74 8	74 24	−0 16
70	76 56	77 42	−0 46
75	79 20	80 53	−1 33
80	83 23	83 58	−0 35
85	86 23	86 0	+0 23
90	90 0	90 0	0 0

The last column but one was calculated by the formula $\cot\theta = \cot x \,.\, (\cot 58° 40')^2$. The differences on an average amount only to 36′.

In determining the quantity of polarized light in the refracted pencil, we must follow the method already explained for the reflected ray, *mutatis mutandis*. The principal section of the analysing rhomb being now supposed to be placed in a plane perpendicular to the plane of reflexion, the quantity of light Q′ polarized in that plane, will be

$$Q' = 1 - 2\cos^2\varphi,$$

the quantity of transmitted light being unity. But

$$\operatorname{Cot}\varphi = \cot x \cos(i - i'),$$

and as $\cot\varphi = \frac{\cos^2\varphi}{\sin^2\varphi}$ and $\sin^2\varphi + \cos^2\varphi = 1$, we have the quotient and the sum of $\sin^2 \varphi$ and $\cos^2 \varphi$ to find them. Hence

$$\operatorname{Cos}^2 \varphi = \frac{(\cot x \cos (i - i'))^2}{1 + (\cot x \cos (i - i'))^2}$$

and by substituting this for $\cos^2 \varphi$ in the former equation, it becomes

$$Q' = 1 - 2 \frac{(\cot x \cos (i - i'))^2}{1 + (\cot x \cos (i - i'))^2}$$

Now since by Fresnel's formula the quantity of reflected light is

$$R = \tfrac{1}{2} \left(\frac{\sin^2 (i - i')}{\sin^2 (i + i')} + \frac{\tan^2 (i - i')}{\tan^2 (i + i')} \right)$$

the quantity of transmitted light T will be

$$T = 1 - \tfrac{1}{2} \left(\frac{\sin^2 (i - i')}{\sin^2 (i + i')} + \frac{\tan^2 (i - i')}{\tan^2 (i + i')} \right)$$

Hence

$$Q' = \left(1 - \tfrac{1}{2} \left(\frac{\sin^2 (i - i')}{\sin^2 (i + i')} + \frac{\tan^2 (i - i')}{\tan^2 (i + i')} \right) \right) \left(1 - 2 \frac{(\cos (i - i'))^2}{1 + (\cos (i - i'))^2} \right)$$

This formula is applicable to common light in which $\cot x = 1$ disappears from the equation; but on the same principles which we have explained in a preceding paper, it becomes for partially polarized rays and for polarized light,

$$Q' = \left(1 - \tfrac{1}{2} \left(\frac{\sin^2 (i - i')}{\sin^2 (i + i')} \cos^2 x + \frac{\tan^2 (i - i')}{\tan^2 (i + i')} \sin^2 x \right) \right) \left(1 - 2 \frac{(\cot x \cos (i - i'))^2}{1 + (\cot x \cos (i - i'))^2} \right)$$

In all these cases the formula expresses the quantity of light really or apparently polarized in the plane of refraction.

As the planes of polarization of a pencil polarized + 45° and − 45° cannot be brought into a state of coincidence by refraction, the quantity of light polarized by refraction can never be mathematically equal to the whole of the transmitted pencil, however numerous be the refractions which it undergoes; or, what is the same thing, refraction cannot produce rays truly polarized, that is, with their planes of polarization parallel.

The preceding analysis of the changes produced on common light, considered as represented by two oppositely polarized pencils, furnishes us with the same conclusions respecting the partial polarization of light by refraction, which we deduced in a preceding paper respecting the partial polarization of light by reflexion. Each refracting surface produces a change in the position of the planes of polarization, and consequently a physical change upon the transmitted pencil by which it has approached to the state of complete polarization.

This position I shall illustrate by applying the formula to the experiments which I have published in the Philosophical Transactions for 1814.

According to the first of these experiments, the light of a wax candle at the distance of ten or twelve feet is wholly polarized by eight plates, or sixteen surfaces of parallel plate glass at an angle of 78° 52′. Now I have ascertained that a pencil of light of this intensity, will disappear from the extraordinary image, or appear to be completely polarized, provided its planes of polarization do not form an angle of less than 88¾° with the plane of refraction for a moderate number of plates, or 88½° for a considerable number of plates, the difference arising from the great diminution of the light in passing through the substance of the glass. In the present case the formula gives

$$\text{Cot}\,\theta = \big(\cos\,(i - i')\big)^{16} \text{ and } \theta = 88^\circ\, 50' ;$$

so that the light should appear to be completely polarized, as it was found to be.

At an angle of 61° 0′ the pencil was polarized by 24 plates or 48 surfaces. Here

$$\text{Cot}\,\theta = \big(\cos\,(i - i')\big)^{48} = 89^\circ\, 36'.$$

At an angle of 43° 34′ the light was polarized by 47 plates or 94 surfaces. Here

$$\text{Cot}\,\theta = \big(\cos\,(i - i')\big)^{94} \text{ and } \theta = 88^\circ\, 27'.$$

It is needless to carry this comparison any further; but it may be interesting to ascertain by the formula the smallest number of refractions which will produce complete polarization. In this case the angle of incidence must be 90°.

Hence $\varphi = 56^\circ\, 29'$ and $\big(\cos\,(i - i')\big)^9$ gives 88° 36′, and $\big(\cos\,(i - i)\big)^{10}$ 89° 4′; that is, the polarization will be nearly complete by the most oblique transmission through 4½ plates or 9 surfaces, and will be perfectly complete through 5 plates or 10 surfaces.

Having thus obtained formulæ for the quantity of light polarized by refraction and reflexion, it becomes a point of great importance to compare the results which they furnish. Calling R the reflected light, these formulæ become

$$Q = R\left(1 - 2\,\frac{\left(\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}{1 + \left(\dfrac{\cos(i+i')}{\cos(i-i')}\right)^2}\right) \text{ and}$$

$$Q' = 1 - R\left(1 - 2\,\frac{(\cos(i-i')\,)^2}{\Phi + (\cos(i-i'))^2}\right).$$

But these two quantities are exactly equal, and hence we obtain the important general law, that,—At the first surface of all bodies, and at all angles of incidence, the quantity of light polarized by refraction is equal to the quantity polarized by reflection. I have said 'of all bodies', because the law is equally applicable to the surfaces of crystallized and metallic bodies, though the action of their first surface is masked or modified by other causes.

It is obvious from the formula that there must be some angle of incidence where $R = 1 - R$, that is, where the reflected is equal to the transmitted light. When this takes place, we have $\sin^2 \varphi = \cos^2 \varphi'$, that is,

The reflected is equal to the transmitted light, when the inclination of the planes of polarization of the reflected pencil to the plane of reflection, is the complement of the inclination of the planes of polarization of the refracted pencil to the same plane;—or if we refer the inclination of the planes to the two rectangular planes into which the planes of polarization are brought,—The reflected will be equal to the transmitted light when the inclination of the planes of polarization of the reflected pencil to the plane of reflection, is equal to the inclination of the plane of polarization of the refracted pencil to a plane perpendicular to the plane of reflection.

In order to show the connection between the phænomena of the reflected and those of the transmitted light, I have given the following Table, which shows the inclination of the planes of polarization of the reflected and the refracted pencil, and the quantities of light reflected, transmitted, and polarized, at all angles of incidence upon glass, *m* being equal to 1.525, and the incident light = 1000.

Angles of Incidence, i.		Angles of Refraction, i'.		Inclination of Plane of Polarization of the Reflected Light, φ'.		Inclination of Plane of Polarization of the Refracted Light, φ.		Quantity of Light Reflected, R.	Quantity of Light transmitted, 1 − R.	Quantity of Light Polarized, Q.
°	′	°	′	°	′	°	′			
0	0	0	0	45	0	45	0	43.23	956.77	0.
2	0	1	$18\frac{2}{3}$	44	57	45	0.7	43.26	956.74	0.07
10	0	6	32	43	51	45	3	43.39	956.61	1.73
20	0	12	58	40	13	45	13	43.41	956.59	7.22
25	0	16	5	37	21	45	21	43.64	956.36	11.6
30	0	19	$8\frac{1}{2}$	33	40	45	31	44.78	955.22	17.24
35	0	22	6	29	8	45	44	46.33	953.67	24.4
40	0	24	56	23	41	46	0	49.10	950.90	32.2
45	0	27	$37\frac{1}{2}$	17	$22\frac{1}{2}$	46	20	53.66	946.33	44.0
50	0	30	9	10	18	46	45	61.36	938.64	57.4
56	45	33	15	0	0	47	29	79.5	920.5	79.5
60	0	34	36	5	$4\frac{1}{2}$	47	$54\frac{1}{2}$	93.31	906.69	91.6
65	0	36	28	12	45	48	42	124.86	875.14	112.7
70	0	38	2	18	32	49	28	162.67	837.33	129.8
75	0	39	18	26	52	50	55	257.56	742.44	152.3
78	0	39	54	30	44	51	48	329.95	670.05	157.6
78	7	39	55	30	53	51	50	333.20	666.80	157.65
79	0	40	4	31	59	52	7	359.27	640.73	157.6
80	40	40	13	33	13	52	$27\frac{1}{2}$	391.7	608.3	156.7
82	4	40	35	36	22	53	$26\frac{1}{3}$	499.44	500.56	145.4
84	0	40	42	38	2	53	57	560.32	439.68	134.93
85	0	40	47	39	12	54	22	616.28	383.72	123.7
85	$50\frac{2}{3}$	40	$50\frac{2}{3}$	40	12	54	44	666.44	333.56	111.11
86	0	40	51	40	$22\frac{7}{10}$	54	48	676.26	323.74	108.67
87	0	40	54	41	32	55	16	744.11	255.89	89.8
88	0	40	$57\frac{1}{2}$	41	23	55	43	819.9	180.1	65.9
89	0	40	58	43	51	56	14	904.81	95.19	36.3
90	0	40	58	45	0	56	29	1000.	0.	0.

It is obvious from a consideration of the principle of the formula for reflected light, that the quantity of polarized light is nothing at 0° because the force which polarizes it is there a minimum. At the maximum polarizing angle, Q is only 79° because the glass is incapable of reflecting more light at that angle, otherwise more would have been polarized. The value of Q then rises to its maximum at 78° 7′, and descends to its minimum at 90°; but the polarizing force has not increased from 56° 45′ to 78° 7′, as the value of φ' shows. It is only the quantity of reflected light that has increased, which occasions a greater quantity of light to disappear from the extraordinary image of the analysing rhomb.

The case however is different with the refracted light. The value of Q′ has one minimum at 0° and another at 90°, while its maximum is at 78° 7′.

while the force has its minimum at 0° and its maximum at 90°, where its effect is a minimum only because there is no light to polarize. At the incidence of 78° 7′, where the quantities Q, Q' reach their maxima, the reflected light is exactly one half of the transmitted light; $\sin^2 \varphi' = \cos^2 \varphi$ and $\tan \varphi' = \cos \varphi$.

At 85° 50′ 40″, where the transmitted light is one half of the reflected light, the deviation $(i - i') = 45°$, and the quantity of polarized light is one third of the transmitted light, one sixth of the reflected light, and one ninth of the incident light. $\mathrm{Sin}^2 \varphi' : \cos^2 \varphi =$ reflected light : transmitted light, and $\cot \varphi' = \sin (i - i')$.

At 45° we have $(i + i') + (i - i') = 90°$ and $\varphi' = (i - i')$,

$$\mathrm{Tan}\ (i - i') = \frac{\cos (i + i')}{\cos (i - i')}, \text{ and } \tan (i - i')^2 = \frac{(\sin (i - i'))^2}{(\sin (i + i'))^2}$$

At 56° 45′, the polarizing angle, the formula for reflected light becomes $R = \frac{1}{2} \left(\sin^2 (i - i')\right)^2$; but at this angle we have $i' = 90° - i$. Hence we obtain the following simple expression in terms of the angle of incidence, for the quantity of light reflected by all bodies at the polarizing angle.

$$R = \tfrac{1}{2} (\cos 2i)^2.$$

I have already mentioned the experiment of M. ARAGO with plates of glass, in which he found that "at every possible inclination" the quantity of light polarized by transmission was equal to the quantity polarized by reflexion. This conclusion he extends to single surfaces; but it is remarkable that the law is true of single surfaces in which he did not ascertain it to be true, while it is incorrect with regard to plates in which he believes that he has ascertained it to be true. As the consideration of this point does not strictly belong to the present branch of the inquiry, I shall reserve it for a separate communication, "On the action of the second surfaces of transparent plates upon Light."

Allerly, December 29, 1829.

X. *On the action of the second surfaces of transparent plates upon light.*

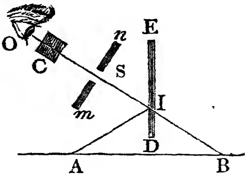
By David Brewster, *LL.D. F.R.S. Lond. & Edin.*

Read February 25, 1830.

In a paper on the Polarization of Light by Reflexion, published in the Philosophical Transactions for 1815, I showed that the Law of the Tangents was rigorously true for the second surfaces of transparent bodies, provided that the sine of the angle of incidence was less than the reciprocal of the index of refraction. The action of the second surfaces of plates at angles of incidence different from the maximum polarizing angle, was studied by M. Arago, who conducted his experiments in the following manner.

"With respect to this phænomenon," says M. Arago, "a remarkable result of experiment may here be noticed; that is, that *in every possible inclination* $A = A'$*.

Fig. 1.



"Let us suppose that a plate of glass E D (Fig. 1.) is placed in the position that the figure represents before a medium A B of a uniform tint; for instance, a sheet of fine white paper. The eye placed at O, will receive simultaneously the ray I O reflected at I, and the ray B I O transmitted at the same point. Place at *m n* an opaque diaphragm blackened, and perforated by a small hole at S. Lastly, let the eye be furnished with a doubly refracting crystal C, which affords two images of the aperture.

"If now, by means of a little black screen placed between B and I, we stop the ray B I which would have been transmitted, the crystal properly placed will give an ordinary image $= A + \frac{1}{2} B$, and an extraordinary image

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* A is the light polarized by reflexion, and A' that polarized by refraction.

$= \frac{1}{2}B$. But if the screen were placed between A and I, and the ray A I were intercepted, we should still have two images of the hole, and their intensities would be $\frac{1}{2}B'$ and $A' + \frac{1}{2}B'$ respectively. Consequently, without any screen, if the whole of the reflected light A I O, and the transmitted B I O are allowed to arrive at the eye, we shall have for the ordinary image $A + \frac{1}{2}B + \frac{1}{2}B'$, and for the extraordinary image $\frac{1}{2}B + A' + \frac{1}{2}B'$.

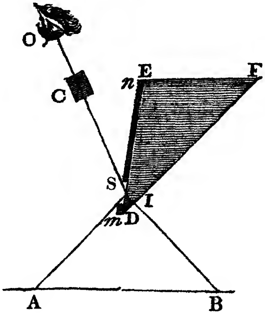
"Now it appears from actually making the experiment, that the two images are perfectly equal, *whatever may be the angle formed by the ray* A I *with the plate of glass* which can only be because A is always equal to A′. Consequently

"The quantity of polarized light contained in the pencil transmitted by a transparent plate is exactly equal, to the quantity of light polarized at right angles, which is found in the pencil reflected by the same plate."

We have no doubt that M. Arago obtained these results, particularly near the polarizing angle, at which limit they are rigorously true; but at all other angles of incidence they are wholly incorrect. When we consider, indeed, the nature of the experiment which has been lauded for its elegance and ingenuity, we shall see reason to pronounce its results as nothing more than coarse estimates, in which the apparent equality of the two images is the effect either of imperfect observation or of some unrecognized compensation.

If we make the experiment in the manner shown in Fig. 2. with a colourless and well annealed prism of glass E F D, in place of a plate of glass; and make the ray B I enter the surface F D perpendicularly at I, we get rid of all sources of error, and we obtain, what is really wanted, the result for a single surface. In this case the experiment is not disturbed by the light reflected from the inner surfaces of the prism, which is all thrown off from the pencil which enters the eye.

Fig. 2.

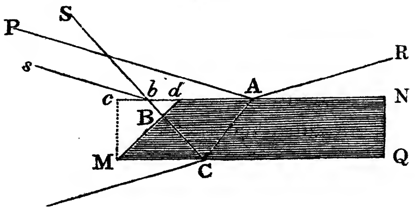


In M. Arago's form of the experiment, part of the ray B I (Fig. 1.) undergoes reflexions within the plate, and there comes along with it to the eye, at O, a portion of light polarized in the plane of reflexion: in like manner the part of the pencil A I that enters the plate, undergoes partial reflexions, and the part reflected from the first surface carries along with it another portion of light polarized in the plane of reflexion, so that four portions of light polarized in the plane of reflexion reach the eye, while only two portions reach it polarized at right angles to the plane of reflexion, viz. those which are polarized by the refraction of each of the surfaces of the plate. Now the part of the pencil A I which suffers a first reflexion from each of the surfaces of the plate, is, as we shall presently show, defective in polarized light compared with that which has experienced two refractions, so that it requires the above additional quantities to produce a compensation with the transmitted pencil B O. If this is not the true cause of the apparent compensation, that is, if M. Arago took means to exclude the reflected pencils which seem to have produced the compensation, we must then ascribe the equality of the two images to inaccuracy of observation.

But even if we admit that M. Arago's experimental results are correct with regard to plates, it necessarily follows that they cannot be true with regard to surfaces; for it is obvious from the slightest consideration of the subject, that the phænomena of the one can never be interchangeable with those of the other.

In order to demonstrate these views by an analysis of the changes which the intromitted light experiences from the two refractions and the intermediate reflexion of a transparent plate, I took a plate of glass of the shape M N (Fig. 3.) having an oblique face M *d* cut upon one of its ends. A ray of light R A, polarized + 45° and − 45°, was made to fall upon it at A, at an angle of incidence of nearly 83°, so that the inclination of the planes of polarization of the reflected ray A P was about 36½°. Now the ray A C after reflexion in the direction C S, without any refraction at B, where it emerges perpendicularly to M *d*, would also have had the inclination of its planes of polarization equal to 36½° if there had been no intermediate refraction at A; but this refraction alone being

Fig. 3.



capable of producing an inclination of 53° or a rotation of $53° - 45° = 8°$, and this rotation being in an opposite direction from that produced by the second reflexion at C, the inclination of the planes of polarization for the ray C S is nearly $44\frac{1}{2}°$, the reflexion at C having brought back the ray A C almost exactly into the state of natural light.

Without changing either the light or the angle, I cemented a prism M*cd* on the face M *d*, so that *c d* was parallel to *d* N, and I found that the second refraction at *b*, equal to that at A, changed the inclination of the planes of polarization to 53°; that is, the two refractive actions at A and *b* had overcome the action of reflexion at C, and the pencil *bs* actually contained light polarized perpendicular to the plane of reflexion.

In order to put this result to another test, I took a plate M *c* N Q (Fig. 3.) of the same glass, which separated the pencil *b s* reflected at the second surface, from the parallel pencil A P reflected from the first surface, and I found that at an angle of 83°, the value of the inclination I or φ for the ray was about $37\frac{1}{2}°$, while the value of I for the ray *b s* was nearly 55°, an effect almost equal to the refractive action of a plate at 83° of incidence.

When the pencil R A is incident on the first surface at the polarizing angle or 56° 45′, the rotation produced by refraction at A is about 2°, or the inclination $I = 45° + 2° = 47°$; but the maximum action of the polarizing force at C is sufficient to make $I = 0°$ whether x is 45° or 47°. Hence CB is completely polarized in the plane of reflexion, and the refractive action at *b* is incapable of changing the plane of polarization when $I = 0°$: the reason is therefore obvious why the two rotations at A and *b*, of 2° each, produce no effect at the maximum polarizing angle.

If we now call

φ = Inclination to the plane of reflexion produced by the 1st refraction at A,

φ' = Inclination produced by the reflexion at C,

φ'' = Inclination produced by the 2nd refraction at *b*,

We shall have

$$\operatorname{Cot} \varphi = \cos (i - i') \text{ ; or } \tan \varphi = \frac{1}{\cos (i - i')}$$

$$\text{Tan } \varphi' = \tan x \left(\frac{\cos(i+i')}{\cos(i-i')} = \frac{\cos(i+i')}{(\cos(i-i'))^2}\right.$$

$$\text{Cot } \varphi'' = \cot x\ (\cos(i-i')) = \frac{(\cos(i-i'))^3}{\cos(+i')}$$

These formulæ are suited to common light where $x = 45°$, but when x varies they become

$$\text{Cot } \varphi = \cot x\ (\cos(i-i'))$$

$$\text{Tan } \varphi' = \tan x \left(\frac{\cos(i+i')}{(\cos(i-i'))^2}\right)$$

$$\text{Cot } \varphi'' = (\cot x \left(\frac{(\cos(i-i'))^3}{\cos(i+i')}\right).$$

Resuming the formula for common light, viz. $\cot \varphi'' = \frac{(\cos(i-i'))^3}{\cos(i+i')}$, it is obvious that when $(\cos(i-i'))^3 = \cos(i+i')$, $\cot \varphi'' = 1$, and $\varphi'' = 45°$; that is, the light is restored to common light.

In glass where $m = 1.525$ this effect takes place at 78° 7′; a little below 78° in diamond; and a little above 80° in water.

At an angle below this, φ becomes less than 45°, and the pencil contains light polarized in the plane of reflexion; while at all greater angles φ is above 45°, and the pencil contains light polarized perpendicular to the plane of reflexion. Hence we obtain the following curious law.

"A pencil of light reflected from the second surfaces of transparent plates, and reaching the eye after two refractions and an intermediate reflexion, contains at all angles of incidence from 0° to the maximum polarizing angle, a portion of light polarized in the plane of reflexion. Above the polarizing angle the part of the pencil polarized in the plane of reflexion diminishes till $\cos(i+i') = (\cos(i-i'))^3$, when it disappears, and the whole pencil has the character of common light. Above this last angle the pencil contains a quantity of light polarized perpendicularly to the plane of reflexion, which increases to a maximum and then diminishes to zero at 90°."

Let us now examine the state of the pencil C S′ that has suffered only one refraction and one reflexion. Resuming the formula $\tan \varphi' = \frac{\cos(i+i')}{(\cos(i-i'))^2}$, it is evident that when $(\cos(i-i'))^2 = \cos(i+i')$, $\varphi' = 45°$, and consequently the light is restored to common light. This takes place in glass at an angle of 82° 44′. At all angles beneath this the pencil contains light polarized in the plane of reflexion; but at all angles above it, the pencil contains light polarized perpendicular to the plane of reflexion, the quantity increasing from 82° 44′ to its maximum, and returning to its minimum at 90°.

By comparing these deductions with the formula and table for reflected light given in my paper On the Laws of the Polarization of Light by Refraction, the following approximate law will be observed. When

$\text{Cos}\,(i - i') = \cos\,(i + i')$ All the incident light is reflected.

$(\text{Cos}\,(i - i'))^2 = \cos\,(i + i')$ Half the incident light is reflected.

$(\text{Cos}\,(i - i'))^3 = \cos\,(i + i')$ A third of the incident light is reflected.

$(\text{Cos}\,(i - i'))^n = \cos\,(i + i')$ An nth part of the incident light nearly is reflected.

This law deviates from the truth by a regular progression as n increases, and always gives the value of the reflected light in defect. Thus

Angles of Incidence.	Values of n.	Differences.
82° 44′	2	0
78 34	3	12
75 38	4	21
68 56	8	38
66 4	11	43
61 22	20	50

Let us now apply the results of the preceding analysis to M. Arago's experiment shown in Fig. 1. Suppose the angle of incidence to be 78° 7′, and let the light polarized by reflexion at A (Fig. 3.) be $= m$, and that polarized by one refraction also $= m$. Then since the pencil bs is common light, the polarized light in the whole reflected pencil AP, bs is $= m$, whereas the light polarized by the two refractions is $= 2m$; so that M. Arago's experiment makes two quantities appear equal when the one is double that of the other. If the angle exceeds 78° 7′, the oppositely polarized light in the pencil bs will neutralize a portion of the polarized light in the pencil AP, and the ratio of the oppositely polarized rays which seem to be compensated in the experiment, may be that of $3m$ or even $4m$ to 1.

Having thus determined the changes which light undergoes by reflexion from plates, it is easy to obtain formulæ for computing the exact quantities of polarized light at any angle of incidence, either in the pencil CBS or *bs*.

The primitive ray RA being common light, AC will not be in that state, but will have its planes of polarization turned round a quantity x by the refraction at A; so that $\cot x = \cos(i - i')$. Hence we must adopt for the measure of the light reflected at C the formula of FRESNEL for polarized light whose plane of incidence forms an angle x with the plane of reflexion. The intensity of AC being known from the formula for common light, we shall call it unity, then the intensity I of the two pencils polarized $-x$ and $+x$ to the plane of reflexion will be

$$\mathrm{I} = \frac{\sin^2(i - i')}{\sin^2(i + i')}\cos^2 x + \frac{\tan^2(i - i')}{\tan^2(i + i')}\sin^2 x \quad \text{and}$$

$$\mathrm{Q} = \mathrm{I}\left(1 - 2\,\frac{\left(\frac{\cos(i + i')}{(\cos(i - i'))^2}\right)^2}{+1\left(\frac{\cos(i + i')}{(\cos(i - i'))^2}\right)^2}\right)$$

In like manner if we call the intensity of CB $= 1$, we shall have

$$\operatorname{Tan} x = \frac{\cos(i + i')}{(\cos(i - i'))^2}$$

and the intensity I of the transmitted pencil *bs*

$$\mathrm{I} = 1 - \frac{\sin^2(i - i')}{\sin^2(i + i')}\cos^2 x + \frac{\tan^2(i - i')}{\tan^2(i + i')}\sin^2 x \quad \text{and}$$

$$\mathrm{Q} = \left(\mathrm{I}\ 1 - 2\,\frac{\left(\frac{(\cos(i - i'))^3}{\cos(i + i')}\right)^2}{1 + \left(\frac{(\cos(i - i'))^3}{\cos(i + i')}\right)^2}\right)$$

I shall now conclude this paper with the following Table computed from the formulæ in pages 148, 149, and showing the state of the planes of polarization of the three rays AC, CS, and *bs*.

Angle of Incidence on the First Surface.	Angle of Refraction at First Surface, and Angle of Incidence on Second Surface.	Inclination of Plane of Polarization of A C Fig. 3.	Inclination of Plane of Polarization of C S Fig. 3.	Inclination of Plane of Polarization of *bs* Fig. 3.
° ′	° ′	° ′	° ′	° ′
0 0	0 0	45 0	45 0	45 0
32 0	20 33	45 34	32 20	32 51
40 0	25 10	45 58	24 12	24 56
45 0	27 55	46 17	17 49	18 38
56 30	33 30	47 22	0 0	0 0
67 0	37 34	48 57	18 20	20 50
70 0	38 30	49 33	23 34	27 6
75 0	39 46	50 45	32 22	37 48
78 37	40 29	51 49	38 10	44 59
79 0	40 33	51 56	38 49	45 46
80 0	40 42	52 16	40 27	47 46
83 0	41 5	53 21	44 39	53 40
86 30	41 23	54 47	50 58	60 13
90 0	41 58	56 29	56 29	66 19

Allerly, December 31*st*, 1829.

XI. *Observations made with the invariable pendulum (No. 4.* Jones*), at the Royal Observatory, Cape of Good Hope, for the purpose of determining the compression of the earth. By the* Rev. Fearon Fallows, F.R.S. *Astronomer of the Cape Observatory. Communicated by the Lords Commissioners of the Admiralty.*

Read February 18, 1830.

THE important problem of ascertaining the ellipticity of the earth has, for a long time past, drawn forth the talents and labours of some of the most scientific men in Europe; and it still continues to be deemed an object of especial regard by all who feel an interest in the promotion of natural knowledge. To attempt to depict the strenuous exertions, the innumerable fatigues, the ardour for the improvement of science, which actuated so many illustrious persons in endeavouring to discover the true figure of the earth, would only be a waste of time; as learned Societies have always recognised and stamped the due meed of merit to each, and invariably appreciated and published to the world the valuable results which have with so many difficulties been obtained.

The nations of Europe, emuloûs of each other in a work which particularly distinguishes the acquirements of modern times, have encouraged and laudably given their protection and sanction to eminent individuals engaged in undertakings so conducive to the honour of an enlightened community.

Whatever difference of opinion may exist as to the respective values to be attached to observations in the admeasurement of an arc of the meridian, or by the variation in the lengths of the seconds pendulum made in different latitudes, there can be no doubt that the former has this paramount advantage; that certain fixed points are determined, by which the geography of a country is considerably promoted, and at length brought as nearly as possible to perfection: whilst the latter method possesses a superiority in being able to concentrate under the immediate eye of the observer the results of his inquiries, which are readily obtained, and easily performed and accomplished.

The Lords Commissioners of the Admiralty having furnished this observatory with the invariable pendulum (No. 4. Jones), which had for several years been most strictly examined by Captain Sabine at Mr. Browne's, in Portland Place, London, and subsequently by the same learned gentleman in the course of his inquiries in different parts of the earth, I wished at the earliest opportunity to have a series of experiments made upon it, which might unite every possible degree of accuracy on the part of the observers, as well as the utmost stability in the structure intended for supporting the pendulum. From many unforeseen circumstances, I found at length that the completion of this observatory would be delayed for a considerable time, though previously I had, too fondly perhaps, anticipated finishing it at a much earlier period; and I therefore took advantage of a small out-house (then of no further use) within a short distance of the building, to erect a stout brick pier, well bonded, for the support of the small transit instrument; the same which I used in forming my catalogue of 273 southern stars reduced to the beginning of 1824.

Temporary chases were made for meridian observations; and the recess for the clock (Molyneux's *) and pendulum was soon constructed in a most substantial manner within a few feet of the transit. The top of this recess consisted of a thick strong Robben-Island stone, perforated in the middle, and chiselled out at the upper part, for the reception of the brass plate containing the agate planes upon which the knife-edges of the pendulum rest. This plate was securely fastened to the stone with cement; and I found that when the agates were once truly adjusted to level, they remained (with one exception only, and I think this might be attributed to some error in one of the small levels,) during the whole of the observations that succeeded. It is hardly worthy of remark that at the end of each day's work the knife-edges were screwed up from the agates, and remained so till the commencement of another series on the first favourable opportunity. The clock (adjusted to sidereal time and corrected for rate by the stars,) was placed in the recess, having the bottom of the case resting upon a large block of stone embedded in well-wrought clay, and the back of the case was tightly screwed to a strong piece of well seasoned

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* The new clock, not the old one.

wood let into the wall. I have little doubt but that the good going of the clock was by these means in a great measure secured. On each side of the pendulum, somewhat above the middle of it, were suspended two of JONES's thermometers. The disk was formed of a small circle of card paper about three-tenths of an inch in diameter, this measure being found to be the best and most convenient. Instead of the wooden stand for supporting the small telescope, through which the coincidences, or rather the disappearances and re-appearances of the pendulum-rod and the disk are noted, as well as the magnitude of the arc of vibration, a small thick brick pier was raised so as to receive the plate to which the telescope is attached. The usual adjustments of the clock, pendulum and telescope, being completed, we were now ready to commence.

I here deem it right to notice the very able assistance which I received from Captain RONALD: his exertions were truly praiseworthy; I cannot recommend them too highly. I likewise avail myself of paying a just tribute to the aid which was afforded by Lieutenant JOHNSON, of the Honourable East India Company's Service, now superintendant of the observatory at St. Helena. This gentleman being on a visit to me, very kindly at my request took an active part in all the observations.

The sheets accompanying this short paper must be considered as the united labours of Captain RONALD, Mr. JOHNSON, and myself, and the responsibility of each as to accuracy must depend upon the papers signed with the observer's name. As far as I am able to judge of these things, the near agreement of three independent series of observations is no small argument of their accuracy.

It must not for a moment be conceived that I reckon our results as in any way final in the determination of the compression of the earth, inasmuch as it would be advisable to begin a new series of observations (say) in the midst of winter, or what would be better, in different seasons of the year, lest the coefficient for temperature might require some correction; though even this test can hardly be supposed, from the experiments made by Captain SABINE on this very pendulum in London, to be required. I have ever been of opinion, (how far correct or not, I leave others to judge,) that the invariable pendulum ought to be a standard instrument in an observatory; that it should be swung at all seasons of the year; that it would be proper to forward it on authorized occasions to the various fixed observatories now situated in the northern and southern hemispheres; that the instrument should return again to the same stations as before, and the observations be renewed; that, finally, after each circuit it should undergo a strict examination at the spot where it was first tried, in order that it might proceed again as before. Should any harm take place from improper packing or accidental circumstances, the evil would be soon discovered, and the instrument repaired. The pendulum is of a very delicate construction, and consequently it is the more necessary that it should be as often as possible compared at those points where it has previously been used.

Formulæ used in computing the Observations.

a = greater, and b the lesser arc observed at the beginning and end of each set of coincidences.
t = mean of the thermometers immediately adjacent to the pendulum.
H = height of the barometer.
τ = height of the attached thermometer.

It must be remarked that the sidereal day is reduced to a mean solar day for comparison in London.

$$\text{Log (reduct. for arc)} = 9.55132 + \log(a+b) + \log(a-b) - \log\{\log a - \log b\}$$

$$\text{Reduct. for temp.} = (t - 62^\circ) \times 0.421.$$

$$\left.\begin{array}{l}\text{Log (reduct. for vacuum)} = 9.31083 + \log \text{H}. \\ \qquad - \{\log(1 + {\cdot}\overline{002083} \,.\, \overline{t - 53^\circ}) + \log(1 + {\cdot}\overline{0001} \,.\, \overline{\tau - 53^\circ}\,\}\end{array}\right\}$$

The specific gravity of the pendulum is assumed . . .	8.6	SABINE.
The expansion for 1° of the pendulum	0.421	—
The temperature assumed as the standard for spec. grav. =	53°	—
The temperature assumed as the standard for pendulum =	62°	—
Specific gravity of the air (water = 1)	$\frac{1}{835}$	—

The height of barometer assumed as standard for spec. grav. = 29.27 inches.

The height of the pendulum above the mean level of the sea in Table Bay was found to be 32 feet nearly, and above the low-water mark 34 feet, on the day when it was determined. Hence

Vibrations.
For 34 feet at the Cape . . . . Reduction = 0.14
For 83 feet in London . . . . Reduction = 0.34
Difference = 0.20

This quantity 0.20 vibration is additive to the difference between the number of vibrations (in a mean solar day) at Mr. Browne's and the Cape.

Brief statement of the final results.

1st,—Captain Ronald.

Observations made in London.—Pendulum (No. 4. Jones).

Vibrations in a Mean Solar Day.
Mean of 100 coincidences . . . = 86164.62

Observations made at the Cape of Good Hope.

Vibrations in a Mean Solar Day.
Mean of 270 coincidences . . . . = 86097.64
Mean of 160 —— . . . . = 86097.88
Mean of 200 —— . . . . = 86097.74
True Mean of 630 —— . . . . = 86097.73

2nd,—Mr. Johnson.

Vibrations in a Mean Solar Day.
Mean of 240 coincidences . . . . = 86097.792
Mean of 180 —— . . . . = 86097.898
True Mean of 420 —— . . . . = 86097.83

3rd,—Mr. FALLOWS.

		Vibrations in a Mean Solar Day.
Mean of 401	coincidences	= 86097.67
Mean of 419	—	= 86097.66
Mean of 507	—	= 86097.70
Mean of 81	—	= 86097.69
True Mean of 1408	—	= 86097.68

Summary at the Cape.

			Vibrations in a Mean Solar Day.
Captain RONALD.	True Mean of 630	coincidences	= 86097.73
Mr. JOHNSON	True Mean of 420	—	= 86097.83
Mr. FALLOWS	True Mean of 1408	—	= 86097.68
	True Mean of 2458	—	= 86097.72

From this last conclusion, compared with that obtained by Captain SABINE at Mr. BROWNE's, in Portland Place, and which only differs $\frac{1}{50}$th of a vibration in a day from Captain RONALD's subsequent determination, the compression of the earth is very readily ascertained.

Let n = the difference in the number of vibrations of the pendulum at London and the Cape. Then for

$$\text{Compression}\begin{cases}\frac{1}{280} \;\ldots\ldots\ldots\; n = 65.752\\ \frac{1}{290} \;\ldots\ldots\ldots\; n = 67.344\\ \frac{1}{300} \;\ldots\ldots\ldots\; n = 68.828\end{cases}$$

London.	No. of vibrations in a mean solar day .	= 86164.64	SABINE.
Cape.	No. of vibrations in a mean solar day .	= 86097.72	
		66.92	
Reduction due to difference of level from low-water mark		= 0.20	
True difference in the No. of Vib^{ns} in a mean solar day .		= 67.12	

This quantity gives the compression of the earth . . = $\frac{1}{288.5}$

I am aware that this compression somewhat differs from that obtained in the Southern Hemisphere by Sir THOMAS BRISBANE, at Paramatta, though it is nearly in accordance with M. FREYCINET'S observations in Cape Town. The documents, however, are here given, and I have every hope that they may meet the public eye, and undergo the usual test of candid and liberal criticism.

FEARON FALLOWS.

Royal Observatory, Cape of Good Hope,
May 19*th*, 1829.

P. S. The latitude of the Observatory is not yet ascertained by actual observation with the new mural circle. This instrument has been attached to its pier for several months past; but from some discrepancies in reading, the cause of which is not yet discovered, I am under the necessity of assuming 33° 55′ 56″ as a very near approximate latitude*. The examination of the mural will go on, and the conclusions be forwarded home when some definite opinion may be formed of it.

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* I connected the Observatory with my former temporary one in Cape Town, by a survey over the intermediate ground.

APPENDIX.

Observations made by Captain RONALD with the invariable pendulum (No. 4. of SABINE), in Portland Place, London. Lat. 51° 31′ 8″.4.

1825.	Barom.	Therm.	No. of Coin.	Disapp.	Re-app.	Time of Coincidence.	Arc.
	inches.	°		m s	m s	h m s	°
July 31	29.950	70.48	1	31 21	31 28	1 31 24.5	1.185
	29.934	70.50	11	28 03	28 18	3 28 10.5	.610
Aug. 1	29.944	69.75	1	13 35	13 45	11 13 40.0	1.110
	29.942	70.25	11	10 34	10 49	1 10 41.5	.575
"	29.942	70.35	1	33 55	34 01	1 33 58.0	1.320
	29.941	70.65	11	30 31	30 45	3 30 38.0	.670
2	29.950	71.00	1	49 29	49 36	10 49 32.5	1.225
	29.944	71.90	11	45 59	46 15	0 46 7.0	.640
"	29.944	71.90	1	58 01	58 08	0 58 4.5	1.305
	29.944	71.80	11	54 26	54 41	2 54 33.5	.665
3	29.952	71.00	1	9 39	9 44	11 9 41.5	1.370
	29.952	71.80	11	6 02	6 14	1 6 8.0	.695
"	29.953	71.80	1	18 47	18 55	1 18 51.0	1.180
	29.924	72.00	11	15 16	15 29	3 15 22.5	.605
4	29.584	72.20	1	52 51	52 57	0 52 54.0	1.310
	29.574	72.00	11	49 11	49 24	2 49 17.5	.665
5	29.500	70.30	1	33 38	33 44	10 33 41.0	1.290
	29.520	70.75	11	30 18	30 30	0 30 24.0	.640
"	29.522	70.80	1	36 24	36 31	0 36 27.5	1.370
	29.530	71.05	11	32 51	33 03	2 33 57.0	.685
6	29.628	67.20	1	46 11	46 16	11 46 13.5	1.315
	29.616	68.60	11	43 21	43 34	1 43 27.5	.650
Mean..	29.804						

Observations computed.

Observations.	Mean Temp.	Interval.	Vib^{ns} in 24 Mean Solar Hours.	Corr. for Arc.	Reduct. to 70° 82′ Fahr.	Reduced Vib^{ns} in 24 Mean Solar Hours at 62° Fahr.
		h m s				
1	70.49	1 56 46.0	86153.354	+1.277	−0.139	86154.492
2	70.00	1 57 01.5	86153.899	1.124	0.345	86154.678
3	70.50	1 56 40.0	86153.143	1.568	0.135	86154.576
4	71.45	1 56 34.5	86152.949	1.378	+0.265	86154.592
5	71.85	1 56 29.0	86152.754	1.534	0.434	86154.722
6	71.40	1 56 26.5	86152.666	1.685	0.244	86154.595
7	71.90	1 56 31.5	86152.843	1.260	0.455	86154.558
8	72.10	1 56 23.5	86152.560	1.541	0.539	86154.640
9	70.52	1 56 43.0	86153.249	1.467	−0.126	86154.590
10	70.92	1 56 29.5	86152.772	1.665	+0.042	86154.479
11	67.90	1 57 14.0	86154.336	+1.521	−1.229	86154.629
Mean =	70.82					86154.595
					Correction for rate........	+0.441
					Correction for buoyancy....	+5.873
					Reduction to 62° Fahr.....	+3.713
						86164.622*

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* NOTE.—The accordance of this result with that obtained by Captain SABINE, is sufficient to show that the instrument remained unaltered, and warrants the comparison of any future observations with those obtained by former observers; subject, of course, to any change, which it may have undergone since that period, or that it may yet meet with, which can only be satisfactorily ascertained, when another series shall have been made with it, in London, where its rate is probably better known, than at any other of Captain SABINE's stations.

Mr. **Browne**, at whose house these observations were made, favoured me with the rate of the clock, as follows:

1825.	Cumming with Molyneux set =		
		s	
July 31	12 p.m.	+0.2	
August 1	noon	0.4	+ 0.2
	12 p.m.	0.6	0.2
2	noon	0.8	0.2
	12 p.m.	1.0	0.2
3	noon	1.2	0.2
	12 p.m.	1.4	0.2
4	noon	1.6	0.2
	12 p.m.	1.8	0.2
5	noon	2.0	0.2
	12 p.m.	2.2	0.2
6	noon	+2.4	+ 0.2

Molyneux with Mean Time.

	s
July 30th, fast ……	0.26
Aug. 6th, …………	0.55
	7)0.29
	0.042 per diem.
Cumming gains	0.400
Gains on Mean Time. .	0.442=rate.

Observations made with the pendulum (No. 4, of **Sabine**). Observatory, Cape of Good Hope. Lat. 33° 55′ 56″. By Captain **Ronald**.

1828.	Barometer.	Thermometers. A.	Thermometers. B.	No. of Coin.	Disapp.	Re-app.	Time of Coincidence.	Arc.
	inches.	°	°		m s	m s	h m s	°
Nov. 23	29.906	72.2	73.2	1	4 53	5 9	12 5 1.0	1.03
	29.917	74.7	75.7	31	42 5	42 17	14 42 11.0	.48
”	29.917	75.7	74.6	1	50 2	50 15	14 50 8.5	1.11
	29.947	71.3	72.3	31	27 13	27 26	17 27 19.5	.51
24	29.956	67.8	68.4	1	56 22	56 35	12 56 28.5	1.215
	29.983	71.2	71.8	31	33 57	34 9	15 34 3.0	.520
”	29.986	70.2	70.9	1	14 13	14 27	16 14 20.0	1.27
	29.951	69.3	69.9	31	51 50	52 2	18 15 56.0	.52
”	29.951	69.3	69.9	1	57 45	57 59	18 57 52.0	1.27
	29.964	68.1	68.9	31	35 23	35 36	21 35 29.5	.53
25	29.966	66.3	67.2	1	55 49	56 1	9 55 55.0	1.175
	29.998	67.5	68.2	31	33 44	33 59	12 33 51.5	.495
”	29.998	67.5	68.1	1	41 20	41 33	12 41 26.5	1.32
	30.006	70.6	71.4	31	19 5	19 17	15 19 11.0	.54
”	29.997	72.7	73.4	1	25 52	26 8	16 26 0.0	1.200
	29.966	72.3	73.0	31	3 8	3 22	19 3 15.0	.515
”	29.963	71.7	72.6	1	19 7	19 21	19 19 14.0	1.20
	29.966	71.9	72.8	31	56 31	56 44	21 56 37.5	.50
Mean. .	29.963							

Observations computed. The clock making on the 23rd, 86636.826*, on the 24th, 86636.916*, and on the 25th, 86637.024* sidereal vibrations in a mean solar day.

Number of Observations.	Mean Temp.	Interval.	Vibns in 24 Mean Solar Hours.	Correc. for Arc.	Reduc. to 62° Fahr.	Reduced Vibns in 24 Mean Solar Hours at 62° F.
		h m s				
1	73°.95	2 37 10.0	86085.584	+0.891	+5.031	86091.506
2	73.47	2 37 11.0	86085.643	1.024	4.829	86091.496
3	69.80	2 37 34.5	86087.102	1.164	3.284	86091.550
4	70.02	2 37 36.0	86087.189	1.232	3.376	86091.794
5	69.05	2 37 37.5	86087.276	1.249	2.968	86091.493
6	67.30	2 37 56.5	86088.486	1.077	2.231	86091.794
7	69.40	2 37 44.5	86087.790	1.329	3.115	86092.234
8	72.85	2 37 15.0	86086.073	1.138	4.568	86091.779
9	72.25	2 37 23.5	86086.569	+ 1.114	+ 4.315	86091.998
Mean ..	70.90				Mean....	86091.738
					Correction for buoyancy	+ 5.900
					Number of Vibrations in vacuo, in 24 Mean Solar Hours..	86097.638

Rate of the Sidereal Clock.

Nov. 22nd.			Nov. 24th.			Nov. 25th.		
Stars.	Rates.		Stars.	Rates.		Stars.	Rates.	
	s			s			s	
α Pegasi	+0.10	Compared with Observations of the 19th and 21st.	α Arietis ..	+0.10	Compared with Observations of the 22nd.	α Pegasi	+0.35	Compared with Observations of the 22ud and 24th.
α Andromedæ	.07		α Ceti	.29		α App. Sc. ..	.40	
γ Pegasi	.04		γ Eridani..	.42		β Arietis	.45	
β Arietis	.12		Aldebaran .	.35		α ——....	.24	
α ——....	.13		Mean..	+ .290		ρ Ceti	.49	
γ Ceti	.17					δ ——......	.36	
α ——	.28					γ ——......	.43	
γ Eridani....	.37					α ——......	.69	
Aldebaran ..	+ .23					γ Eridani....	.55	
Mean..	+ .168					Aldebaran ..	+ .63	
						Mean..	+ .459	

Here the rate appears to have been increasing, and therefore a proportion was made for each day's observations, thus:

		s
For the observations of the 23d, Rate	+	0.270
24th, . . .	+	.360
25th, . . .	+	.468

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* Note.—In these numbers, both in this and the two following series, the rate of the clock is included, being = 86636.556 + rate.

Observations with the pendulum (Continued).

1829.	Attached Therm.	Barom.	Thermometers.		No. of Coin.	Disapp.	Re-app.	Time of Coincidence.	Arc.
			A.	B.					
	°	inches.	°	°		m s	m s	h m s	
Jan. 7	72.0	30.035	72.4	73.0	1	51 16	51 31	15 51 23.5	1.12
	73.5	30.020	73.0	73.6	21	36 20	36 34	17 36 27.0	.62
”	77.5	29.964	76.2	76.8	1	34 22	34 36	21 34 29.0	1.18
	77.0	29.919	74.8	75.2	21	19 10	19 25	23 19 17.5	.66
”	77.5	29.919	75.6	76.0	1	30 4	30 18	23 30 11.0	1.37
	76.5	29.900	74.5	75.1	21	14 46	15 2	1 14 54.0	.73
8	69.0	29.952	69.0	69.6	1	45 56	46 11	15 46 3.5	1.30
	69.6	29.951	70.6	71.1	21	31 3	31 18	17 31 10.5	.71
”	72.0	29.951	70.6	71.2	1	36 41	36 56	17 36 48.5	1.26
	72.5	29.932	72.0	72.6	21	21 45	21 58	19 21 51.5	.69
”	72.5	29.932	72.0	72.7	1	27 44	27 58	19 27 51.0	1.23
	74.0	29.925	72.6	73.4	21	12 43	12 58	21 12 50.5	.66
”	74.0	29.925	72.3	73.0	1	24 10	24 22	21 24 16.0	1.49
	73.0	29.926	72.4	73.1	21	8 59	9 15	23 9 7.0	.83
”	73.0	29·926	72.3	72.9	1	14 46	15 2	23 14 54.0	1.19
	73.0	29.918	70.9	71.5	21	59 45	0 0	0 59 52.5	.66
Mean..	73.54	29.943							

On the morning of the 9th the clock was found stopped, which prevented the continuance of this series.

Observations computed. The clock making on the 7th, 86636.306, and on the 8th, 86636.215 sidereal vibrations in a mean solar day.

Number of Observations.	Mean Temp.	Interval.	Vib^{ns} in 24 Mean Solar Hours.	Correc. for Arc.	Reduc. to 62° Fahr.	Reduced Vib^{ns} in 24 Mean Solar Hours at 62° F.
	°	h m s				
1	73.00	1 45 3.5	86086.539	+1.206	+4.631	86092.376
2	75.75	1 44 48.5	86085.228	1.349	5.789	86092.366
3	75.30	1 44 45.5	86084.746	1.794	5.599	86092.139
4	70.08	1 45 7.0	86086.756	1.607	3.402	86091.765
5	71.60	1 45 3.0	86086.408	1.513	4.042	86091.963
6	72.67	1 44 59.5	86086.058	1.418	4.492	86091.968
7	72.70	1 44 51.0	86085.359	2.145	4.505	86092.009
8	71.90	1 44 58.5	86086.015	+1.363	+4.168	86091.546
Mean ..	72.88				Mean....	86092.016
					Correction for buoyancy......	+ 5.866
Number of Vibrations in vacuo, in 24 Mean Solar Hours....						86097.882

Rate of the Sidereal Clock.

Jan. 7th. Stars.	Rates.	
	s	
α Ceti	−0.32	Compared with Observations of the 6th.
α Persei....	.10	
γ Eridani ..	.30	
ο ——— ..	.28	
χ ——— ..	.04	
43d —— ..	.40	
ψ ——— ..	.30	
γ Cali Sc. ..	.47	
γ Orionis ..	.52	
δ ——— ..	.08	
ε ——— ..	.28	
ζ ——— ..	.12	
β Aurigæ ..	— .04	
Mean	— .250	

Jan. 8th. Stars.	Rates.	
	s	
α Ceti	−0.40	Compared with Observations of the 6th & 7th.
γ Eridani ..	.43	
γ Cali Sc...	.10	
Capella	.50	
Rigel......	.30	
γ Orionis ..	.20	
ζ ——— ..	.30	
β Aurigæ ..	— .50	
Mean	— .341	

Observations with the pendulum (Continued).

1829.	Attached Therm.	Barom.	Thermometers.		No. of Coin.	Disapp.	Re-app.	Time of Coincidence.	Arc.
			A.	B.					
	°	inches.	°	°		m s	m s	h m s	
Jan. 16	78.0	29.986	80.1	80.8	1	57 22	57 34	20 57 28.0	1.41
	78.0	29.998	73.2	73.7	21	41 55	42 9	22 42 2.0	.77
”	78.0	29.998	74.0	74.6	1	47 16	47 30	22 47 23.0	1.30
	76.0	29.990	72.7	73.2	21	32 2	32 16	0 32 9.0	.71
”	76.0	29.990	72.8	73.3	1	37 44	37 58	0 37 51.0	1.11
	76.0	29.990	72.4	72.9	21	22 40	22 55	2 22 47.5	.59
17	70.0	29.988	71.0	71.7	1	32 32	32 44	13 32 38.0	1.46
	72.0	30.014	71.3	71.8	21	17 32	17 47	15 17 39.5	.73
”	72.0	30.014	71.3	71.8	1	20 39.5	20 54	15 20 46.25	1.42
	74.0	30.026	71.8	72.5	21	5 39	5 55	17 5 47.0	.75
”	74.0	30.026	72.0	72.7	1	9 4	9 18	17 9 11.0	1.22
	75.5	30.025	72.6	73.1	21	54 6	54 21	18 54 13.5	.64
”	75·6	30.025	73.0	73.5	1	57 1	57 14	18 57 7.5	1.24
	75.6	30.023	72.9	73.4	21	41 59	42 13	20 42 6.0	.70
”	75.6	30.023	73.6	74.1	1	45 54	46 8	20 46 1.0	1.35
	76.0	30.006	73.1	73.7	21	30 47	31 0	22 30 53.5	.72
”	76.0	30.006	74.0	74.6	1	35 49	36 2	22 35 55.5	1.30
	75.0	29.990	73.3	73.8	21	20 43	20 57	0 20 50.0	.70
”	75.0	29.990	73.6	74.8	1	24 34	24 48	0 24 41.0	1.44
	72.0	29.988	73.0	73.6	21	9 25	9 40	2 9 32.5	.74
Mean..	75.01	30.005							

Observations computed. The clock making on the 16th, 86636.057, and on the 17th, 86636.027 sidereal vibrations in a mean solar day.

Number of Observations.	Mean Temp.	Interval.	Vib^{ns} in 24 Mean Solar Hours.	Corr. for Arc.	Reduc. to 62° Fahr.	Reduced Vib^{ns} in 24 Mean Solar Hours at 62° F.
	°	h m s				
1	76.95	1 44 34.0	86083.707	+1.890	+6.294	86091.891
2	73.62	1 44 46.0	86084.762	1.607	4.892	86091.261
3	72.85	1 44 56.5	86085.681	1.146	4.568	86091.395
4	71.45	1 45 1.5	86086.088	1.890	3.978	86091.956
5	71.85	1 45 0.75	86086.022	1.867	4.147	86092.036
6	72·60	1 45 2.5	86086.175	1.370	4.463	86092.008
7	73.20	1 44 58.5	86085.826	1.501	4.715	86092.042
8	73.62	1 44 52.5	86085.301	1.700	4.892	86091.893
9	73.92	1 44 54.5	86085.476	1.589	5.018	86092.083
10	73.75	1 44 51.5	86085.214	+1.879	+4.947	86092.040
Mean ..	73.38				Mean....	86091.860
					Correction for buoyancy	+ 5.876
Number of Vibrations in vacuo, in 24 Mean Solar Hours.....						86097·736

Rate of the Sidereal Clock.

Jan. 16th.			Jan. 17th.		
Stars.	Rates.		Stars.	Rates.	
	s			s	
12 Eridani..	−0.67	Compared with Observations of the 15th.	α Persei ..	−0.60	Compared with Observations of the 15th and 16th.
17 ——..	.40		17 Eridani..	.50	
δ ——..	.52		19 —— ..	.50	
ζ Orionis ..	.33		δ —— ..	.42	
β Aurigæ ..	.70		γ Orionis ..	.65	
δ Geminor..	.28		δ —— ..	.59	
β Can. Min.	.38		ε —— ..	.58	
Castor	.56		ζ —— ..	.74	
Procyon....	− .65		μ Columbæ	.60	
Mean..	− .499		β Aurigæ ..	.33	
			θ Leporis ..	.50	
			ε Can. Maj.	.50	
			δ ——	.48	
			δ Geminor.	.57	
			β Can. Min.	.38	
			Procyon....	.47	
			Pollux	− .58	
			Mean..	− .529	

Abstract of Captain Ronald's observations.

London. Mean of 110 coincidences . = 86164.62

Cape. . Mean of 270 coincidences . . . . = 86097.64

—— . —— 160 . . . . = 86097.88

—— . —— 200 . . . . = 86097.74

True Mean of 630 coincidences at the Cape = 86097.73

Cape of Good Hope: Observer, Mr. Johnson.

1828.	Attached Therm.	Barom.	Therm. A.	Therm. B.	No. of Coin.	Disapp.	Re-app.	Time of Coin.	Arc.	Remarks.
Dec.	°	inches.		°		m s	m s	h m s		
15	68.5	30.224	67.3	67.7	1	15 48	16 4	15 15 56.0	1.20	The Agate Planes were thrown out of level, and again adjusted before commencing these observations.
	68.7	30.220	67.9	68.2	21	1 9	1 27	17 1 18.0	0.67	
„	68.8	30.220	68.3	68.6	1	22 22	22 38	17 22 30.0	1.28	
	69.5	30.215	68.6	68.9	21	7 41	8 0	19 7 50.5	0.68	
„	69.5	30.215	68.8	69.0	1	24 9	24 27	19 24 18.0	1.07	
	69.5	30.204	68.4	68.7	21	9 31	9 53	21 9 42.0	0.56	
„	69.5	30.204	68.7	69.2	1	20 38	20 54	21 20 46.0	1.32	
	69.0	30.205	68.2	68.7	21	5 53	6 13	23 6 3.0	0.70	
16	69.0	30.118	67.8	68.3	1	50 26	50 43	15 50 34.5	1.20	High wind from S. E. during the whole of this day.
	69.5	30.110	67.3	67.7	21	35 45	36 3	17 35 54.0	0.71	
„	69.5	30.092	67.9	68.3	1	46 45	47 1	17 46 53.0	1.20	
	69.5	30.076	67.7	68.0	21	32 3	32 21	19 32 12.0	0.72	
„	69.5	30.076	68.0	68.4	1	48 27	48 44	19 48 35.5	1.23	
	70.0	30.048	67.9	68.3	21	33 44	34 1	21 33 52.5	0.77	
„	70.0	30.048	68.0	68.4	1	44 45	45 2	21 44 53.5	1.21	
	68.5	30.028	67.8	68.2	21	30 1	30 19	23 30 10.0	0.78	
17	68.5	30.028	67.5	68.0	1	39 34	39 50	14 39 42.0	1.20	
	69.5	30.022	67.8	68.3	21	24 51	25 9	16 25 0.0	0.72	
„	69.5	30.022	68.2	68.7	1	35 45	36 1	16 35 53.0	1.12	
	70.5	30.016	68.5	69.0	21	20 58	21 15	18 21 6.5	0.68	
„	70.5	30.016	69.0	69.5	1	37 33	37 50	18 37 41.5	1.12	
	71.5	30.009	69.5	69.8	21	22 41	22 58	20 22 49.5	0.67	
„	71.5	30.009	68.5	68.8	1	34 1	34 17	20 34 9.0	1.30	
	70.0	30.004	69.0	69.5	21	19 8	19 25	22 19 16.5	0.79	

Observations computed.

No. of Observ.	Mean Temp.	Interval.	Uncorrected Number of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° Fahr.	Red. Vib^{ns} in 24 Mean Solar Hours at 62° Fahr.
	°	h m s		−	+	+	
1	67.78	1 45 22.0	86088.392	0.39	1.394	2.432	86091.828
2	68.60	1 45 20.5	86088.275	0.39	1.524	2.779	86092.188
3	68.73	1 45 24.0	86088.568	0.39	1.052	2.833	86092.063
4	68.70	1 45 17.0	86087.960	0.39	1.622	2.821	86092.013
5	67.78	1 45 19.5	86088.198	0.21	1.461	2.433	86091.882
6	67.97	1 45 19.0	86088.137	0.21	1.482	2.513	86091.922
7	68.15	1 45 17.0	86087.960	0.21	1.609	2.589	86091.948
8	68.10	1 45 16.5	86087.920	0.21	1.597	2.568	86091.875
9	67.90	1 45 18.0	86088.059	0.21	1.482	2.484	86091.815
10	68.63	1 45 13.5	86087.660	0.21	1.300	2.791	86091.541
11	69.45	1 45 8.0	86087.180	0.21	1.284	3.136	86091.390
12	68.95	1 45 7.5	86087.120	0.21	1.753	2.925	86091.588
Mean..	68.39					Mean	86091.838
						Correction for buoyancy	+5.956
						Number of Vibrations in vacuo, in 24 Mean Solar Hours ..	86097.794

Rate of the Sidereal Clock.

Dec. 14th.		Dec. 15th.	
	s		s
By α Arietis......	−0.53	By α Ceti........	−0.74
Aldebaran	0.26	Aldebaran	0.28
Capella	0.50	Capella	0.50
Rigel	0.12	Rigel	0.14
β Tauri	0.34	β Tauri	0.44
ζ Orionis......	0.60	δ Orionis......	0.30
β Aurigæ	0.74	ζ ———......	0.39
Mean Rate..	−0.44	Mean Rate..	−0.39

Mean Rate on the 18th by Mr. Fallows's Observations 0.03

Remark.—On the 16th and 17th the weather was unfavourable for observation; the quantity assumed for the rate of the clock on those days is a mean of its rate on the 15th and 18th: probably on the 17th the rate was less than that which has been assigned.

Observations with the pendulum (Continued).

1829.	Attached Therm.	Barom.	Therm.		No. of Coin.	Disapp.	Re-app.	Time of Coin.	Arc.	Remarks.
			A.	B.						
	°	inches.	°	°		m s	m s	h m s		
Jan. 20.	68.5	29.972	68.6	69.0	1	9 46	9 55	17 9 50.5	1.67	The Pendulum was not stopped between these Observations.
	73.7	30.017	73.0	73.6	21	54 19	34 35	18 54 27.0	1.12	
”	73.7	30.017	72.8	73.4	1	59 34	59 49	18 59 41.5	1.12	
	71.7	30.010	71.7	72.0	21	44 22	44 37	20 44 29.5	0.83	
”	71.7	30.010	71.8	72.0	1	1 55	2 5	21 2 0.0	1.60	The Pendulum was not stopped between these Observations.
	73.5	30.003	73.0	73.5	21	46 18	46 33	22 46 25.5	1.31	
”	73.5	30.003	73.2	73.7	1	51 31	51 45	22 51 38.0	1.31	
	74.5	30.003	73.5	74.0	21	36 6	36 22	24 36 14.0	0.90	
21.	67.0	30.137	68.8	69.0	1	14 28	14 37	15 14 32.5	1.68	The Pendulum was not stopped between these Observations.
	70.0	30.145	69.4	69.9	21	59 25	59 41	16 59 30.0	0.91	
”	70.0	30.145	69.4	69.9	1	4 41	4 57	17 4 49.0	0.90	
	72.0	30.145	70.0	70.4	21	50 1	50 17	18 50 9.0	0.50	
”	72.0	30.145	70.6	71.0	1	7 6	7 17	19 7 11.5	1.60	The Pendulum was not stopped between these Observations.
	73.0	30.137	70.2	70.6	21	52 3	52 17	20 52 10.0	0.87	
”	73.0	30.137	70.5	70.8	1	57 18	57 34	20 57 26.0	0.86	
	72.0	30.116	70.3	70.6	21	43 34	43 49	22 43 41.5	0.48	
”	72.0	30.116	70.7	71.3	1	59 40	59 49	22 59 44.5	1.70	
	71.0	30.114	69.9	70.2	22	49 51	50 9	0 50 0.0	0.87	

Observations computed.

No. of Observ.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° Fahr.	Reduced No. of Vibns in 24 Mean Solar Hours at 62° Fahr.
	°	h m s					
1	71.05	1 44 36.5	86084.431	+0.54	+3.148	+3.810	86091.929
2	72.50	1 44 48.0	86085.440	+0.54	1.547	4.421	86091.948
3	72.57	1 44 25.5	86083.460	+0.54	3.458	4.454	86091.912
4	73,60	1 44 36.0	86084.392	+0.54	1.978	4.883	86091.793
5	69.27	1 45 0.5	86086.529	−0.410	2.666	3.061	86091.846
6	69.93	1 45 20.0	86088.216	−0.410	0.781	3.338	86091.924
7	70.60	1 44 58.5	86086.373	−0.410	2.425	3.620	86092.008
8	70.55	1 45 15.5	86087.823	−0.410	0.716	3.600	86091.729
9	70.52	1 50 15.5	86086.529	−0.410	+2.609	+3.587	86092.315
		Mean....					86091.934
		Correction for buoyancy					+5.944
Number of Vibrations in vacuo, in 24 Mean Solar Hours ..							86097.878

Rate of the Sidereal Clock.

Jan. 20th.	s	Jan. 21st.	s
By β Tauri........	+0.44	By β Tauri........	−0.44
δ Orionis	0.47	δ Orionis	0.40
ζ ———	0.50	ζ ———	0.34
α ———	0.82	α ———	0.48
Procyon	0.64	Mean Rate..	−0.41
Pollux	0.88		
Mean Rate..	+0.54		

Remark.—From the above result, it appears that the rate varied nearly a second in the space of 24 hours: there is reason to suppose that the change was owing to some sudden derangement which the clock might have suffered on the night of the 20th. From the tolerable accordance with each other of the pendulum observations of the 21st, it may be presumed that the rate was equable during that day.

Abstract of Mr. Johnson's observations.

Mean of 240 coincidences	=	86097.792
Mean of 180 coincidences	=	86097.878
True mean of 420 coincidences	=	86097.829

General Remark.—During both series of experiments, the observatory was kept in the same state, the North shutter being open during the day, and the door of the recess, in which the clock and pendulum stood, constantly closed, with the exception of the two small openings made in it for the purpose of showing the dial plate, and the graduated arc of the pendulum. During the second series, the weather was particularly favourable, both for observations of the transit instrument and pendulum.

Cape of Good Hope; Observer Mr. Fallows.

First Series.

1829.	Attached Therm.	Barom.	Therm. A.	Therm. B.	No. of Coin.	Disapp.	Re-app.	Time of Coin. by the Sidereal Clock.	Arc.	Remarks.
	°	inches.	°	°		m s	m s	h m s	°	
Dec. 2	66.5	30.205	68.2	69.1	1	50 41	50 56½	16 50 48¾	1.38	This day very favourable for observation; little wind—the Observatory chase shut, except when observing. The Thermometers read off before the North chase was opened.
	67.0	30.205	68.0	68.9	31	28 31	28 51	19 28 41	0.51	
”	67.1	30.205	68.0	68.9	1	33 50	34 4½	19 33 57¼	1.48	
	67.5	30.207	67.1	68.0	31	11 44	12 1	22 11 52½	0.61	
3	66.0	30.205	67.1	68.0	1	38 1	38 16	14 38 8½	1.38	
	68.5	30.205	69.6	70.5	31	15 56	16 13	17 16 4½	0.57	
”	68.7	30.205	70.1	71.2	1	42 43½	42 59	17 42 51¼	1.20	The weather fine during the whole of the day.
	68.5	30.175	70.2	71.2	31	20 24	20 44½	20 20 34¼	0.50	
”	68.6	30.174	70.2	71.3	1	30 6	30 22	20 30 14	1.22	
	68.0	30.151	68.0	69.0	31	7 57	8 15	23 8 6	0.49	
4	65.0	30.102	64.2	65.0	1	3 18	3 32	12 3 25	1.38	
	67.5	30.139	66.9	67.6	31	41 31	41 49	14 41 40	0.57	
”	67.6	30.140	66.9	67.6	1	47 24	47 40	14 47 32	1.20	
	68.5	30.165	69.1	70.0	31	25 26½	25 41½	17 25 34	0.51	The South-east wind springing up. All the doors closed. What is called at Cape Town a South-easter, ought to be designated South-by-east.
”	68.6	30.166	69.1	70.0	1	31 10	31 26	17 31 18	1.35	
	68.5	30.120	68.3	69.1	31	9 0	9 18	20 9 9	0.58	
”	68.5	30.120	67.0	67.8	1	19 40	19 54	20 19 47	1.41	
	68.0	30.108	67.3	68.0	31	57 33	57 52	22 57 42½	0.58	
5	64.0	30.125	63.0	63.6	1	40 18	40 34	10 40 26	1.42	The weather rainy; wind due North; no stars last night, from clouds.
	64.5	30.127	64.0	64.8	31	18 32	18 49	13 18 40½	0.68	
”	64.5	30.127	64.1	64.8	1	24 13	24 28	13 24 20½	1.29	
	66.4	30.127	65.8	66.4	31	2 31	2 46	16 2 38½	0.56	
”	66.3	30.127	66.3	67.0	1	13 24	13 32	16 13 28	1.72	
	68.0	30.200	68.4	69.2	42	49 5½	49 22½	19 49 14	0.50	The sky clearing up.
”	68.0	30.200	68.1	68.9	1	54 25	54 41	19 54 33	1.13	
	68.0	30.115	68.0	68.5	31	32 20	32 36	22 32 28	0.49	
Mean	67.23	30.1594								

First Series computed.

1829.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° Fahr.	Reduced Vibns in 24 Mean Solar Hours.
	°	h m s					
	68.550	2 37 52¼	86087.78	−0.52	+1.354	+2.758	86091.37
	68.000	2 37 55¼	86087.94	0.52	1.682	2.526	86091.63
	68.800	2 37 56	86087.98	0.41	1.464	2.863	86091.50
	70.540	2 37 43	86087.24	0.41	1.114	3.595	86091.54
	69.625	2 37 52	86087.76	0.41	1.121	3.210	86091.68
	65.925	2 38 15	86089.08	0.26	1.464	1.652	86091.94
	68.400	2 38 2	86088.34	0.26	1.362	2.694	86092.14
	69.125	2 37 51	86087.70	0.26	1.442	3.000	86091.88
	67.525	2 37 55½	86087.98	0.26	1.564	2.326	86091.61
	63.850	2 38 14½	86089.06	0.26	1.730	0.779	86091.31
	65.275	2 38 18	86089.26	0.26	1.326	1.379	86091.69
	67.725	3 35 46	86088.24	0.26	1.797	2.410	86092.15
	68.375	2 37 55	86087.94	−0.26	+1.017	+2.684	86091.38
Mean..	67.824					Mean	86091.69
						Buoyancy	+5.98
(No. of Coincidences = 401)						Number of Vibrations in vacuo, in 24 Mean Solar Hours.. ..	860 97.67

Rate of the Sidereal Clock.

Dec. 2nd.		Dec. 3rd.		Dec. 4th to Dec. 9th.	
	s		s		s
By α Arietis......	−0.38	By α Ceti........	−0.24	By α Ceti........	−0.33
α Ceti........	0.52	Capella	0.30	Capella	0.26
Rigel	0.50	β Tauri	0.50	Rigel	0.22
β Tauri	0.70	δ Orionis......	0.58	β Tauri	0.23
δ Orionis	0.50	ζ ——.....	0.36	δ Orionis......	0.23
ζ ——......	0.55	β Aurigæ	−0.50	β Aurigæ	0.27
β Aurigæ	−0.46			Sirius	−0.28
		Mean Rate ..	−0.41		
Mean Rate..	−0.52			Mean Rate..	−0.26

Second Series.

1829.	Attached Therm.	Barom.	Thermometers. A.	Thermometers. B.	No. of Coin.	Disapp.	Re-app.	Time of Coincid. by the Sidereal Clock.	Arc.
	°	inches.	°	°		m s	m s	h m s	°
Dec. 18	66.1	30.022	65.2	65.7	1	15 26	15 38	11 15 32	1.78
	69.5	30.053	68.1	68.7	53	49 17	49 35	15 49 26	0.39
”	70.0	30.053	71.2	71.7	1	0 9	0 13	16 0 11	1.87
	72.0	30.016	72.2	72.4	59	4 21	4 37	21 4 29	0.40
”	72.1	30.015	72.2	72.4	1	25 47	26 4	21 25 55½	1.12
	71.0	30.024	71.0	71.5	16	44 30	44 47	22 44 38½	0.76
”	71.0	30.025	71.0	71.4	1	55 5	55 22	22 55 13½	0.89
	71.5	30.054	70.1	70.5	18	24 26	24 42	0 24 34	0.55
19	67.5	30.165	68.0	68.3	1	56 39	56 47	12 56 43	1.72
	72.0	30.185	71.5	72.0	58	16 5	16 18	17 56 11½	0.37
”	72.0	30.185	71.6	72.2	1	1 37	1 45	18 1 41	1.79
	72.5	30.102	70.6	71.2	54	39 43	39 57	22 39 50	0.39
”	72.5	30.101	71.2	71.7	1	44 54	45 11	22 45 2½	0.92
	71.0	30.086	69.8	70.2	20	24 48	25 7	0 24 57½	0.58
20	67.5	30.023	67.2	67.8	1	19 24	19 31	11 19 27½	1.77
	72.0	29.995	73.8	74.2	63	44 56	45 7	16 45 1½	0.34
”	72.5	29.994	74.9	74.9	1	51 39	51 47	16 51 43	1.74
	75.0	29.909	76.8	77.1	52	18 26	18 44	21 18 35	0.44
”	75.0	29.908	77.0	77.0	1	30 29	30 46	21 30 37½	1.31
	74.5	29.901	74.8	75.3	36	33 38	33 55	0 33 46½	0.50
Mean..	71.360	30.036							

Second Series computed.

1829.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° FAHR.	Reduced Vib^ns in 24 Mean Solar Hours.
		h m s					
	66.925	4 33 54	86088.34	−0.03	+1.628	+2.073	86092.04
	71.875	5 4 18	86086.14	0.03	1.757	4.157	86092.02
	71.775	1 18 43	86086.28	0.03	1.427	4.115	86091.79
	70.600	1 29 20½	86087.04	0.03	0.849	3.621	86091.48
	69.950	4 59 28½	86086.90	0.17	1.505	3.347	86091.54
	71.400	4 38 9	86086.30	0.17	1.641	3.957	86091.73
	70.725	1 39 55	86087.40	0.17	0.906	3.673	86091.81
	70.750	5 25 34	86086.59	0.08	1.499	3.684	86091.69
	75.925	4 26 52	86084.68	0.08	1.689	5.862	86092.15
	76.025	3 3 9	86084.68	−0.08	+1.248	+5.905	86091.75
Mean..	71.595					Mean....	86091.76
						Buoyancy............	+5.90
(No. of Coincidences = 419)			Number of Vibrations in vacuo, in 24 Mean Solar Hours ..			..	86097.66

Rate of the Sidereal Clock.

Dec. 18th.		Dec. 19th.		Dec. 20th.	
	s		s		s
By β Aurigæ	−0.04	By α Ceti........	−0.28	By α Ceti	+0.03
Sirius	−0.15	Aldebaran	0.00	Aldebaran	−0.02
Procyon......	+0.05	β Tauri	0.00	Rigel	0.22
Pollux	0.00	δ Orionis......	0.12	β Tauri	0.04
		ζ ———......	−0.46	δ Orionis......	0.07
				ζ ———......	−0.20
Mean Rate..	−0.03	Mean Rate..	−0.17	Mean Rate..	−0.08

Third Series.

1829.	Attached Therm.	Barom.	Therm. A.	Therm. B.	No. of Coin.	Disapp.	Re-app.	Time of Coin. by the Sidereal Clock.	Arc.	Remarks.
	°	inches.	°	°		m s	m s	h m s	°	
Dec. 23	72.0	30.094	71.9	72.2	1	26 48	27 4	22 26 56	1.29	Closed the clock door; the face open.
	71.0	30.070	70.0	70.3	28	48 33	48 52	0 48 42½	0.58	
24	66.0	30.095	65.0	65.5	1	45 0	45 5	11 45 2½	1.84	The light on the right not good, so opened the clock door. Door shut while swinging.
	70.0	30.094	69.1	69.6	62	5 59	6 10	17 6 4½	0.34	
”	71.1	39.094	71.9	72.4	1	10 37	10 52	17 10 44½	1.31	Clock door still left open.
	74.0	30.072	74.3	75.0	24	10 14	10 33	19 10 23½	0.68	
”	74.0	30.072	76.0	76.6	1	16 1	16 17	19 16 9	0.92	There may be a slight mistake in the first arc 0.92; the pencil mark on the slate being dim.
	74.0	30.055	75.0	75.7	31	53 20	53 32	21 53 26	0.40	
”	74.0	30.054	75.4	76.0	1	59 12	59 26	21 59 19	1.38	Clock door not quite closed.
	73.0	30.080	72.9	73.1	33	46 48	49 4	0 46 56	0.50	
25	74.0	30.196	73.1	73.7	1	46 45	46 57	19 46 51	1.64	
	73.0	30.185	73.0	73.5	59	49 52	50 14	0 50 3	0.48	
26	72.0	30.010	71.5	72.0	1	23 25	23 38	12 23 31½	1.60	Clock door shut.
	77.0	30.060	76.2	76.9	65	58 42	58 59	17 58 50½	0.28	
”	79.0	30.061	79.9	79.9	1	9 1	9 19	18 9 10	0.98	Lower shutter of the clock open.
	82.0	30.020	81.6	81.9	21	53 29	53 45	19 53 37	0.58	
”	82.0	30.019	82.0	82.0	1	0 50	1 6	20 0 58	1.16	
	81.0	30.003	80.4	80.6	32	42 30	42 46	22 42 38	0.52	
”	81.0	30.002	80.2	80.4	1	53 4	53 22	22 53 13	1.12	
	79.0	29.970	78.8	79.3	25	58 15	58 33	0 58 24	0.66	
27	74.0	29.942	74.0	74.0	1	40 47	40 55	12 40 51	1.83	Lower shutter of the clock open during this day.
	79.0	29.965	79.8	80.1	65	15 26	15 36	18 15 31	0.28	
”	80.0	29.975	80.0	80.1	1	21 4	21 22	18 21 13	1.28	The weather very favourable.
	80.0	29.976	80.6	80.8	28	41 54	42 12	20 42 3	0.58	
”	81.0	29.976	81.0	81.3	1	47 44	48 0	20 47 52	1.34	
	80.0	29.930	80.0	80.1	29	13 43	14 1	23 13 52	0.60	
”	80.0	29.928	80.0	80.2	1	23 52	24 10	23 24 1	1.13	
	78.0	29.962	78.0	78.5	19	57 46	58 8	0 57 54	0.68	
Mean	76.107	30.034								

Third Series computed.

1829.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° Fahr.	Reduced Vib^{ns} in 24 Mean Solar Hours.
	°	h m s					
	71.100	2 21 46	86086.56	−0.01	+1.361	+3.831	86091.74
	67.300	5 21 2	86087.82	+0.14	1.587	2.231	86091.78
	73.400	1 59 39	86085.38	+0.14	1.567	4.799	86091.89
	75.825	2 37 17	86085.72	+0.14	0.675	5.820	86092.35
	74.350	2 47 37	86085.22	+0.14	1.336	5.199	86091.89
	73.325	5 3 12	86085.12	+0.14	1.640	4.768	86091.67
	74.150	5 35 20	86085.36	−0.02	1.167	5.115	86091.62
	80.825	1 44 27	86083.60	−0.02	0.974	7.925	86092.45
	81.250	2 41 40	86082.82	−0.02	1.098	8.104	86092.00
	79.675	2 5 11	86082.88	−0.02	1.241	7.441	86091.54
	76.975	5 34 40	86084.30	−0.02	1.428	6.304	86092.01
	80.375	2 20 50	86082.92	−0.02	1.348	7.736	86091.98
	80.600	2 26 0	86082.72	−0.02	1.464	7.831	86091.99
	79.175	1 33 53	86082.86	−0.02	+1.302	+7.231	86091.37
Mean..	76.301					Mean	86091.85
						Buoyancy	+5.85
(No. of Coincidences = 507)						Number of Vibrations in vacuo, in 24 Mean Solar Hours.. ..	86097.70

Rate of the Sidereal Clock.

Dec 23rd.		Dec. 24th.		Dec. 25th, 26th, & 27th.	
	s		s		s
By α Ceti	−0.02	By α Ceti ...... :	+0.02	By Rigel	+0.09
Aldebaran	−0.01	Aldebaran	0.26	β Tauri	+0.01
Rigel	−0.05	Rigel	0.17	δ Orionis......	−0.10
β Tauri	−0.01	β Tauri	0.02	ζ ———......	−0.08
δ Orionis......	+0.01	δ Orionis......	0.00	Mean Rate..	−0.02
ζ ———......	−0.08	ζ ———......	+0.35		
Mean Rate ..	−0.01	Mean Rate..	+0.14		

Fourth Series.

1830.	Attached Therm.	Barom.	Therm. A.	Therm. B.	No. of Coin.	Disapp.	Re-app.	Time of Coin. by the Sidereal Clock.	Arc.	Remarks.
	°	inches.	°	°		m s	m s	h m s	°	
Jan. 22	67.5	30.088	67.8	68.2	1	16 11	16 29	16 16 20	1.40	Shut the clock door, and kept the Northern chase open.
	68.0	30.085	67.2	67.4	11	8 47	9 5	17 8 56	1.03	
"	68.0	30.085	67.8	68.1	1	14 4	14 20	17 14 12	1.01	
	69.0	30.085	68.2	68.6	11	6 47	7 3	18 6 55	0.76	
"	69.0	30.085	69.2	69.7	1	12 4	12 20	18 12 12	0.72	
	70.0	30.087	69.8	70.2	11	4 48	5 4	19 4 56	0.52	
"	72.0	30.087	72.0	72.3	1	18 30½	18 36½	19 18 33½	1.91	
	72.0	30.084	71.5	71.7	11	10 47	11 2	20 10 54½	1.36	
"	72.0	30.084	72.6	73.2	1	16 1	16 17	20 16 9	1.34	Closed the clock door, the Northern chase still open.
	72.0	30.062	71.2	71.5	11	8 31	8 47	21 8 39	0.97	
"	72.0	30.062	71.7	71.9	1	13 45½	14 2½	21 13 54	0.93	
	72.0	30.055	70.5	70.8	11	6 20	6 36	22 6 28	0.71	
"	72.5	30.037	71.2	71.5	1	10 21	10 37	23 10 29	1.47	
	72.5	30.035	70.2	70.4	12	8 1½	8 18½	0 8 10	1.06	
"	72.5	30.035	71.2	71.5	1	13 17	13 33	0 13 25	1.02	
	72.5	30.043	70.0	70.3	11	5 51½	6 8½	1 6 0	0.78	
Mean..	70.844	30.063								

Fourth Series computed.

1830.	Mean Temp.	Interval.	Uncorrected No. of Vibrations.	Clock's Rate.	Correc. for Arc.	Reduc. to 62° Fahr.	Reduced Vib^{ns} in 24 Mean Solar Hours.
	°	h m s					
	67.650	0 52 36	86087.52	−0.93	+2.481	+2.379	86091.45
	68.175	0 52 43	86088.76	0.93	1.275	2.600	86091.70
	69.725	0 52 44	86088.90	0.93	0.624	3.252	86091.85
	71.875	0 52 21	86084.94	0.93	4.340	4.157	86092.51
	72.125	0 52 30	86086.48	0.93	2.168	4.263	86091.98
	71.225	0 52 34	86087.18	0.93	1.095	3.984	86091.33
	70.825	0 57 41	86085.86	0.93	2.600	3.715	86091.24
	70.750	0 52 35	86087.34	−0.93	+2.053	+3.684	86092.15
Mean..	70.294					Mean....	86091.78
						Buoyancy..............	+5.91
(No. of Coincidences = 81)			Number of Vibrations in vacuo, in 24 Mean Solar Hours..			..	86097.69

Rate of the Sidereal Clock.

Jan. 22nd.	
	s
By Castor........	−0.90
ε Geminor.....	0.96
Sirius	0.82
δ Geminor.....	0.90
Castor........	0.80
Procyon	1.02
Pollux........	−1.08
Mean Rate..	−0.93

Abstract of Mr. Fallows's observations.

Mean of 401 coincidences . . . = 86097.67
Mean of 419 coincidences . . . = 86097.66
Mean of 507 coincidences . . . = 86097.70
Mean of 81 coincidences . . . = 86097.69

True Mean of 1408 coincidences . . . = 86097.68

NOTE BY CAPTAIN SABINE.

The observations which Mr. FALLOWS has communicated to the Society in this memoir, having been corrected for buoyancy and expansion, before the volume of the Phil. Trans. had reached the Cape, in which the true value of those corrections is assigned from experiments with a pendulum of the same materials and figure as that employed by Mr. FALLOWS, I have re-computed his results with the correct elements of reduction, and find the retardation of the vibrations at the Cape, compared with those in London, to be 67.15 per diem, instead of 67.12, the difference between Mr. FALLOWS'S calculation and mine amounting only to three hundredths of a vibration per diem.

The small amount of the difference, on the employment of the more correct elements of reduction, is an illustration of the remark, with which I concluded the paper on the reduction to a vacuum of an invariable pendulum (Phil. Trans. 1829, page 236.); that in relative experiments, computed before the true reduction to a vacuum was known, and in which a correction for expansion was employed, derived directly from the vibration of the pendulum at the same spot in different temperatures, (as is the case in Mr. FALLOWS'S calculation,) a compensation takes place of the errors of the respective reductions for expansion and resistance, leaving the only uncompensated error in the final result, that arising from barometric differences, which in all cases of comparison between stations not far removed from the level of the sea, cannot be otherwise than extremely small.

In Mr. FALLOWS'S calculation he has taken the rate in London of the invariable pendulum which Captain RONALD took out to the Cape, solely from my observations with it: if, however, Captain RONALD'S observations with the same pendulum in London be added to mine, and a true mean be taken corresponding to the number of observations of each observer, the retardation is precisely that stated by Mr. FALLOWS; namely, 67.12 vibrations per diem.

XII.—*Statement of the principal circumstances respecting the united Siamese Twins now exhibiting in London.* *By* George Buckley Bolton, *Esq., Member of the Royal College of Surgeons, and of the Medical and Chirurgical Society of London. Communicated by the President.*

Read April 1, 1830.

THE youths who are the subject of the following memoir, were born in May 1811 in the kingdom of Siam, at Maklong, a small village sixty miles distant from the capital, Bankok. They are the offspring of Chinese parents, and have been named, the one Chang, and the other Eng. Bankok is situated on the river Minam, forty miles from its mouth, between the Burmese and Chinese empires, in latitude 13° north, and longitude 101° east. Siam is tributary to the latter empire.

The king of Siam soon heard of the birth of Chang and Eng, and at first designed to have them put to death, conceiving them to be monsters, and imagining that the existence of such beings portended some evil to his kingdom. But afterwards hearing that they were harmless, and would probably be able to support themselves by labour, he allowed them to remain unmolested. Mr. Robert Hunter, a British merchant resident at Siam six years ago, saw the twins for the first time in a fishing-boat on the river Minam. They were naked from the hips upwards, were very thin in their persons, and it being then dusk, he mistook them for some strange animal. Shortly afterwards he endeavoured to prevail on the Siamese government to allow them to visit England, but could not at that time succeed. However, in March 1829, Captain Coffin and Mr. Hunter conjointly obtained this permission for a certain period, having gained the ready acquiescence of their mother, on the condition of a provision being made for her during the absence of her sons.

The mother is stated by Captain Coffin to be about five feet seven inches in height, well formed, with large hips, and, for her country, a strong woman, though of lax fibre. She was thirty-five years of age when her twins were born, and had previously given birth to several other children, none of whom had any malformation. She declared to Captain COFFIN that she suffered less during her pregnancy with these than on any similar occasion, and also that her labour was not attended with the least difficulty. She further stated that the twins were born with the head of one between the legs of the other, and were rather small infants. All her other children, except these and two others, a brother and sister, are dead.

The united twins left Siam in the beginning of April 1829, with Captain COFFIN and Mr. HUNTER, on board the American ship Sachem, and arrived at Boston in the United States in the following August, where they remained eight weeks, during which period they excited the greatest interest among scientific and professional men.

They embarked at New York for England in the October following, and arrived in London on the evening of the 19th of November 1829. On the 24th of November an invitation was given to the most distinguished persons of the medical faculty in London, to view them at the Egyptian Hall, Piccadilly, and on the 26th of November I was desired to become their sole medical attendant.

These youths are both of the same height, namely, five feet two inches; and their united weight is one hundred and eighty pounds. They are much shorter, and appear less advanced in puberty, than youths of this country at the age of eighteen years; but the average stature of their countrymen is less than that of Europeans. Many of our ordinary twins bear a stronger resemblance to each other in countenance than is observed in these youths. Their bodies and limbs are well formed, but the spine of CHANG, who habitually holds his arm over the shoulder of ENG, is considerably curved laterally, an effect which is apparently the result of this long continued habit. They have not the broad and flat forehead so characteristic of the Chinese race, but resemble the lower class of the people of Canton in the colour of their skins, and in the forms of their noses, lips, eyes, and ears. The left eye of CHANG is weaker than the right; but this is reversed in the case of ENG, so that each sees best with the eye nearest his brother. Their bodies are much paler now than they were on their first arrival in England. Their genital organs are, like all their other external parts, regularly formed; but the youths are naturally modest, and evince a strong repugnance to any close investigation on this subject.

The band of union is formed in the following manner.—At the lowest part of the sternum of each boy, the ensiform cartilage is bent upwards and forwards, meeting the other in the middle of the upper part of the band, where moveable joints exist, which admit of vertical as well as lateral motion; each junction appearing to be connected by ligamentous structures. It is difficult to define precisely where the respective cartilages from each body meet, and whether a slip from one of the cartilages of the false ribs enters into the structure of these parts; but it is certain that the ensiform cartilages have assumed an extended and altered figure. This cartilaginous portion occupies the upper region of the band. The outline of the band is convex above, and arched below. Under the cartilage, while they stand in their ordinary posture, are large hernial sacs opening into each abdomen, and into which, on coughing, congenital herniæ are forced; probably, in each boy formed by a portion of the transverse arch of the colon: generally, however, and under ordinary circumstances, these herniæ are not apparent. Whether there is a communication between the two abdominal cavities, or a distinct peritoneal sac belonging to each hernia, is by no means obvious; and this is a point of vital importance, if ever, by their mutual desire, a surgical separation should be contemplated. If, however, any such operation hereafter be strongly requested by both the youths, when arrived at years of discretion, and after they have been fully apprised of its danger, it will be essential that some preliminary steps be taken to provide against the exposure of either or both of the abdominal cavities.

When these herniæ protrude, their respective contents are pushed forwards as far as the middle of the band. The entire band is covered with common integument; and when the boys face each other, its length at the upper edge is one inch and three quarters, and at the lower, not quite three inches. From above downwards, it is three inches and a quarter, and its greatest thickness is one inch and five-eighths. In the centre of the lower part of this band, which presents a thin edge, formed only by skin and cellular substance, there is the cicatrix of a single navel, showing where the umbilical cords or cord had entered, and which I have no doubt contained two sets of vessels*. Small blood-vessels and nerves must of course traverse the substance of the band, but no pulsation can be detected in it.

Captain COFFIN and Mr. HUNTER were informed by the mother of the twins, that soon after their birth, and during the period of infancy, this band was much larger in proportion to the size of their bodies than it is at the present time: it had then no hard cartilaginous feel at its upper margin; it was also larger in circumference, and the bodies of the twins were nearer in contact; but from continued stretching it has become elongated, and its circumference has diminished. In their own country they were employed to row a boat, for which purpose both stood at the stern, each using a one-handed oar, an exercise which must have assisted greatly in stretching the band. It is now remarkably strong, and possesses little sensibility; for they have been formerly pulled by a rope fastened to it, without complaining of pain, or expressing any uneasiness. In the month of February last one of them fell out of bed while asleep, and hung by the band for some time, and when both awoke, they alike stated, that they experienced no pain in the band from this accident. Mr. HALE, their constant attendant, has lifted one of them from the ground, allowing the other to hang by the band with his feet raised from the floor; yet the whole weight of one of the boys thus suspended did not occasion pain to either, or even excite their displeasure. The circumstance of the small degree of sensibility possessed by the band, tends to corroborate the opinion I entertain of the possibility of effecting a separation of the twins by a surgical operation.

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* It has been asserted, that "these twins are the produce of a single ovum, and grew upon one placenta, by one umbilical cord;" but of this there does not appear to be any evidence. By permission of the Board of Curators, I have had an opportunity of examining a preparation of united female twins, now in the museum of the Royal College of Surgeons in London. The union extends from the lower part of the sternum of each twin to the navel; and there is one umbilical cord common to both. On dissection, the following appearances were observed.—The umbilical vein in its course towards the twins, is divided into two nearly equal sized branches, the division taking place at about one inch and three quarters from the umbilicus; one branch passing upwards in front to the porta of the anterior liver, and the other behind to its proper liver. The number of arteries are four, two from each fœtus, and are included in the same theca with the umbilical vein as far as the body, retaining the appearance of an ordinary funis.

To the ordinary touch there is not any middle line where the sense of feeling common to both the boys terminates; but it is difficult to ascertain the precise point where the inosculation of the one individual with the other takes place: and this is not discoverable by making punctures with a needle, for each boy shrinks from a puncture whenever it is made in any part of a vertical line drawn down the middle of the band. It is therefore obvious, that the nerves of the common skin covering the band maintain a sensitive communication with each of the two youths; and it is reasonable to infer, that a similar communication subsists between the small arteries and veins, which mutually nourish the middle portion of the band. If, however, slight punctures be made at the distance of half an inch from the centre of the band, then the sensation is only felt by the individual belonging to the side punctured.

From these evidences it may be concluded, that the united twins would be subject to certain distempers in common, although each possesses a distinct existence, and even different constitutional peculiarities.

On the suggestion of Doctor ROGET, a silver teaspoon was placed on the tongue of one of the twins, and a disk of zinc on the tongue of his brother: when the metals thus placed were brought into contact, the youths both cried out "Sour, sour." This experiment was repeated several times with the same result, and was reversed by exchanging the positions of the metals, when a similar effect was produced.

These experiments prove that the galvanic influence passes from one individual to the other, through the band which connects their bodies, and thus establishes a galvanic circuit with the metals when these are brought into contact.

They habitually face in one direction, and place themselves side by side, ENG to the right and CHANG on the left, but are able to turn and remain in the opposite position. They always walk in the posture first described, although there is no other reason for this than established habit, as they are physically able to move in a reverse direction. Their united strength is great, for they can with perfect ease throw down a powerful man. At Philadelphia they also carried without inconvenience a person weighing rather more than twenty stone for a considerable distance. Their activity is remarkable; they run with great swiftness, and elude pursuit so admirably, that in sportive exercises they can with great difficulty be caught by a single person. They have each the power to bend their bodies in all directions, and turn their heels over their shoulders. They also often playfully tumble head over heels while on their bed, without occasioning the slightest pain or inconvenience in the band. The same degree of personal dexterity is evinced by each youth when playing at battledore and shuttlecock; and in all the bodily actions common to both, such as running and jumping, a remarkable consent or agreement is displayed without any apparent conference. These concurrences appear to be the necessary result of long continued and extraordinary intimacy.

In their respective physical constitutions, however, several differences occur. The boy on the left, CHANG, possesses the more vigorous bodily health of the two; but their intellectual abilities appear equal, for they are alike proficients in the games of chess and draughts, although they object to compete with each other. In the game of whist, however, they rather prefer not to be partners.

The tongue of ENG is at all times whiter than that of CHANG, and his digestion is more easily deranged by unsuitable diet. I have never heard that CHANG has passed a single day without alimentary discharges, but the contrary has often occurred to ENG. In general they both obey the calls of nature at the same time, and this happens even when these result from the operation of medicines.

It having occurred to me, that the odour given by asparagus to the urine would be a test of the extent of the circulation of the blood through both the twins, on the 22nd of March I gave that vegetable to CHANG with his dinner, not allowing any to be given to his brother. On examining their urine four hours after this meal, that of CHANG had distinctly the peculiar asparagus smell, but the urine of his brother was not influenced by it. The next day this experiment was reversed, and therefore with reversed results. These trials sufficiently prove a fact which was otherwise apparent,—that the sanguineous communication between the united twins is very limited.

On the 9th of December they were both attacked with bronchial catarrhs, became pale and languid, coughed severely, and complained of pain in their throats; each of them had also slight pains during strong inspirations. Their skins were dry and cold, respirations hurried, pulses ninety beats in a minute, rather hard and small; the tongue of ENG was glazed and pallid as usual, CHANG's became furred and dry. The bowels of both had been naturally relieved the day previous, and each was directed to take such medicines as experience had shown to be proper in the malady now common to both. Under this discipline and suitable diet, together with the additional clothing of leather waistcoats, and a leather coverlet for their bed, then considered to be required on account of the inclement winter, they both regained their ordinary state of health.

These incidents are recorded merely to show that they have been treated as two distinct persons, although from the very unusual circumstance of their conjunction, the same causes of disorder are presented to both, and similar consequences have thence ensued.

They have had the measles, and at eight years of age the confluent small-pox, distinct marks of the latter disease being still visible on their faces. When they were attacked with variola, a brother and two sisters had also that malady so severely as to occasion their deaths. While lying on their backs in bed, during their late illness, I counted the pulse of ENG in this position; it was sixty-three, soft and regular. I then went round the bed to examine CHANG, but in the mean time both had had occasion to move, and returned immediately to their former position. Although these movements were effected quietly, and did not occupy half a minute, producing no coughing, yet on counting CHANG's pulse, it was eighty, and soft. I then again counted ENG's, which was also eighty, soft and regular. After a lapse of ten minutes, both having remained perfectly still, on counting their pulses I found them at seventy-two.

I have submitted these occurrences in order to show the reciprocity of their symptoms under similar conditions of disorder.

In a healthy state, their ordinary pulses are generally alike, but CHANG's is the strongest; they are both easily excited, and when in one the pulse has been, from this cause, raised to ninety, that of the more passive brother has remained at seventy-two. They always take their meals together, objecting to be seen while thus engaged. Neither will eat or drink what the other dislikes, though they occasionally take different sorts of food at the same time, such as meat or fish. When the appetite of the one is satiated, the other is also satisfied. In their habits they are very cleanly and delicate, and mutually assist each other in dressing. They are exceedingly affectionate and docile, and grateful for every kindness shown them. It is not often that they converse with each other, although their dispositions and tempers agree, and their tastes and opinions are similar. Sometimes they engage in distinct conversations with different persons at the same time, upon totally dissimilar subjects. Both are very fond of music, and are equally interested in dramatic performances.

It does not appear that they have ever had any serious quarrel, except on one occasion, which occurred, as their mother reported, when they were eight years of age. While on their passage to America, one of them wished to bathe, as was their custom, to which the other objected, the day being cold; a slight dissension ensued, but Captain COFFIN soon reconciled the difference.

They always fall asleep at the same moment, and it is impossible to wake one without also arousing the other. When they were at Boston, Doctor SKEY, Surgeon General to the British Army, entered their bed-room at midnight on three successive nights when both were asleep: on each occasion he touched one and was answered by the other, both awaking at the same instant, inquiring why they were disturbed.

The experiment has also been repeated in this country, and with the same result. On my tickling one of them, the other told me to desist, though he stated that he did not feel the touch, and it was quite clear that he could not see me tickle his brother.

On their voyage to England one of them had the tooth-ache, during three days and nights, and suffered great pain, with loss of sleep, which last evil was shared by his brother, both remaining awake. On the 16th of December Mr. HALE went into their bed-room when they were asleep. ENG was restless, and tossing about in bed, while CHANG was screaming. He awoke them, and on inquiring what ailed them, ENG replied that he was dreaming about his mother, and CHANG said that a man was cutting off the long hair from his head. These different dreams appear to have occurred simultaneously.

The preceding instances of their mutual consent in many physical and moral particulars, may be accounted for by their constant moral and physical intimacy, which unquestionably is the source of more impressions common to both, than ever happen to two distinct individuals. They are at present very much attached to each other, but judging from what is now become a very common subject of discourse between them, it is not an unreasonable conjecture, that some female attachment, at a future period, may occur to destroy their harmony, and induce a mutual and paramount wish to be separated.

They are remarkably quick in intellect, and possess great imitative powers. They also observe very minutely whatever is presented to them, and comment upon the subject to their friends. On the 29th of November, a gentleman visited them with me, and recommended Captain COFFIN to have them taught their letters, and to write. By way of experiment he marked with a pencil, on a card, a large A; he then pronounced the letter, which sound the boys exactly imitated. He afterwards formed a B and a C; but while doing this, CHANG interrupted him, wishing to obtain the pencil; and both not only repeated the sounds of the three letters, but imitated their forms, CHANG even making a pun on the letter C; for on being asked if he knew its form and pronunciation, he replied, laughing, "Yes, I *see* you."

A person who had lost an eye, visiting them at their exhibition-room in New York, they inquired of their attendant what he had paid for his admission; and on being informed that it was the same as other persons, they remarked that half of it should be returned, as he had not had the same advantage as the others.

These extraordinary individuals are the most remarkable instances on record of perfect and distinctly formed human beings united together, who have attained the age of puberty in a state of sound bodily health: hence an authentic account of their moral and physical habitudes will probably be deemed valuable*.

On concluding this report, I wish it to be known, that I have neither instituted, nor permitted to be made, any unjustifiable experiments upon these youths; considering myself bound, by professional responsibility, as well as by a sense of national justice, to resist all such improper proposals. For these reasons, and also because I am averse to the administration of mercury, unless it be imperatively demanded, I have not had an opportunity of knowing whether the mercurial influence would pervade the one youth, if applied exclusively to the other. As, however, the capillary blood-vessels of each unquestionably inosculate with those of his companion in the uniting band, it must be obvious, that certain constitutional diseases, and many diffusible medicinal substances, would, partially at least, pervade both the united twins, though only one of them were exposed to their influences.

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* There is a case of a double female monster, born at Szony in Hungary, October 26th, 1701, who died February 23rd, 1723, at Presburg, in the convent of the Nuns of St. Ursula, recorded in the Philosophical Transactions for 1757, page 311, by JUSTUS JOANNES TORKOS, M.D. F.R.S.

In addressing these particulars and observations to the Royal Society, I have intentionally confined myself to a narration of facts, and have abstained from discussions of a speculative or hypothetical character.

I cannot here deny myself the pleasure of stating the kindness which has at all times been evinced towards these youths by Captain Coffin, Mr. Hunter, and Mr. Hale: the unwearied anxiety manifested by these gentlemen for their welfare and happiness, and the liberal manner in which they have uniformly afforded the means of investigating so curious an object of philosophical inquiry, entitle them equally to the thanks of the philanthropist and the lover of science.

3 *King Street, St. James's Square,*
March 29*th*, 1830.

The Plate exhibits two reduced views of the uniting band; also a representation of a section of the fac simile in wax.

Fig. 1. represents the front view of the ordinary standing attitude of the youths.

Fig. 2. exhibits the opposite side; or what may be termed the ordinary back view.

Fig. 3. represents a section through the middle of the band in its relaxed or unstretched state.

Fig. 2

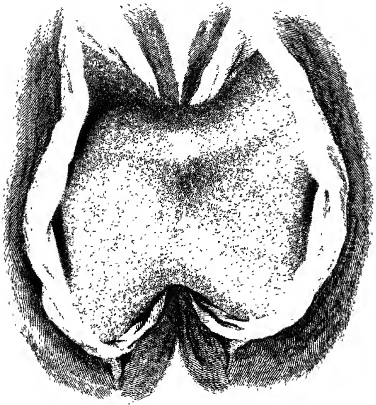


Fig. 3

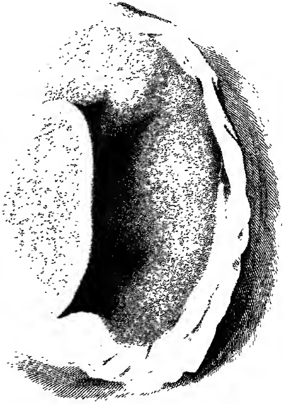
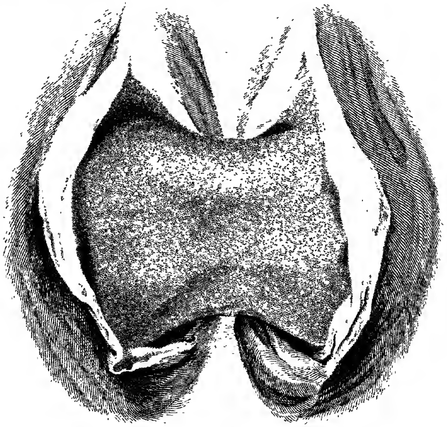


Fig. 1



[illegible]

XIII. *On some properties in achromatic object-glasses applicable to the improvement of the microscope. By* Joseph Jackson Lister, *Esq. Communicated by Dr.* Roget, *Secretary.*

Read January 21, 1830.

THE improvement of the achromatic compound miscroscope having been an occasional object of my leisure for several years past, my attention has in consequence been attracted to some properties of object-glasses of short focus and large aperture, which, so far as I am aware, have not been before noticed, and which, I flatter myself, may be applied to increase its powers and the ease of its manufacture.

In offering these, accompanied by some other miscellaneous remarks, it may be explanatory to introduce them by a short notice of the several achromatic object-glasses for the microscope which have originated apparently independent of each other within a few years past.

The first produced in England were the triple ones constructed in 1824 and 1825 by Tulley, who had been incited to the undertaking by Dr. Goring: of these an account is already before the public. Tulley has since adopted for his triple object-glass to be used singly the focal length of 0.9 inch, applying another of not quite 0.5 inch focus before it when high magnifying power is required; and he obtains with these an image of great sharpness and perfection. His first glasses have all the merit of a new invention, having been executed without the knowledge that any thing of the kind previously existed, though other achromatic glasses had before been made in France by Selligue, by Fraunhofer at Munich, and by Amici at Modena.

The glasses of Selligue's microscope, of which a report was made by M. Fresnel to the Royal Academy of Sciences in 1824, were composed of a plano-concave lens of flint-glass, and a double convex of crown or plate, with their inner curves cemented together. Four of these, of from $1\frac{1}{2}$ to $1\frac{7}{8}$ inch each in focal length, were made to screw before each other, so as to be used together or alone in the manner long practised with single lenses.

The chromatic aberration was thus in a considerable degree corrected, but the glasses were fixed in their cells with the convex side foremost, which is their worst position; and the spherical error was in consequence enormous, showing itself even through the contracted opening, to which it was necessary on that account to limit them.

Yet, inferior as was the instrument of SELLIGUE, the happy idea of combining achromatic object-glasses, now generally adopted and to which their present superiority is owing, seems to have occurred to no one else till put in practice by him; and the very simple structure of his glasses will be shortly seen not to be incompatible with the finest microscopic vision.

CHEVALIER of Paris having manufactured some of these instruments, appears to have observed the great error in the position of SELLIGUE's glasses; he retained their construction, but turned their plane sides foremost; and making them of shorter focal length and more correctly achromatic, produced in 1825 a microscope far superior to the former. His deepest glasses are not more than 0.4 inch in focal length, and two of these were united in his earlier instruments for his highest power; but this was the only combination retained in them, and all his glasses were restricted to apertures too small to show difficult test objects.

He has since increased the number of his glasses to be used together, and otherwise improved their performance; but if I may judge from microscopes of his which I have recently seen in this country, he does not yet derive from the construction that he has adopted, all the advantage which it may afford.

I am unacquainted with the date of the first production of FRAUNHOFER's glasses; they resemble the French in having their flint lens plano-concave, but they are not cemented, and the inner surfaces are not in contact, each object-glass being adapted by the curves of its convex lens for being used alone. My friend Mr. BROWN has kindly lent me a series of five glasses of this description, purchased by him at Munich a few months ago from the establishment of UTZSCHNEIDER and FRAUNHOFER, which range from 1.8 to 0.43 inch in focal length, and are of excellent workmanship: they screw before one another in the manner of SELLIGUE's. It appears from an account of the microscope of those artists, printed in 1829, that it is only lately the glasses have been combined together, and that of the shortest focus added to the former series of four; and also that they are intended to be used in the order of their focal lengths, the shorter towards the object. Each of these glasses singly admits but a small pencil of from 8° to 15° of light, and when so used, their defining power is necessarily not very great; but their combinations have much more, and the different effects of these will be again adverted to.

The eminent professor of Modena, besides inventing his well-known reflecting microscope, was engaged about the year 1815 with achromatic object-glasses; but as they did not equal his reflector, he laid the work aside, till he was induced to resume it in 1824, from reading the report already mentioned on the microscope of SELLIGUE: from this AMICI took the thought of combining his object-glasses, and pursued it with great success, making them double, with the curves of the two lenses of each planned for the place it is to occupy, and for obtaining a good image either with the back glass alone, or with a second or a third in front.

He brought with him, when he visited London in 1827, some glasses of this description of very fine performance; and I have been informed by him that he has since executed a combination of "2.7 lines in focal length and 2.7 lines in aperture," which considerably excels them.

The glasses which have been enumerated possess very different degrees of merit, chiefly dependent on the extent to which they are divested of chromatic and spherical aberration, and particularly, in connection with this, on the focal angle of their aperture; for it has been well established, that a large pencil from the object is absolutely essential to that brilliancy and distinctness of image which characterize a fine achromatic.

When the rays received by the most perfect object-glass from any indefinitely small bright portion of an object in the centre of its field are brought together at its conjugate focus, the image formed by them, though it appears a sharply defined point if moderately magnified, is really a spot or small circle, and will show as such if the microscope is sufficiently overcharged with power in the eye-glass. These circles bear a considerable analogy to the spurious disks of stars; and like them they will be found to be much enlarged by diminishing the aperture of the object-glass.

They are enlarged also, without contracting the aperture of the glass, by increasing the intensity of the illumination, whereas by darkening the object beyond a certain point they may be rendered ill defined, and be at length dissolved.

These peculiarities are most observable on some opaque objects, (the reflection from a very small microscopic globule of quicksilver* offers perhaps the best example,) but the same effects are produced on the light received from transparent ones; and the consequent blunting and mingling together of their minute details when the object-glass admits but a small pencil of light, gives rise to various fallacious appearances. One of the most remarkable is the spottiness which some surfaces assume, not unfrequently so much resembling small globules as to have been mistaken for them; an optical illusion having thus been the basis of some ingenious speculations on organic matter.

Such appearances have little place with the finer achromatics, the large angle of whose pencil and its accurate correction enable us to magnify their image greatly, still discovering something new in our object, before we are checked by the circles of diffusion of the effective rays; but these at last, whether proceeding from the causes mentioned or from others to be hereafter noticed, form in every microscope the boundary to defining power, except where faulty materials or workmanship give it an earlier limit.

It is the marginal rays which contribute especially to render visible close and delicate lines, such as those on the scales of lepidopterous insects, and some of the most difficult of these are even best seen when the central light is intercepted†.

A glass that is far from correct in its figure will sometimes show lines of this description sharply, while the outline of the scale is indistinct, and the contrary; but one in which both aberrations are destroyed should give the outline and the lines distinct together. Even some good glasses, however, have a defect, against which we should be on our guard; that in certain directions of the light they are liable to show lines on an object which do not really exist.

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* To obtain such, I have always placed a globule on a piece of black glass, and scattered it into many by a smart stroke, some of which will be extremely minute.

† The blue down of the Menelaus and the white of the Cabbage Butterfly (Morpho Menelaus and Pieris Brassicæ) well deserve the place they have acquired as standard tests, especially on account of the respective characters of their transverse tracings. Some of the small oval scales from the body of the Twenty-plumed Moth (Alucita hexadactyla) are among the closest and most elegant in their lines that I have seen; others are very easily resolvable, and the down has such diversity of form as often to afford within a short space a ready variety of excellent tests. The same is to be said of the scales of Podura plumbea, of which all are difficult, and some seem to defy all powers of definition.

It may be observed, that with most objects there is such difference between individual specimens of the same kind, that in general the only safe way to determine between two good microscopes, is to apply the same specimen to both under the same circumstances of power and light.

It is observable, that provided the aperture of the glass remains open, the central pencil of light admitted behind many transparent objects may be limited to a very small one without greatly impairing their sharpness; parallel lines or spots spread closely over a flat surface often remaining plainly visible in this case, which at a far less amount of contraction by a stop behind the object-glass cannot by any management be made to appear. The reason of this seems to be, that both reflection and refraction of a part of the rays take place at such objects, by which the pencil is spread out on leaving them to a much increased angle in its progress to the glass.

The relation between the aperture of microscopic object-glasses, even of the same focal length, and the pencil of light admitted by them, will vary much, according to differences in their thickness, their combination &c.; and as aperture is valuable only in proportion to the pencil it admits, the latter would seem to be the circumstance the more deserving attention of the two. It is so often erroneously estimated, that I will mention a simple mode of ascertaining it, which will be found pretty accurate.

Fix a piece of paper on a table, and on it place the microscope with its body horizontal, and one of the eye-pieces on; set a candle on a level with it a few yards distant; then having directed the body of the instrument so far on one side of the candle, as that the light from it shall bisect the field vertically, leaving half of it dark, trace on the paper a line corresponding to the side of one of the legs. Now, taking the focus of the object-glass as a pivot, turn the microscope horizontally to the other side of the candle till the opposite half of the field only is illuminated, and mark again on the paper the position of the side of the leg. The measure of the angle traversed shown by the two lines is that of the pencil of light.

In the remarks which follow, the term correction is used to imply the effect produced by the denser concave lens of a compound object-glass upon the aberration of its convex. Thus as in a simple convex lens, the rays which pass through it near the circumference have their foci shorter than the more central rays, and the colours of the violet side of the spectrum into which each ray is refracted have also their foci shorter than those of the red side; if either of these errors is but partially removed by the concave lens, the glass is said to be under-corrected as to that aberration, and over-corrected if the opposite error is produced by it.

A large focal pencil free from all aberration is evidently the great requisite for the object-glass of the compound microscope; a second point desirable to be attained is, that the field should be flat and well defined throughout; and a third, that the light admitted should as much as possible be only such as goes to form the picture, and should not be intercepted or diffused over the field by too many reflections.

The prominent obstacle to obtaining a sufficient pencil for high powers by one object-glass of large aperture and deep curves, is that the correction for the spherical figure by the concave lens is greater for the rays of the circumference than its due proportion to that for the more central ones; so that when such a glass is corrected for the mean of the pencil, if we suppose its disk divided into a central space and three rings surrounding it, the rays which pass through the central space and those of the second ring from it will arrive at their focus when those of the first ring will have just crossed the axis, and those of the marginal ring will not quite have reached it. The injury resulting to the defining power is in similar glasses inversely as the squares of their focal lengths, as far as regards this cause of error, as well as those which arise from incorrigible colour and defects of workmanship. The effects upon the pencil which have been before described, must not be included under the same law. This excess of correction in the marginal rays increases after a certain point so rapidly with a small enlargement of the aperture of the glass, as soon to prescribe a limit, beyond which it cannot be carried without injury to the picture.

With glasses of more contracted aperture and at the several surfaces of which the marginal refraction is moderate, the effect alluded to is comparatively inconsiderable; and consequently by dividing the refraction among two or more such glasses corrected for the rays that pass through them, the pencil received may be enlarged without impediment, and the light and distinctness greatly increased, thus constituting the important advantage of combination.

The triple object-glass is thus very superior to a double one when each is used singly, and the union of two triple ones has been already proved in England to be eminently effective. Tulley's 0.9 inch glass singly admits a pencil of near 20°, and his combination, one of 38°.

These triple glasses and the double ones of Professor Amici are adapted by the form of their curves each to its respective place; but the foreign double glasses which have their flint lens plano-concave, and particularly those of Utzschneider before mentioned, are made much on one model, and intended to be each good alone. It might seem but reasonable to infer from this, that they would be unfit to be combined; and accordingly when screwed together, most of the numerous practicable changes of his series of five glasses, of which as many as four may be united for use, have much indistinctness from spherical error; and this is I think the case with all those combinations which the maker contemplated. Some peculiarities, however, observed two years ago in Chevalier's object-glasses, led me to undertake a close examination of these, which were liberally placed at my disposal for the purpose, in the hope of discovering the cause for a discrepancy which appeared in their effects.

I found that with a part of the combinations, the image of any bright point that was at some distance from the centre of the field had a faint light or coma stretching outwards from it; with others the coma was as much inwards.

The spherical aberration was in general much over-corrected, but in some triple and quadruple combinations the opposite error showed itself; and out of the whole number one triple and one quadruple were remarkably beautiful and distinct.

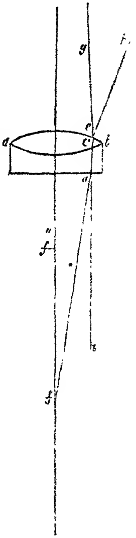
The result of this investigation was to disclose or confirm to me the existence of some properties of the double object-glass, which have not I believe been hitherto recorded, and which it is now my purpose to describe only in connection with the subject before us,—the improvement of the microscope.

With this in view I would premise that the plano-concave form for the correcting flint lens, which was probably adopted at first for its simplicity, has in that quality a strong recommendation; particularly as it obviates the danger of error which otherwise exists in centering the two curves, and thereby admits of correct workmanship for a shorter focus. To cement together also the two lenses of the glass, diminishes by very nearly half, the loss of light from reflexion, which is considerable at the numerous surfaces of a combination. I have thought the clearness of the field and brightness of the picture evidently increased by doing this; it prevents any dewiness or vegetation from forming on the inner surfaces; and I see no disadvantage to be anticipated from it, if they are of identical curves and pressed closely together, and the cementing medium permanently homogeneous.

These two conditions then, that the flint lens shall be plano-concave, and that it shall be joined by some cement to the convex, seem desirable to be taken as a basis for the microscopic object-glass, provided they can be reconciled with the destruction of the spherical and chromatic aberrations of a large pencil.

Now in every such glass that has been tried by me, which has had its correcting lens of either Swiss or English flint glass, with a double convex of plate, and has been made achromatic by the form given to the outer curve of the convex, the proportion has been such between the refractive and dispersive powers of its lenses, that its figure has been correct for rays issuing from some point in its axis not far from its principal focus on its plane side, and either tending to a conjugate focus within the tube of a microscope, or emerging nearly parallel.

Let *a b* be supposed such an object-glass, and let it be roughly considered as a plano-convex lens, with a curve *a c b* running through it, at which the spherical and chromatic errors are corrected, which are generated at the two outer surfaces; and let the glass be thus free from aberration for rays *f d e g* issuing from the radiant point *f*; *h e* being a perpendicular to the convex surface, and *i d* to the plane one. Under these circumstances the angle of emergence *g e h* much exceeds that of incidence *f d i*, being probably almost three times as great.



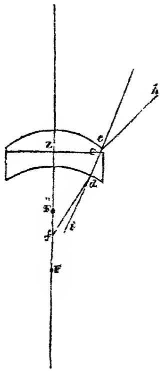
If the radiant is now made to approach the glass so that the course of the ray *f d e g* shall be more divergent from the axis, as the angles of incidence and emergence become more nearly equal to each other, the spherical aberration produced by the two will be found to bear a less proportion to the opposing error of the single correcting curve *a c b*; for such a focus therefore the rays will be over-corrected.

But if *f* still approaches the glass, the angle of incidence continues to increase with the increasing divergence of the ray, till it will exceed that of emergence which has in the mean while been diminishing, and at length the spherical error produced by them will recover its original proportion to the opposite error of the curve of correction. When *f* has reached this point f'', (at which the angle of incidence does not exceed that of emergence so much as it had at first come short of it,) the rays again pass the glass free from spherical aberration.

If *f* be carried from hence towards the glass, or outwards from its original place, the angle of incidence in the former case, or of emergence in the latter, becomes disproportionately effective; and either way the aberration exceeds the correction.

These facts have been established by careful experiment: they accord with every appearance in such combinations of the plano-convex glasses as have come under my notice, and may I believe be extended to this rule;—that in general an achromatic object-glass, of which the inner surfaces are in contact, or nearly so, will have on one side of it two foci in its axis, for the rays proceeding from which it will be truly corrected at a moderate aperture; that for the space between these two points, its spherical aberration will be over-corrected, and beyond them either way under-corrected.

I am not aware that an exception is to be made for any quality of glass or curves that are likely to be used for the microscope: but I apprehend a case may occur, if the flint glass is convexo-concave and the convex lens united to its concave side, that neither of the aplanatic pencils may converge after traversing the glass, and that their foci for a radiant may be on opposite sides of it, the principle however of the two foci remaining unaltered.



To try this principle under a great change of circumstances, and to prove in what manner it was applicable to another most simple form of the object-glass, having made a magnified tracing of the curves of one of UTZSCHNEIDER'S*, and drawn a ray through it from its longer aplanatic focus, which was ascertained after cementing the lenses; I laid down a figure of the flint lens inverted, and by means of the angles of the other diagram projected a plano-convex z to be joined to its flat side. The new glass proved as achromatic as the original, though the single radius of the plano-convex was more than one-third longer than corresponded with the curves of the former double convex lens: the two aplanatic foci showed themselves as before; the longer, however, not in the place indicated by the ray drawn: but at a point F, about two-thirds more distant from the glass, and from which both surfaces of the flint lens bent the pencil outwards; the shorter focus F'' too becoming about half the length of the other, and the angle of incidence of its rays not equalling that of their emergence.

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* See Figure p. 194.

The longer aplanatic focus may be found when one of the plano-convex object-glasses is placed in a microscope, by shortening the tube if the glass shows over-correction, if under-correction by lengthening it, or by bringing the rays together should they be parallel or divergent, by a very small good telescope. The shorter focus is got at by sliding the glass before another of sufficient length and large aperture that is finely corrected, and bringing it forwards till it gives the reflexion of a bright point from a globule of quicksilver, sharp and free from mist, when the distance can be taken between the glass and the object.

The longer focus is the place at which to ascertain the utmost aperture that may be given to the glass, and where, in the absence of spherical error, its exact state of correction as to colour is seen most distinctly.

The correction of the chromatic aberration, like that of the spherical, tends to excess in the marginal rays; so that if a glass, which is achromatic with a moderate aperture, has its cell opened wider, the circle of rays thus added to the pencil will be rather over-corrected as to colour.

The same tendency to over-correction is produced, if, without varying the aperture, the divergence of the incident rays is much augmented; as in an object-glass placed in front of another: but, generally, in this position a part only of its aperture comes into use; so that the two properties mentioned neutralize each other, and its chromatic state remains unaltered. If, for example, the outstanding colours were observed at the longer focus to be green and claret, which show that the nearest practicable approach is made to the union of the spectrum, they usually continue nearly the same for the whole space between the foci, and for some distance beyond them either way.

The places of these two foci and their proportions to each other, depend on a variety of circumstances. In several object-glasses that I have had made for trial,—plano-convex, with their inner surfaces cemented, their diameters the radius of the flint lens, and their colour pretty well corrected,—those composed of dense flint and light plate have had the rays from the longer focus emerging nearly parallel; and this focus has been not quite three times the distance of the shorter from the glass: with English flint the rays have had more convergence, and the shorter focus has borne a rather less proportion to the longer.

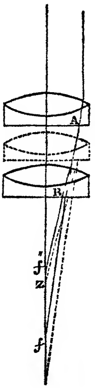
If the inner surfaces are not cemented, a striking effect is produced by minute differences in their curves. It may give some idea of this, that in a glass of which almost the whole disk was covered with colour from contact of the lenses, the addition of a film of varnish so thin that this colour was not destroyed by it, caused a sensible change in the spherical correction.

I have found that whatever extended the longer aplanatic focus, and increased the convergence of its rays, diminished the relative length of the shorter. Thus by turning to the concave lens the flatter instead of the deeper side of a convex lens, whose radii were to each other as 31 to 35, the pencil of the longer aplanatic focus, from being greatly divergent, was brought to converge at a very small distance behind the glass; and the length of the shorter focus which had been one-half that of the longer, became but one-sixth of it.

The direction of the aplanatic pencils appears to be scarcely affected by differences in the thickness of glasses, if their state as to colour is the same.

One other property of the double object-glass remains to be mentioned; which is, that when the longer aplanatic focus is used, the marginal rays of a pencil not coincident with the axis of the glass are distorted, so that a coma is thrown outwards; while the contrary effect of a coma directed towards the centre of the field is produced by the rays from the shorter focus. These peculiarities of the coma seem inseparable attendants on the two foci, and are as conspicuous in the achromatic meniscus, as in the plano-convex object-glass.

Of several purposes to which the particulars just given seem applicable, I must at present confine myself to the most obvious one. They furnish the means of destroying with the utmost ease both aberrations in a large focal pencil, and of thus surmounting what has been hitherto the chief obstacle to the perfection of the microscope. And when it is considered that the curves of its diminutive object-glasses have required to be at least as exactly proportioned as those of a large telescope, to give the image of a bright point equally sharp and colourless, and that any change made to correct one aberration was liable to disturb the other, some idea may be formed of what the amount of that obstacle must have been. It will however be evident, that if any object-glass is but made achromatic, with its lenses truly worked and cemented so that their axes coincide, it may with certainty be connected with another possessing the same requisites and of suitable focus, so that the combination shall be free from spherical error also in the centre of its field. For this the rays have only to be received by the front glass B, from its shorter aplanatic focus *f*, and transmitted in the direction of the longer correct pencil *f* A of the other glass A. It is desirable that the latter pencil should neither converge to a very short focus, nor be more than very slightly, if at all, divergent; and a little attention at first to the kind of glass used will keep it within this range, the denser flint being suited to the glasses of shorter focus and larger angle of aperture.



The adjustment for the microscope is then perfected, if necessary, by slightly varying the distance between the object-glasses; and after that is done, the length of the tube which carries the eye-pieces may be altered greatly without disturbing the correction; opposite errors which balance each other being produced by the change.

If the two glasses, which in the diagram are drawn as at some distance apart, are brought nearer together, (if the place of A for instance, is carried to the dotted figure,) the rays transmitted by B in the direction of the longer aplanatic pencil of A, will plainly be derived from some point (z) more distant than f'', and lying between the aplanatic foci of B; therefore (according to what has been stated) this glass, and consequently the combination, will then be spherically over-corrected. If on the other hand the distance between A and B is increased, the opposite effects are of course produced.

In combining several glasses together, it is often convenient to transmit an under-corrected pencil from the front glass, and to counteract its error by over-correction in the middle one.

Slight errors in colour may in the same manner be destroyed by opposite ones; and on the principles described, we not only acquire fine correction for the central ray, but by the opposite effects at the two foci on the transverse pencil, all coma can be destroyed, and the whole field rendered beautifully flat and distinct.

The occurrence of two good combinations among the numerous ones of Utzschneider's glasses will now appear only what might be expected; and more are to be obtained from them by varying the distance of the glasses from each other.

I have very lately seen one of Chevalier's combinations of three glasses, each about 0.4 inch in focus, inferior indeed to these, and of contracted aperture, but which I ought to notice because its image is pretty well corrected with the glasses in the order given to them by the maker. At the same time, I do not suppose him to be acquainted with the principle which constitutes the key to the effect; otherwise he would hardly have failed to apply it more effectually.

The achromatic meniscus that has been described enters well into combination, and may perhaps be useful as the front glass of three, but its power is small in proportion to its curves. It admits too of being cemented at the back of an achromatic plano-convex; and they may make together a powerful compound glass to go before another.

Though the plano-concave form has been proposed for the flint lens, in the larger glasses it might perhaps occasionally be relinquished with benefit.

Some attention has yet to be given to obtain the best effect from combination; so that with the largest pencil that may be found desirable, and sufficient clear space before the front of the glass, the field may be aplanatic, and the focus short. Already, however, the three first plano-convex glasses that have been made for me by Tulley, only for preliminary experiments, the shortest of them of 0.7 inch focus, have produced at an aperture of 50° the most distinct microscopic vision that I have yet met with; and I anticipate no serious impediment to the carrying defining power much further.

These statements are intended only as a notice, for practical purposes and wholly detached from theory, of facts that have been in part very recently ascertained. In investigating them I have depended chiefly on magnified measurements and diagrams, which, though not strictly correct, may perhaps be as well adapted as other far more exact but difficult methods, for constructing the minute and complex object-glass of a microscope. It is to this that the observations made particularly apply; and should they bring more within reach than previously, the requisites which have been enumerated, I trust they may not be unacceptable to the enlightened optician; especially if in this department he unite, like TULLEY, the zeal of the amateur with the skill of the superior artist.

I intend soon to put to the test of experiment, whether or not the principle of the two aplanatic pencils may be applied to telescopes, in cases in which it is requisite to restrict their length, so as to enlarge their aperture with a corresponding increase of light and distinctness. In the mean time, it would give me pleasure to see that principle demonstrated, if it deserve it, by some abler hand than mine, and treated in a more rigorous manner than my own limited acquaintance with mathematical science qualifies me to undertake.

XIV. *On the pendulum.* *By* J. W. Lubbock, *Esq. F.R.S.*

Read March 11, 1830.

CAPTAIN KATER was the first who made use of Huygens's theorem with respect to the convertibility of the centres of suspension and oscillation to eliminate the moment of inertia, and to obtain the length of the simple pendulum by measuring the distance between the knife edges or axes of suspension. But this very ingenious method of determining the length of the simple pendulum must be considered as a first approximation, which is true only when many circumstances which might affect the truth of the result are not taken into account, but of which the following investigation shows that when the experiments are conducted with care, the effect is insensible. It is, however, desirable to ascertain carefully the limits of the errors which may rise from the circumstances to which I have alluded, and to render the theory of Captain Kater's pendulum as perfect as the method of observation. Laplace has given a complete theory of the apparatus used by Borda in the Connaissance des Temps; and he has shown that in the apparatus of Captain Kater, the distance between the knife edges is equal to the length of the simple pendulum, when they are considered as cylinders of small curvature, provided their radii of curvature are equal; which theorem is also proved in Professor Whewell's Dynamics. But no one I believe has yet discussed all the circumstances which affect the accuracy of Captain Kater's method; and I have therefore attempted to do this in the following paper, in which I have treated the question with the utmost generality, taking the case of all possible deviations and of axes any how placed, provided only that they are synchronous.

I have taken the pendulum used by Mr. Baily, and described by him in the Philosophical Magazine of last February, to afford a numerical example, and I have given the errors which would arise in the length of the simple pendulum corresponding to given deviations of the knife edges: it is difficult to make the results intelligible without the use of symbols; but I may add, that the effect of a small deviation of one of the knife edges in azimuth is quite insensible: this is not the case with a deviation in altitude: a deviation of a degree in altitude increases by 3 the vibrations in twenty-four hours: a deviation from horizontality in the agate planes has a more sensible influence than either of the former deviations: a deviation in horizontality in the agate planes of 10′ increases by about 6 the vibrations in twenty-four hours: both these deviations have the effect of rendering the distance between the knife edges greater than the true length of the simple pendulum. I have also considered the case in which the agate planes are fixed on the pendulum and vibrate on a fixed knife edge; and I find, as might be expected, that the length of the simple pendulum is equal to the distance between the planes.

Let Ox, Oy, Oz be rectangular coordinate axes meeting in the point O in the plane $(x\,y)$, upon which the pendulum rests; let the plane $(x\,z)$ be vertical and the plane $(x\,y)$ nearly horizontal, and let the axis of rotation coincide with the line Ox. Let g be the force of gravity, ε the angle which a vertical line makes with the axis Oz, a the distance of the centre of gravity from the line Ox, and $M(a^2 + k^2)$ the moment of inertia of the pendulum about the axis Ox; then, according to the analysis of M. Poisson, (Traité de Mécanique, p. 116.) the length of the simple pendulum which oscillates in the same time is $\frac{a^2 + k^2}{a \cos \varepsilon_{,}}$.

Let G $x_{,}$, G $y_{,}$, G $z_{,}$ be the three principal axes which intersect each other in the point G, and let the equations to the axis Ox referred to the coordinate axes G $x_{,}$, G $y_{,}$, G $z_{,}$, which are fixed in the pendulum and move with it, be

$$y_{,} = x_{,} \tan \delta + \beta$$

$$z_{,} = x_{,} \frac{\tan \delta'}{\cos \delta} + \gamma.$$

δ and δ' are small angles which may be considered as the deviations of the knife edge in azimuth and altitude.

$$\text{If} \quad a\,y = b\,x + \beta$$

$$a\,z = c\,x + \gamma$$

are the equations to any straight line (ϱ) in space, the equations to a straight line perpendicular to this line, and passing through the origin, are

$$\left\{\begin{array}{l} a x + b y + c z = 0 \\ \gamma (a y - b x) = \beta (a z - c x) \end{array}\right\}$$

and the shortest distance from the origin to the given line

$$= \sqrt{\frac{(\beta b - \gamma c)^2 + \beta^2 c^2 + \gamma^2 a^2}{a^2 (a^2 + b^2 + c^2)}}$$

The equation to a plane passing through the origin and the given line is

$$\gamma (a y - b x) = \beta (a z - c a)$$

and the equations to the intersection of this plane with the plane ($z\ y$) are

$$\gamma y = \beta^2$$
$$x = 0$$

If $a' y = b' x + \beta'$
$a' z = c' x + \gamma'$

be the equations to any other straight line (ϱ') in space

$$\cos \varrho \varrho' = \frac{a a' + b b' + c c'}{\sqrt{a^2 + b^2 + c'} \sqrt{a'^2 + b'^2 + c'^2}}$$

Hence the cosine of the angle formed by the line ϱ and the intersection of the plane

$$\gamma (a y - b x) = \beta (a z - c x)$$

with the plane $z\ y$

$$= \frac{\beta b + \gamma c}{\sqrt{a^2 + b^2 + c^2} \sqrt{\beta^2 + \gamma^2}}$$

the sine of the same angle

$$= \sqrt{\frac{(\beta b - \gamma c)^2 + \beta^2 c^2 + \gamma^2 a^2}{\sqrt{a^2 + b^2 + c^2} \sqrt{\beta^2 + \gamma^2}}}$$

These equations being premised; let C be the point in the axis or knife edge, O x, where a perpendicular let fall upon it from the centre of gravity G cuts it; let C′ be the point where the plane ($z\ y$) cuts the axis O x; and let C″ be the point where one of the surfaces of the pendulum, supposed a parallelepiped, cuts the same knife edge; and let G″ be the point in this surface where a perpendicular let fall from G cuts it.

If half the thickness of the pendulum be called t

$$\text{G C} = \text{G}'' \text{C}'' \sin \text{C C}' \text{G} - t \cos \text{C C}' \text{G}$$

$$\text{sine } \mathrm{C\,C'\,G}^2 = \frac{\{\beta \sin\delta \cos\delta' - \gamma \sin\delta'\}^2 + \beta^2 \sin\delta'^2 + \gamma^2 \cos\delta^2 \cos\delta'^2}{\beta^2 + \gamma^2}$$

$$\cos \mathrm{C\,C'\,G} = \frac{\beta \sin\delta \cos\delta' + \gamma \sin\delta'}{\sqrt{\beta^2 + \gamma^2}}, \quad \text{if } \mathrm{G\,C'} = a'$$

$\beta = a' \sin\lambda$, $\gamma = a' \cos\lambda$, λ being a small angle

$\sin \mathrm{C\,C'\,G}^2 = \{\sin\lambda \sin\delta \cos\delta' - \cos\lambda \sin\delta'\}^2 + \sin\lambda^2 \sin\delta'^2 + \cos\delta^2 \cos\delta'^2$

$\cos \mathrm{C\,C'\,G} = \sin\lambda \sin\delta \cos\delta' + \cos\lambda \sin\delta'$

neglecting $\sin\lambda \sin\delta$ and $\sin\lambda^2 \sin\delta'$

$\cos \mathrm{C\,C'\,G} = \sin\delta'$, $\sin \mathrm{C\,C'\,G} = \cos\delta'$

$$\mathrm{G\,C} = \mathrm{G''\,C''} \cos\delta' - t \sin\delta'$$

Let ε, ε', ε'' be the angles which the line Ox makes with the coordinate axes G $x_{,}$, G $y_{,}$, G $z_{,}$; and A, B, C the moments of inertia of the pendulum about these axes: then by a well known theorem, if G C $= a$
the length of the simple pendulum

$$= \frac{M a^2 + A \cos\varepsilon^2 + B \cos\varepsilon'^2 + C \cos\varepsilon''^2}{M a \cos\varepsilon_{,}}$$

$$\cos\varepsilon = \cos\delta' \cos\delta, \ \cos\varepsilon' = \cos\delta' \sin\delta, \ \cos\varepsilon'' = \sin\delta'$$

δ and δ' may be considered as the deviations of the knife edge in azimuth and altitude.

C being the point in axis Ox where a perpendicular from G cuts it, the index at foot indicates the knife edge, the length of the simple pendulum if $\varepsilon_{,} = 0$

$$= \mathrm{G\,C_1} + \frac{A \cos\varepsilon_1{}^2 + B \cos\varepsilon'_1{}^2 + C \cos\varepsilon''_1{}^2}{M\,\mathrm{G\,C_1}}$$

let $A = M k^2$, $B = M k'^2$, $C = M k''^2$, if the knife edges (1) and (2) are isochronous

$$\mathrm{G\,C_1} + \frac{k^2}{\mathrm{G\,C_1}} - \frac{k^2 \sin\varepsilon_1{}^2 - k'^2 \cos\varepsilon'_1{}^2 - k''^2 \cos\varepsilon''_1{}^2}{\mathrm{G\,C_1}}$$

$$= \mathrm{G\,C_2} + \frac{k^2}{\mathrm{G\,C_2}} - \frac{k^2 \sin\varepsilon_2{}^2 - k'^2 \cos\varepsilon'_2{}^2 - k''^2 \cos\varepsilon''_2{}^2}{\mathrm{G\,C_2}}$$

whence

$$k^2 = \mathrm{G\,C_1} \times \mathrm{G\,C_2} + \frac{\mathrm{G\,C_2}}{\mathrm{G\,C_2} - \mathrm{G\,C_1}} \{k^2 \sin\varepsilon_{,}^2 - k'^2 \cos\varepsilon_{,}'^2 - k''^2 \cos\varepsilon''_1{}^2\}$$

$$- \frac{\mathrm{G\,C_1}}{\mathrm{G\,C_2} - \mathrm{G\,C_1}} \{k^2 \sin\varepsilon_2{}^2 - k'^2 \cos\varepsilon'_2{}^2 - k''^2 \cos\varepsilon''_2{}^2\}$$

The length of the simple pendulum is

$$\mathrm{G\,C}_1 + \mathrm{G\,C}_2 + \frac{k^2(\sin \varepsilon_1{}^2 - \sin \varepsilon_2{}^2) - k'^2(\cos \varepsilon'_1{}^2 - \cos \varepsilon'_2{}^2) - k''^2(\cos \varepsilon''_1{}^2 - \cos \varepsilon''_2{}^2)}{\mathrm{G\,C}_2 - \mathrm{G\,C}_1}.$$

$$\mathrm{G\,C} = \mathrm{G''\,C''} \cos \delta' - t \sin \delta'$$

$$= \mathrm{G''\,C''} \left\{1 - 2 \sin \frac{\delta'^2}{2}\right\} - t \sin \delta'$$

The apparent length of the simple pendulum $= \mathrm{C''_1\,C''_2}$

The true length of the simple pendulum is

$$\mathrm{G''\,C''_1} + \mathrm{G''\,C''_2} - 2\,\mathrm{G''\,C''_1} \sin \frac{\delta'^2_1}{2} - 2\,\mathrm{G''\,C''_2} \sin \frac{\delta'^2_2}{2} - t \sin \delta'_1 - t \sin \delta'_2$$

$$+ \frac{k^2(\sin \varepsilon_1{}^2 - \sin \varepsilon_2{}^2) - k'^2(\cos \varepsilon'_1{}^2 - \cos \varepsilon'_2{}^2) - k''^2(\cos \varepsilon''_1{}^2 - \cos \varepsilon''_2{}^2)}{\mathrm{G\,C}_2 - \mathrm{G\,C}_1}$$

The angle $\mathrm{C''_1\,G''\,C''_2} = \lambda_1 - \lambda_2$

$$\mathrm{C_1''\,C_2''} = \mathrm{G''\,C_1''} + \mathrm{G''\,C_2''} - \frac{2\,\mathrm{C_1''\,G''} \times \mathrm{C_2''\,G''}}{\mathrm{C_1''\,C_2''}} \left\{\sin \frac{(\lambda_1 - \lambda_2)}{2}\right\}^2$$

The true length of the pendulum is

$$\mathrm{C_1''\,C_2''} + \frac{2\,\mathrm{C_1\,G} \times \mathrm{C_2\,G}}{\mathrm{C_1\,C_2}} \left\{\sin \frac{(\lambda_1 - \lambda_2)}{2}\right\}^2 - 2\,\mathrm{G\,C}_1 \left\{\sin \frac{\delta'_1}{2}\right\}^2 - 2\,\mathrm{G\,C}_2 \left\{\sin \frac{\delta'_2}{2}\right\}^2$$

$$- t \sin \delta'_1 - t \sin \delta'_2$$

$$+ \frac{k^2(\sin \varepsilon_1{}^2 - \sin \varepsilon_2)^2 - k'^2(\cos \varepsilon_1{}^2 - \cos \varepsilon'_2)^2 - k''^2(\cos \varepsilon''_1{}^2 - \cos \varepsilon''_2{}^2)}{\mathrm{G\,C}_2 - \mathrm{G\,C}_1}$$

The sign of the quantity $t \sin \delta'$ depends upon which surface of the pendulum the distance between the axes is measured, and might be got rid of by measuring the distance between the knife edges on each of the surfaces which are intersected by them, and taking the mean.

I have endeavoured as far as possible to conform to the notation of M. Poisson in the Traité de Mécanique; but this is rendered difficult, because M. Poisson sometimes takes the axis $\mathrm{O}x$ to be vertical, (vol. ii. p. 113,) and sometimes the axis $\mathrm{O}z$ (as vol. ii. p. 185), and he uses the letter ε in two different acceptations, (vol. ii. pp. 119 & 185.)

In the notation of the article on the Pendulum in the Supplement to the Encyclopædia Britannica

$$\varepsilon = \mathrm{X},\ \varepsilon' = \mathrm{Y} \text{ and } \varepsilon'' = \mathrm{Z},\ a = h.$$

The author of this article assumes the equations of the axis of rotation to be

$$x_{,} = a\, z_{,} + \alpha$$
$$y_{,} = b\, z_{,} + \beta$$

and he gives the equation $h = \frac{\sqrt{\alpha^2 + \beta^2}}{\sqrt{1 + a^2 + b^2}}$

this equation is incorrect; it should be

$$h = \frac{\sqrt{\{(\alpha^2 + \beta^2)(1 + a^2 + b^2) - (a\,\alpha + b\,\beta)^2\}}}{\sqrt{(1 + a^2 + b^2)}}$$

It is easy by proper substitutions in the equations which I have given, to ascertain the influence of any deviation of the knife edge; and for this purpose I shall take the pendulum described in the Annals of Philosophy, vol. iv. p. 137. used by Mr. Baily, of which the length is 62 inches, the width 2 inches, and the thickness ·275 inch. One of the knife edges is 5 inches from the extremity; and therefore from well known expressions for the moment of inertia in a parallelepiped,

$$k^2 = \frac{31^2 + 1}{3} = 320{\cdot}6666$$

$$k'^2 = \frac{31^2 + {\cdot}1375^2}{3} = 320{\cdot}339$$

$$k''^2 = \frac{1 + {\cdot}1375^2}{3} = {\cdot}33964$$

If λ, δ, δ' and $\varepsilon_{,} = 0$, $G\,C_1 = 11.2514$, $G\,C_2 = 28.5$, $C_1\,C_2 = 39.7514$

1. When δ, δ' and $\varepsilon_{,} = 0$, the true length of the simple pendulum

$$= C_1''\,C_2'' + \frac{2\,C_1 G \times C_2 G}{C_1\,C_2} \left\{\sin \frac{\lambda}{2}\right\}^2$$

$$= 39{\cdot}7514 + {\cdot}80667 \left\{\sin \frac{\lambda^2}{2}\right\}^2.$$

2. When λ, δ' and $\varepsilon_{,} = 0$, the true length of the simple pendulum

$$= C_1''\,C_2'' + \frac{k^2 - k'^2}{G\,C_2 - G\,C_1} \sin \delta^2$$

$$= 39.7514 + .013137 \sin \delta^2.$$

3. When λ, δ and $\varepsilon_{,} = 0$, the true length of the simple pendulum

$$= C_1'' C_2'' + \frac{k^2 - k''^2}{G C_2 - G C_1} \sin \gamma'^2 - 2 \, G \, C_1 \sin \frac{\delta'^2}{2}$$

$$= 39.7514 + 18.5712 \sin \gamma'^2 - 22.5028 \sin \frac{\gamma'^2}{2}.$$

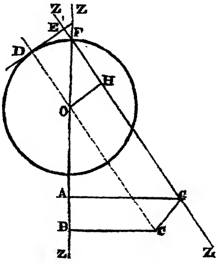
4. When λ, δ and $\gamma' = 0$, the length of the simple pendulum

$$= \frac{C_1 C_2}{\cos \varepsilon_{,}}.$$

The following table shows the increase in the number of vibrations in a day due to different values of λ, δ, γ', and $t_{,}$.

	λ	δ	γ	$\varepsilon_{,}$
30′	.62	.00	1.08	1.85
1°	2.66	.00	4.20	6.54

So that if, for example, $\lambda = 1°$, the number of vibrations in a day is increased by 2·66, and a deviation γ' in altitude of 1° has the effect of making the distance between the knife edges less than the true length of the simple pendulum by .00394, and of increasing the apparent number of vibrations by 4.20 in a day.



I shall now consider the case in which the plane is moveable with the pendulum, and rests upon the knife edge, which is fixed.

Let D E F be a section of the knife edge, F O $= r$ the angle D O F $= \theta$.

A G $= \bar{y}$, O A $= \bar{z}$, G H $= a$

$\bar{y} =$ B C + C G, ultimately when θ is small

$\bar{y} =$ B C $\sin \theta + r \, \theta$, O A = O B ultimately

$\bar{z} =$ B C $\cos \theta$.

It may easily be shown that

$$A \frac{d \theta^2}{d t^2} + M \left\{ \frac{d \bar{z}^2}{d t^2} + \frac{d \bar{y}^2}{d \bar{z}^2} \right\} + 2 \, M g \, \bar{z} = M h$$

(See Poisson, Traité de Mécanique, vol. ii. p. 189.)

$$d \, \bar{y} = \{a \cos \theta + r\} \, d \, \theta$$

$$d \, \bar{z} = - \, a \sin \theta \, d \, \theta$$

$$\{k^2 + (a + r)^2\} \frac{d \theta^2}{d t^2} + 2 \, g \, a \cos \theta = h$$

The length of the simple pendulum is $\frac{k^2 + (a + r)^2}{a}$

If the two knife edges are isochronous, and $r_1 = r_2$

$$\frac{k^2 + (a_1 + r)^2}{a_1} = \frac{k^2 + (a_2 + r)^2}{a_2}$$

$$k^2 = \frac{a_1 (a_2 + r)^2 - a_2 (a_1 + r)^2}{a_2 - a_1}$$

The length of the simple pendulum $= a_2 + a_1 + 2\,r =$ the distance between the planes which vibrate on the knife edges.

Index to the notation.

Ox, Oy, Oz rectangular coordinate axes meeting in the point O. G the centre of gravity, G$x_{\prime}$, G$y_{\prime}$, G$z_{\prime}$ their principal axes which intersect each other in the point G. C the point in the axis Ox, where a perpendicular let fall upon it from the centre of gravity G cuts it, C′ the point when the plane $(z\,y)$ cuts the axis Ox; C″ the point where one of the surfaces of the pendulum cuts the same knife edge, G″ the point in this surface where a perpendicular from G cuts it.

g the force of gravity

$$y_{\prime} = x_{\prime} \tan \delta + \beta$$

$$z_{\prime} = x_{\prime} \frac{\tan \delta}{\tan \delta'} + \gamma$$

the equations to the axis Ox referred to the coordinate axes G$x_{\prime}$, G$y_{\prime}$, G$z_{\prime}$, so that δ and δ' may be considered as the deviations of the knife edge in azimuth and altitude.

G C $= a$, G C′ $= a'$.

$M =$ the mass of the pendulum.

A, B, C the principal moments of inertia.

$A = M k^2$, $B = M k'^2$, $C = M k''^2$.

ε, ε', ε'' the angles which the line Ox makes with the coordinate axes, G$x_{\prime}$, G$y_{\prime}$, G$z_{\prime}$.

$t =$ half the thickness of the pendulum.

$\beta = a' \sin \lambda$, $\gamma = a' \cos \lambda$.

$\bar{x}$, $\bar{y}$, $\bar{z}$ the coordinates of the centre of gravity.

r the radius of curvature of the knife edge.

XV.—*On the theoretical investigation of the velocity of sound, as corrected from M.* DULONG'S *recent experiments, compared with the results of the observations of Dr.* MOLL *and Dr.* VAN BEEK. *By Dr.* SIMONS, *Assistant at the Observatory of the University of Utrecht. Communicated by Captain* HENRY KATER, *Vice-President.*

Read March 18, 1830.

IT has been demonstrated by the ever-to-be-lamented LAPLACE*, that in order to obtain the velocity of sound by calculation, Sir ISAAC NEWTON'S original expression†,

$$V = \sqrt{\frac{g \cdot p}{D}},$$

must be multiplied by the square root of the ratio between the specific heats of atmospheric air under a constant pressure and under a constant volume. In this formula V is the velocity of sound, g the intensity of gravitating force, p the atmospheric pressure, and D the density of the medium through which sound is transmitted; the density of mercury being equal to 1.

The coefficient, which is to multiply the Newtonian formula, has been deduced by M. LAPLACE, first from MM. LAROCHE and BERARDO'S‡ experiments, next from those of MM. CLEMENT and DESORMES§, and finally from the more accurate investigations of MM. GAY-LUSSAC and WELTER.

By introducing this correction, the velocity as deduced from calculation, was found to differ but little from what is actually obtained by experiment. But this difference between calculation and experiment, however small, was always such, that the observed velocity constantly exceeded that which was deduced by calculation.

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* Annales des Physique et de Chimie, t. iii. p. 238. t. xxiii. p. 1. Mécan. Céleste, t. v. p. 119, *seqq.*

† Princip. l. ii. prop. 48.

‡ Annales de Chimie, t. lxxxv. p. 72.

§ Journal de Physique, t. lxxxix. p. 333, *seqq.*

The degree of accuracy with which experiments on the velocity of sound are now conducted, naturally led to a supposition that some of the elements of the theoretical formula were susceptible of a greater degree of correctness; and thus natural philosophers were rather inclined to attribute the difference between experiment and calculation to some deficiency of the analytical expression, than to error in the observation.

It has been shown in a recent volume of the Philosophical Transactions, that the experiments on the velocity of sound, made by Captain Parry in the polar regions, lead to the same conclusions with those made by Drs. Moll and Van Beek, under widely different circumstances; and this coincidence would tend to confirm our doubts as to the correctness of some of the elements of the computation.

A very able experimentalist, M. Dulong, lately published some experiments on the specific heat of the gases*, in the investigation of which he recurs to Laplace's analytical formula. M. Dulong's reasoning is nearly this: If it be admitted that the velocity of sound in atmospheric air is obtained by multiplying the Newtonian formula by the square root of the ratio between the two specific heats of the air, under a constant pressure, and under a constant volume, it must follow, that this ratio, or the coefficient with whose square root the original formula must be multiplied, may also be deduced from the velocity of sound as given by observation. Accordingly, M. Dulong computed this ratio or coefficient from actual observations on the velocity of sound, by the formula

$$K = \frac{V^2}{\frac{g \cdot p}{D}}$$

in which K is the ratio between the specific heat of air under a constant pressure, and the specific heat of air under a constant volume, whilst V is the velocity of sound as obtained by experiment.

The object of this paper is to compare the inquiries of M. Dulong with the experiments on the velocity of sound made by Drs. Moll and Van Beek, which were published in the Philosophical Transactions.

The following Table contains only such of the observations of Drs. Moll and Van Beek, in which the guns were fired on both stations exactly at the same moment, and in which the interval of time between the flash and the report was correctly observed at both extremities of the base.

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* Annales de Physique et de Chimie, t. xli. p. 113.

Date.	No. of the Observation.	Barometrical Pressure reduced to 0° Centigr. and corrected for capillarity. Metres p.	Tension of Aqueous Vapour in Metres T.	Temperature, Centigrade Thermometer. t.	Time elapsed between Flash and Report.
1823.					seconds.
June 27	1	0.74450	0.0095347	12°.08	52.035
	3	0.74455	0.0095474	12.08	51.790
	4	0.74455	0.0094585	12.01	51.695
	5	0.74465	0.0096998	11.94	51.860
	6	0.74455	0.0095347	11.88	51.850
	8	0.74470	0.0094459	11.67	51.950
	9	0.74470	0.0093697	11.60	51.845
	12	0.74475	0.0093697	11.25	51.865
	13	0.74470	0.0092173	11.25	51.945
	17	0.74500	0.0093697	10.47	51.960
	18	0.74505	0.0092173	10.40	52.055
	19	0.74495	0.0092173	10.40	52.025
	23	0.74480	0.0089888	10.28	51.995
	24	0.74470	0.0088364	10.14	51.945
	25	0.74475	0.0088364	10.07	52.020
	26	0.74480	0.0084683	10.00	51.335
June 28	4	0.74830	0.0084638	11.89	52.020
	5	0.74775	0.0083921	11.09	51.525
	6	0.74800	0.0084638	11.53	52.245
	7	0.74820	0.0081763	11.25	32.315
	8	0.74815	0.0081763	10.78	52.215
	9	0.74815	0.0083159	10.42	52.675
	10	0.74835	0.0083921	10.28	51.175
	14	0.74810	0.0081763	10.14	52.445
	15	0.74810	0.0082524	10.42	51.965
	17	0.74815	0.0086206	10.97	51.875
	18	0.74805	0.0086968	10.89	52.240
	19	0.74805	0.0087730	11.12	52.080

Length of base, 17669 metres.

From this Table and the following data, the results obtained by M. Dulong were compared with Drs. Moll and Van Beek's experiments.

The weight of a cube centimetre of mercury at the temperature of 0° Centigrade, is 13.596152 grammes. This is according to the experiments of MM. Biot and Arago, and MM. Dulong and Petit's subsequent investigations. The weight of a cube centimetre of dry atmospheric air, at zero of Centigrade scale, and under the barometric pressure of $0^{m}.760$ is, according to the same observers, 0.001299541 grammes*.

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* Biot, Traité de Physiq. Expér. et Mathém. t. i. p. 402, *seqq.* and p. 387.

Now, if the intensity of the gravitating force at Paris be g', whilst it is g at the place where the experiments are made, we have the weight of a cube centimetre of dry atmospheric air, under a pressure of 760*mm*, and at zero Centigrade, $0.001299541\,\frac{g}{g'}$.

Under the same circumstances the ratio of the density of air to that of mercury, will be:

$$D = \frac{0.001299541}{13.596152} \cdot \frac{g}{g'} = \frac{1}{10462.273} \cdot \frac{g}{g'}.$$

If the barometric pressure becomes p, the temperature t Centigrade, the tension of aqueous vapor T, the density of the air becomes

$$D' = \frac{p - \frac{3}{8}T}{10462.273 \times 0.760\,\{1 + 0.00375.t\}} \cdot \frac{g}{g}$$

substituting this value of D in the formula of the velocity of Sound, and observing that $g = g' \cdot \frac{g}{g'}$ we have,

$$V = \sqrt{\frac{10462.273 \times 0.760\,\{1 + 0.00375.t.\}\,p.g' . \frac{g}{g'}}{\{p - \frac{3}{8}T\} . \frac{g}{g'},}} \times \sqrt{K.}$$

or, $$V = \sqrt{10462.273 \times 0.760\,\{1 + 0.00375.t.\}\frac{g'.p}{p - \frac{3}{8}T}} \times \sqrt{K.}$$

This formula shows that V is independent of the latitude; and thus that the velocity of sound is not directly affected by the geographical position of the place of observation. From the late M. Borda's pendulum experiments, we have the intensity of gravity at Paris, $g' = 9.82827$.

From MM. Gay-Lussac and Welter's experiments, the value of K is deduced $K = 1.3748$.

From the more recent investigations of M. Dulong, we have $K = 1.421$.

Thus, taking K from MM. Gay-Lussac and Welter, the velocity of sound is

$$V = \sqrt{9.82827 \times 10462.273 \times 0.760\,\{1 + 0.00375.t.\}\frac{p}{p - \frac{3}{8}T}} \times \sqrt{1.3748}$$

and taking K from M. Dulong,

$$V = \sqrt{9.82827 \times 10642.273 \times 0.760\,\{1 + 0.00375.t.\}\frac{p}{p. - \frac{3}{8}T}} \times \sqrt{1.421}$$

Finally, supposing V′ the velocity of sound observed at a temperature *t* and a tension T, and V″ the velocity of sound at zero Centigrade and dry air, we have

$$V'' = V' \times \sqrt{\frac{p - \frac{3}{8}\mathrm{T}}{p\,\{t + 0.00375\,.\,t\}}}$$

From these formulæ the experiments of Messrs. **Moll** and **Van Beek** are calculated; the results of which are contained in the following Table.

A Comparative Table of the Velocity of Sound, as deduced by calculation, and obtained by the experiments of Drs. Moll and Van Beek.

Date.	Number of experiments.	Veloc. calc. from the determination of K by MM. Gay-Lussac & Welter in 1″.	Velocity observed by Drs. Moll and VanBeek in 1″.	Diff. of Obs. and Calc. Veloc., taking K from MM. Gay-Lussac and Welter.	Velocity calculated from the determination of K by M. Dulong in 1″.	Difference of Observed and Calculated Velocity taking K from M. Dulong.	Value of K as deduced from the experiments of Drs. Moll and Van Beek.	Observed Velocity reduced to 0° Cent. and in dry air.	Diff. between the observed reduced Velocity, and the mean reduced Velocity.
1823.		m	m	m	m	m		m	m
June27	1	335.590	339.565	+ 5.025	341.182	−1.617	1.4065	331.327	−0.917
	3	339.477	341.172	1.695	345.133	−3.961	1.3885	329.083	−3.161
	4	335.599	341.799	6.200	341.192	+0.607	1.4260	333.497	+1.253
	5	335.519	340.711	5.192	341.110	−0.399	1.4176	332.515	+0.271
	6	335.469	340.777	5.309	341.059	−0.282	1.4186	332.629	+0.385
	8	334.818	340.721	5.303	340.397	−0.276	1.4187	332.634	+0.390
	9	335.287	340.810	5.523	340.873	−0.063	1.4204	332.842	+0.598
	12	335.075	340.678	5.603	340.659	+0.019	1.4211	332.924	+0.680
	13	334.292	340.154	5.862	340.646	−0.492	1.4169	332.424	+0.180
	17	334.604	340.055	5.451	340.180	−0.125	1.4199	332.783	+0.539
	18	334.527	339.435	4.908	340.101	−0.666	1.4154	332.252	+0.008
	19	334.527	339.631	5.104	340.101	−0.470	1.4170	332.444	+0.200
	23	334.453	339.827	5.374	340.026	−0.199	1.4193	332.709	+0.465
	24	334.360	340.154	5.794	339.932	+0.222	1.4228	333.122	+0.878
	25	333.933	339.663	5.730	339.498	+0.165	1.4224	333.067	+0.823
	26	334.245	340.219	5.974	339.814	+0.405	1.4243	333.301	+1.057
June28	4	335.358	339.663	4.305	340.946	−1.283	1.4103	331.652	−0.592
	5	334.893	342.927	8.034	340.474	+2.453	1.4415	335.032	+2.788
	6	335.164	338.200	3.036	340.749	−2.549	1.3998	330.605	−1.539
	7	334.971	337.554	2.583	340.553	−2.999	1.3961	329.973	−2.271
	8	334.688	338.395	3.707	340.262	−1.870	1.4053	331.075	−1.119
	9	334.482	335.440	0.958	340.056	−4.616	1.3827	328.386	−3.858
	10	334.404	345.272	10.868	339.976	+5.296	1.4622	338.090	+6.154
	14	334.301	336.911	2.610	339.872	−2.961	1.3963	330.004	−2.240
	15	334.481	340.023	5.542	340.054	−0.031	1.4207	332.874	+0.630
	17	334.840	340.613	5.773	340.419	+0.194	1.4226	333.094	+0.850
	18	334.798	338.233	3.435	340.377	−2.144	1.4031	330.808	−1.436
	19	334.941	339.272	+ 4.331	340.522	−1.250	1.4106	331.682	−0.562
						Mean number	1.4152	332.244	

The preceding table shows how very near M. Dulong's value of K agrees with the result of experiment. Employing MM. Gay-Lussac's and Welter's co-efficient, the differences between the observed and calculated velocity are constantly affected with the same sign; whereas in taking K from M. Dulong, the differences are sometimes positive and sometimes negative. It is therefore presumed that M. Dulong's labours bring the calculation of the velocity of sound much nearer to the truth than before, and that such differences as are yet remaining between calculation and experiment, may be attributed, with great probability, to errors unavoidable in such complicated observations.

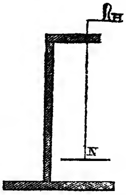
Utrecht, December 30, 1829.

XVI.—*On the elasticity of threads of glass, with some of the most useful applications of this property to torsion balances. By* William Ritchie, *A.M. F.R.S., Rector of the Royal Academy of Tain.*

Read March 18, 1830.

1. From facts connected with crystallization and elasticity, it seems extremely probable, that the atoms of matter do not attract each other indifferently on all sides. There appears to be peculiar points on their surfaces which have a more powerful attraction for each other, than for other points on the same molecule. This property is not peculiar to the atoms of ponderable matter, but seems also to belong to those of light and heat. It is as impossible to prove directly the existence of this property, as it is to prove the existence of atoms themselves; but on account of the satisfactory manner in which it enables us to explain the phænomena of crystallization and elasticity, it is now generally adopted.

2. If the atoms of solid bodies be slightly displaced by any mechanical means, they will endeavour to return to their former state of aggregation when the disturbing cause is removed. This property belongs in very different degrees to different substances. In lead it scarcely exists, and but slightly so in soft copper. In brass, iron and silver, especially when drawn into wires, it exists in a considerable degree. But all these substances have limits beyond which the property does not hold. If for example an iron wire be twisted several times, it will not return exactly to its former state, but remain partially twisted. In threads of glass, on the contrary, there seems to be no limit to this property, whilst the thread remains entire. Let a fine glass thread be suspended from a moveable index, and let a light horizontal needle of wood or straw be fixed to its lower extremity as in the annexed figure. If the thread be now twisted by means of the handle H, whilst the needle N is prevented turning round, and then allowed to untwist itself, the needle will return exactly to its former position after it has ceased to oscillate.



If the vitreous molecules be held together by the attraction of their poles or points of greatest affinity, it is obvious that these points will be displaced by torsion along the whole line of communication. The points of greatest attraction thus displaced, will therefore endeavour to regain their former state of stable equilibrium, and the thread will of course untwist itself till the needle returns to its former position. It would be curious to ascertain if a thread of glass, twisted as much as it can safely bear, and kept in that position for several months or years, would return exactly to its former position, or whether the atoms might not in course of time take up a new state of stable equilibrium.

3. The number of times a thread of glass may be twisted without breaking, will of course depend on its length and diameter. It is almost incredible the number of times a thread of a substance so brittle as glass may be twisted, before the points of greatest attraction of the vitreous molecules be actually removed beyond the sphere of attraction, or in other words, before the thread be broken. I have succeeded in drawing threads of glass of such extreme tenuity, that one of them, not more than a foot long, may be twisted nearly a hundred times without breaking. Hence it is obvious, that if a thread could be drawn so fine as to consist of a single line of vitreous molecules, torsion could have no tendency to displace the points of greatest attraction, and this elementary thread might be twisted for ever without breaking. In that case the compound molecules of glass would only turn round their points of greatest attraction, like bodies revolving on a pivot.

4. It is difficult to prove by direct experiment some of the laws of torsion established by Coulomb, as belonging to metallic wires, on account of the difficulty of procuring threads of glass of a uniform diameter throughout their whole length. It is difficult, for example, to prove by experiments, that the force of torsion of a glass thread is directly as its length, and inversely as the fourth power of its diameter. Fortunately, however, the only property which we are to employ in the construction of the following instruments, can be proved by direct experiment. This property is, within certain limits, common to all elastic threads, namely, "that the force of torsion, or that force with which a thread tends to untwist itself, is directly proportional to the number of degrees through which it has been twisted*."

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* Biot, Traité de Physique, tom. i. p. 486.

This property may be established by the following methods.

1st, Let a horizontal needle of glass, or of any substance not magnetic, be fixed to the lower extremity of a fine thread of glass, and then made to oscillate: it will be found that these oscillations are isochronous, even when the thread has been twisted through several circumferences. Now this isochronism may be proved to belong only to an elastic thread possessing the property enunciated in the preceding proposition.

2ndly, Suspend a magnetic needle in a horizontal position, by a similar glass thread, over the centre of a large circle having its circumference divided into degrees. Twist the thread by means of the key as in the common torsion balance, and note the degrees of torsion and the corresponding deviations of the needle from the magnetic meridian, and it will be found that the sines of the deviations are proportional to the corresponding degrees of torsion;—a property which can only belong to elastic threads possessing the property in question*.

5. This perfect elasticity of torsion belonging to threads of glass, may be applied with decided advantage to the electric and magnetic balances of Coulomb. All that is necessary in those cases is to substitute a thread of glass of the proper degree of fineness, for the silver wires employed with so much success by the ingenious inventor. The application of this property to the construction of a galvanometer and delicate balance, is, I believe, new, and therefore I have ventured to lay a description of them before the Society; but before doing this, it may not be improper to describe the best method of making the glass threads employed in the construction of these instruments.

Heat the end of a clean thermometer tube at the flame of a blowpipe, and draw it out till it be of the thickness required for fixing in the hole in the end of the torsion key, as in the annexed cut. Direct the flame of lamp on the tube at A, till the glass has become sufficiently soft. Remove it from the flame, and draw it out rapidly till it be of the length and fineness required. By separating the thread from the tube, we shall thus have a thread of any degree of fineness, terminated by two thicker portions, which may be securely fixed with cement or sealing-wax as circumstances may require. Simple as this may appear, it requires some practice in manipulation to do it well; it will therefore be necessary to draw several threads of different lengths and degrees of fineness, and afterwards select those best adapted to the instrument to which it is to be applied.

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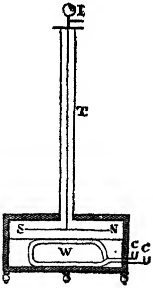
* Biot, Traité de Physique, tom. iii. p. 29.

6. Description and use of the torsion galvanometer.

Take a fine copper wire and cover it with a thin coating of sealing-wax. Roll it about a heated cylinder an inch or two in diameter, ten, twenty, or any number of times, according to the delicacy of the instrument required. Press together the opposite sides of the circular coil, till they become parallel and about an inch or an inch and a half long. Fix the coil in a proper sole, and connect the ends of the wires with two small metallic cups for holding a drop of mercury. Paste a circular disc of paper, divided into equal parts, horizontally on the upper half of the coil, and having a black line drawn through its centre and in the same direction with the middle of the coil. Fix a small magnet, made of a common sewing-needle or piece of steel wire, to the lower end of a fine glass thread, whilst the upper end is securely fixed with sealing-wax in the centre of a moveable index, as in the common torsion balance. The glass thread should be inclosed in a tube of glass, which fits into a disc of thick plate glass, covering the upper side of the wooden box containing the coil and magnetic needle.



The whole will be obvious from the simple inspection of the annexed vertical section of the instruments, in which W is the coil of wire, C C′ the cups, T the glass tube containing the thread, I the index turning in the centre of a divided circle to mark the degrees of torsion, and N S the magnetic needle.

By means of this instrument we may compare with great accuracy the relative quantities of currents of voltaic electricity circulating along the wires of the coil. For this purpose place the needle directly above the line drawn on the paper, and consequently directly above, and in the direction of the wires forming the upper side of the coil. Cause a current of voltaic electricity to circulate along the wires, and the needle will of course be deflected. Twist the glass thread, by turning the index, till the needle be brought to its former position, and note the number of degrees of torsion; untwist the thread, and repeat the experiment with another current; and the quantities of electricity circulating round the wires will be directly proportional to the number of degrees through which the thread has been twisted. By this contrivance it is obvious that the currents always act with the same mechanical advantage on the needle; and consequently their deflecting forces, which are counterbalanced by the elastic force of torsion, must be directly as the number of degrees through which the thread has been twisted. In the common galvanometer, the deflecting force acts with diminished mechanical advantage as the needle deviates from the coil. When it has been deflected nearly 90 degrees from the original position, an additional power will produce scarcely any effect, and consequently the instrument ceases to give indication of a more energetic current.

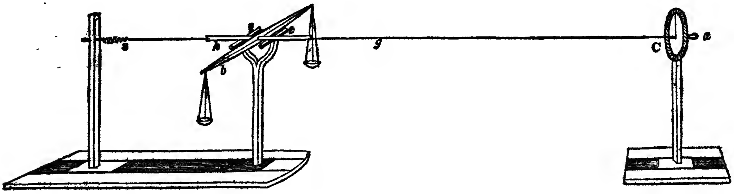
This instrument appears to me well adapted to many interesting investigations connected with voltaic electricity; but these could not properly be introduced in this paper, and may therefore form the subject of another communication.

7. We shall now describe another application of the elasticity of glass threads for ascertaining the weight of very minute portions of matter. The chemist in some of his most refined analyses has frequently to ascertain the weights of minute portions of matter, which by the ordinary process is a work of time and labour. A balance of the most delicate and perfect kind is also required, which, from its expense, frequently prevents the young chemist from prosecuting his experimental researches.

The balance which I am now to propose may be made at a trifling expense, and it will give the weights of small quantities of matter to a degree of accuracy seldom attained by the most perfect hydrostatic balances.

Prepare a small wooden beam, very light and about a foot or fifteen inches long, into the centre of which fix a steel blade having a smooth edge, like the blade of a fine penknife. To one extremity of the blade and in the line of its edge a fine thread of glass is to be securely fixed, whilst the other end of the thread is to be secured with sealing-wax in the centre of a small cylindrical key, passing through the centre of a vertical circle divided into degrees, or any convenient number of equal parts. To the other end of the fulcrum, and also in the line of the edge, a fine thread formed of a few fibres of untwisted silk is to be attached, whilst its other extremity is to be fixed to a spiral spring formed of fine brass wire, in order to keep the glass thread properly stretched. The knife edge is made to rest on two small portions of thermometer tube, placed parallel to each other on the top of an upright support. Portions of a still smaller steel blade are to be fixed, nearly in the same straight line with the edge of the fulcrum and at the ends of the beam, for suspending the scales. In one end of the beam a fine sewing-needle is to be fixed, for the purpose of pointing out, on a divided scale, the position of the beam when nearly horizontal. A similar needle is to be fixed in the cylindrical key, for pointing out the number of degrees of torsion which the thread has suffered.

The annexed figure represents a perspective view of the instrument seen obliquely, in which *b* is a beam, *g* the glass thread, *k* the knife edge fulcrum, *t t'* the thermometer tubes for supporting the beam, *s* the spiral spring with its attached silk thread, *c* the divided circle, and *a* the torsion key.



Having thus described the balance itself, we shall now explain the manner of employing it; first, for the determination of very small weights, and secondly, for heavier ones.

1st. To determine the weight of a small quantity of matter by employing only one weight, suppose a grain.—Twist the glass thread by turning the torsion key through two or three circumferences, according to the degree of torsion which it will bear without the risk of breaking. Put brass filings or other convenient counterpoise, into one of the scales till the beam be brought to a position nearly horizontal, the index of the torsion key pointing to zero on the divided circle. Place the body to be weighed into the scale, which is to be raised by untwisting the glass thread. Turn the torsion key till the elastic force of the thread raise the weight, and carefully observe the point on the scale at which the needle, in the end of the beam, becomes stationary. Note the number of degrees of torsion which the thread has suffered. Remove the weight, and untwist the thread till the beam returns to its horizontal position. Put a small known weight into the same scale, and turn the torsion key till the beam be raised to its former horizontal position, and observe the number of degrees of torsion:—then will the degrees of torsion give the ratio of the known and unknown weights. For example, if the thread suffered a torsion of 1500 degrees to raise the body B, and 1000 to raise one grain, then 1000 : 1500 : : 1 grain : 1.5, the weight of the body. If the body required only 50 degrees of torsion to raise it, then its weight would be $\frac{50}{1000}$ or .05 of a grain.

2ndly. When the body to be weighed is much heavier than a grain, the best way will be to ascertain its weight within a grain by the method of double weighing, and then apply the principle of torsion for ascertaining the fraction of a grain. Suppose the body to weigh nearly 100 grains, twist the thread through two or three circumferences exactly as in the former method. Bring the beam to a horizontal position by small shot or filings. Put the body into one of the scales, and shot or filings into the other, till the body be exactly counterpoised. Remove it, and substitute known weights till they be nearly equal to the weight of the body. Turn round the torsion key till the beam be brought to a horizontal position, and note the degree of torsion.

Put a grain into the scale, and observe the additional number of degrees of torsion necessary to bring the beam to its horizontal position; and we thus get the fraction of a grain which the body exceeds the known weights employed. If for example the body weighed nearly 100 grains, and it required a torsion of 50 degrees to raise the scale when 99 grains had been put into it after the body had been removed, and also that the thread required an additional torsion of 1000 when one grain had been placed in the same scale, then the weight of the body will be $99\frac{50}{1000}$ or 99.05 grains.

8. It is of course necessary to prevent the agitation of the air acting on the balance and its scales, and therefore the whole may be inclosed in a box as in the common balance. It is not necessary, however, to inclose the glass thread and the divided circle, and therefore the thread may be made to pass through a hole in the back of the box, and removed when the balance is not in use. It is useful to have a number of threads of glass of different degrees of

fineness, having small brass tubes cemented on their ends for the purpose of attaching them to the fulcrum and torsion key.

The method now described may appear somewhat tedious, but it is only so in appearance, as the oscillations do not continue so long as in a delicate balance without the torsion thread. In some delicate experiments with this balance, I have used threads of glass about ten feet long, so that in raising a weight of one grain, the glass thread suffered a torsion of at least 5000 degrees. Hence a very small fraction of a grain may be determined with an extraordinary degree of accuracy. From the perfect elasticity of torsion which glass possesses, and from the ease with which threads of any length and fineness may be procured, I am fully convinced that, for all delicate investigations connected with torsion balances, threads of this substance will be found to possess decided advantages.

METEOROLOGICAL JOURNAL,

KEPT BY THE ASSISTANT SECRETARY

AT THE APARTMENTS OF THE

ROYAL SOCIETY,

BY ORDER OF

THE PRESIDENT AND COUNCIL.

MDCCCXXX.

METEOROLOGICAL JOURNAL FOR JULY, 1829.

1829. July.	9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
	Barom.	Attach. Therm.	Barom.	Attach. Therm.		Fahrenheit.		Self-registering.				
						9 A.M.	3 P.M.	Lowest.	Highest.			
● ☿ 1	29.647	65.3	29.513	66.9	55	59.7	61.8	57.6	65.5	0.292	SSE	Overcast—rain.
♃ 2	29.587	73.5	29.631	68.7	51	64.6	66.7	55.7	68.7	0.097	SSW	Fine and clear—brisk wind.
♀ 3	29.609	67.8	29.477	67.7	56	64.5	62.8	57.0	67.8		SSE	Cloudy—showery. Rain, with high wind, at night.
♄ 4	29.631	73.5	29.573	67.8	50	63.5	58.7	52.8	68.6	0.036	WSW	A.M. Fine and clear. P.M. Overcast.
☉ 5	29.585	63.1	29.660	67.1	51	58.4	60.7	50.7	66.7	0.089	W	Cloudy. Evening fine and clear.
☽ 6	29.876	69.7	29.893	69.3	55	66.0	69.7	52.8	71.7	0.125	W	Fine and clear—light wind.
♂ 7	29.878	65.7	29.741	65.9	60	60.7	60.7	55.8	67.7	0.042	WSW	Overcast—brisk wind. Rain.
☿ 8	29.834	70.3	29.786	70.7	55	67.3	69.3	55.8	71.7	0.044	NW	Fine—light clouds.
♃ 9	29.741	67.7	29.809	68.3	51	63.2	65.7	56.7	67.7		NW	Fine—cloudy.
♀ 10	29.913	70.6	29.817	70.2	50	66.7	65.0	53.4	68.8		SW	A.M. Fine. P.M. Lowering. Evening, rain.
♄ 11	29.556	66.4	29.485	67.1	61	62.3	61.6	58.9	65.8	0.022	SSE	Lowering. P.M. Heavy rain.
☉ 12	29.422	67.7	29.497	69.0	62	63.7	68.0	56.7	69.2	0.514	WSW	Fine and clear—cloudy.
☽ 13	29.658	66.3	29.733	70.3	60	63.7	68.5	56.4	69.5	0.100	SSW	Lowering.
♂ 14	29.889	69.2	29.861	70.3	66	66.4	68.7	62.8	73.7		SSW	Cloudy. Heavy rain A.M.
☿ 15	30.014	74.6	29.982	73.0	60	68.7	73.5	58.3	74.7	0.125	W	Fine and clear—light clouds.
○ ♃ 16	29.947	74.3	29.958	72.9	56	66.9	69.5	57.8	73.2		W	Fine. Heavy showers P.M.
♀ 17	29.910	68.3	29.683	68.0	60	64.1	60.3	56.4	64.8	0.125	SSE	Lowering—light wind. Rain P.M.
♄ 18	29.562	68.7	29.589	70.3	64	64.3	66.7	59.7	69.8	0.239	SE	A.M. Lowering—heavy showers. P.M. Fine.
☉ 19	29.785	70.2	29.867	71.7	60	65.9	66.4	59.8	70.2	0.322	NW	Fine—lightly cloudy.
☽ 20	30.097	73.0	30.088	71.3	54	66.3	70.6	51.7	71.1	0.047	NW	Fine and clear.
♂ 21	30.254	72.3	30.238	71.6	57	66.9	73.8	54.0	74.7		WSW	Fine—light clouds.
☿ 22	30.260	74.3	30.205	73.7	62	69.8	76.2	59.3	78.3		W	Fine and clear—cloudy.
♃ 23	30.187	68.7	30.153	72.4	62	63.7	71.4	60.7	73.3		W	A.M. Overcast. P.M. Fine.
♀ 24	30.030	75.2	29.954	74.3	66	68.0	71.7	59.3	72.0		E	Fine—cloudy.
♄ 25	29.873	70.6	29.897	74.5	65	65.8	74.6	64.2	75.4	0.183	WSW	A.M. Thunder and lightning, from 1h to 2h, with hail and rain.
☉ 26	30.028	65.3	30.060	65.6	57	57.8	58.9	56.0	60.6		N	Overcast. Shower at noon.
☽ 27	30.101	65.1	30.102	66.9	56	59.3	64.0	48.7	66.8	0.008	NNE	Fine—light clouds.
♂ 28	30.098	70.7	29.999	67.7	51	66.3	67.9	51.7	70.3		WSW	Fine and clear.
☿ 29	29.642	64.2	29.664	68.0	59	60.1	69.2	58.4	69·7	0.153	SE	A.M. Showers, early. Fine—cloudy.
● ♃ 30	29.748	69.6	29.755	69.3	53	64.3	67.7	51.5	69.7		NNE	Fine—light clouds. Thunder and lightning at 4 P.M. with heavy rain.
♀ 31	29.977	60.7	30.070	66.7	58	58.5	64.0	52.3	65.3	0.347	NNW	A.M. Cloudy. P.M. Clear—brisk wind.
	Mean 29.850	Mean 69.1	Mean 29.830	Mean 69.6	Mean 57.5	Mean 64.1	Mean 66.9	Mean 56.2	Mean 69.8	Sum 2.910		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M.	3 P.M.
	29.760	29.737

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge .... =83 feet 2½ in.

.... above the mean level of the Sea (presumed about)............... =95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House .... =79 feet 0 in.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR AUGUST, 1829.

1829. August.	9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
	Barom.	Attach. Therm.	Barom.	Attach. Therm.		Fahrenheit.		Self-registering.				
						9 A.M.	3 P.M.	Lowest.	Highest.			
♄ 1	30.248	67.5	30.242	67.6	63	63.3	66.0	51.3	68.7		NNW	A.M. Fine and clear. P.M. Lowering —distant thunder.
☉ 2	30.289	66.7	30.262	68.7	57	65.7	72.7	54.3	73.7		W	Fine and clear—light clouds and wind.
☽ 3	30.122	66.8	29.965	66.7	59	65.5	63.0	57.7	67.7		SW var.	Rain—light brisk wind.
♂ 4	29.826	67.8	29.791	66.8	55	61.0	62.6	50.7	64.4	0.183	W	Cloudy—brisk wind. Showery P.M.
☿ 5	29.912	63.7	30.010	66.7	57	61.8	65.9	53.8	65.8	0.039	NNW	Fine—light clouds and breeze. P.M. Cloudy.
♃ 6	30.078	64.0	30.070	68.2	60	62.7	66.7	54.3	69.4		W	Lowering.
♀ 7	30.184	64.7	30.207	68.9	61	61.7	70.6	60.3	71.7	0.114	E	A.M. Light rain, early. P.M. Fine.
♄ 8	30.240	68.4	30.188	71.4	64	67.0	76.5	58.7	76.5		E	Fine and clear—light clouds.
☉ 9	30.115	67.4	30.041	71.2	66	66.3	71.2	57.6	71.7		SSW	A.M. Cloudy. P.M. Clear—light clouds. Breeze.
☽ 10	29.935	66.2	29.921	69.3	60	60.7	69.3	60.5	70.2	0.406	W	A.M. Heavy rain, early. P.M. Fine and clear.
♂ 11	30.123	64.7	30.144	67.4	53	61.7	67.7	54.8	69.3	0.112	W	Fine—light clouds.
☿ 12	30.137	65.8	30.065	69.4	61	65.7	70.7	58.3	71.8		S	Fine and clear—light wind.
♃ 13	29.778	66.3	29.728	69.7	66	66.2	67.3	61.4	72.8		SSW	A.M. Fine. P.M. Lowering. Light wind.
○ ♀ 14	29.505	65.7	29.452	68.2	64	64.4	68.2	61.3	69.8	0.097	SE	Cloudy—heavy rain.
♄ 15	29.577	60.8	29.628	61.8	56	56.5	57.8	53.3	60.4	0.180	W var.	Rain. Light brisk wind.
☉ 16	29.967	58.7	30.053	61.7	54	57.6	61.0	51.0	62.6	0.292	NNW	Fine and clear—cloudy. Brisk wind.
☽ 17	30.180	56.8	30.155	63.7	51	57.0	65.3	46.7	66.4		W	A.M. Fine & cloudless. P.M. Broken clouds.
♂ 18	29.847	59.3	29.753	62.7	58	58.3	62.4	55.4	64.3	0.092	SSW	Overcast—rain. Light wind.
☿ 19	29.526	63.7	29.476	65.3	60	63.7	66.3	58.0	67.3	0.147	WSW	Fine and clear—cloudy. Showery P.M.
♃ 20	29.284	60.7	29.500	62.7	59	59.7	61.3	55.7	63.3	0.083	SSW	Rain. Evening clear.
♀ 21	29.841	60.0	29.902	62.3	52	63.3	62.6	51.8	63.3	0.114	NNW	Fine—light clouds.
♄ 22	29.924	58.7	29.791	62.3	53	59.3	61.4	52.2	63.4		S	Cloudy. At night, strong high wind with heavy rain.
☉ 23	29.514	63.8	29.504	67.5	63	63.0	66.2	59.5	68.3	0.286	SSW	Cloudy—brisk wind. Rain A.M.
☽ 24	29.347	63.3	29.525	63.6	59	60.4	62.7	59.3	62.9	0.128	W	Lightly cloudy and showery.
♂ 25	29.922	58.7	29.999	62.3	52	58.0	62.5	49.7	63.4	0.161	WSW	Fine and clear—brisk wind.
☿ 26	30.060	59.6	29.914	62.6	57	59.3	61.2	51.3	62.7		S	Cloudy. At night, strong wind.
♃ 27	29.626	62.4	29.560	63.3	48	60.4	60.9	57.8	65.7	0.008	WSW	Fine and clear. Evening, brisk wind.
♀ 28	29.521	58.4	29.650	61.6	56	58.4	60.3	53.8	61.0	0.272	NW	Light clouds and brisk wind. At night, rain and strong wind.
● ♄ 29	29.918	60.0	30.499	64.3	59	59.0	64.0	54.7	64.2	0.125	N	Clear—light clouds—strong brisk wind.
☉ 30	30.193	57.2	30.173	61.0	52	55.7	61.3	52.3	62.3		NNW	Fine and clear—light wind.
☽ 31	30.100	56.7	30.068	60.7	56	56.3	59.7	53.3	60.7		NNW	Cloudy—light wind.
	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Sum		
	29.898	62.7	29.911	65.5	57.8	61.3	65.0	55.2	66.6	2.839		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M.	3 P.M.
	29.823	29.828

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge........................ =83 feet 2¼ in.

.... above the mean level of the Sea (presumed about) .... =95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House .... =79 feet 0 in.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR SEPTEMBER, 1829.

1829. Septemb.	9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
	Barom.	Attach. Therm.	Barom.	Attach. Therm.		Fahrenheit.		Self-registering.				
						9 A.M.	3 P.M.	Lowest.	Highest.			
♂ 1	29.998	58.3	29.965	64.0	57	57.4	63.5	54.3	64.2	0.044	NNE	Cloudy—light wind. Evening, light rain.
☿ 2	29.955	60.6	29.980	65.6	59	59.7	63.8	56.4	66.5	0.008	NNW	Cloudy and foggy. Evening clear.
♃ 3	30.176	57.4	30.510	61.3	49	55.3	62.4	52.2	63.2		NW	Fine and clear—lightly cloudy.
♀ 4	30.009	58.0	29.945	61.7	50	56.6	62.0	53.6	63.3		W	Fine and clear—cloudy.
♄ 5	29.712	58.2	29.581	62.0	56	58.3	60.3	51.9	63.5		SSE	Overcast—light wind. At night, light rain.
☉ 6	29.531	61.7	29.499	63.8	59	60.8	63.3	54.8	65.2	0.005	WSW	Fine—lightly overcast.
☽ 7	29.663	59.8	29.658	64.3	54	58.5	63.7	52.3	64.3	?	S	Fine and clear—light clouds. Showery P.M.
♂ 8	29.496	61.3	29.563	63.4	59	61.3	60.7	54.4	65.2	?	S var.	Heavy rain, early A.M. Fine and clear—brisk wind.
☿ 9	29.671	59.7	29.682	64.2	55	58.7	64.2	52.7	65.5	0.389?	WSW	Fine and clear—light wind.
♃ 10	29.387	62.7	29.357	66.5	61	62.0	66.7	58.8	66.7	0.389?	SE	A.M. Cloudy. P.M. Fine and clear. Light brisk wind.
♀ 11	29.606	60.0	29.689	64.6	55	59.4	62.8	55.4	65.2	?	WSW	Fine and clear. Evening rainy.
♄ 12	29.663	59.2	29.630	63.3	54	57.3	60.3	50.3	63.7	0.203	W	Fine and clear—cloudy. Light showers.
○ ☉ 13	29.565	56.0	29.442	62.4	53	55.7	61.4	47.5	62.7	0.011	SSW	Fine and clear. Heavy rain at night.
☽ 14	29.208	57.3	29.300	61.6	48	54.4	56.6	52.4	61.6	0.556	W	A.M. Fine. P.M. Showery.
♂ 15	29.668	53.7	29.765	60.4	45	53.0	59.1	44.9	61.3	0.050	W	Very clear. P.M. Showery: at 4h. thunder and lightning.
☿ 16	29.561	54.7	29.535	57.2	52	52.5	54.7	49.5	54.7	0.408	N	Foggy. Light brisk wind.
♃ 17	29.868	55.3	29.807	59.2	50	50.6	57.9	44.8	58.3	0.033	WNW	Overcast. At night, rain.
♀ 18	29.175	56.7	29.161	61.2	56	56.7	59.7	45.0	63.8	0.389	ESE	Fine—lightly cloudy.
♄ 19	29.472	56.7	29.631	56.8	54	54.5	55.2	51.7	56.7		NW	Overcast.
☉ 20	29.929	55.9	29.898	58.7	51	51.7	58.4	45.7	58.8	0.089	NW	Fine—light clouds.
☽ 21	29.761	56.7	29.798	61.3	54	56.4	61.0	51.4	62.3	0.011	W	Fine and clear—light clouds.
♂ 22	29.880	55.7	29.812	60.5	53	55.7	60.0	48.4	61.4	0.056	WSW	Fine—lightly cloudy.
☿ 23	29.834	56.7	29.844	60.6	53	53.9	59.4	51.2	62.7	0.019	WSW	Fine—hazy.
♃ 24	29.957	55.7	29.913	59.6	51	53.7	58.2	51.6	59.7		WSW	A.M. Strong haze. P.M. Fine.
♀ 25	30.125	53.7	30.146	58.7	53	52.7	58.4	48.3	58.8		N	Fine. Strong fog at night.
♄ 26	30.232	51.6	30.168	59.4	49	51.2	59.3	46.2	62.0		N	Strong haze.
☉ 27	29.892	60.7	29.797	61.3	58	61.8	60.3	50.4	62.0		WSW	Fine and clear—light wind. Showers at noon.
● ☽ 28	29.876	52.9	29.873	58.0	46	51.4	54.2	46.3	56.7	0.044	NW	Fine—light clouds.
♂ 29	29.983	51.0	29.986	54.6	42	48.6	54.0	44.3	54.3		N	Fine—light clouds.
☿ 30	30.213	50.2	30.212	54.8	43	44.3	54.2	42.4	54.3		N	A.M. Strong fog. P.M. Fine & clear.
	Mean 29.769	Mean 56.9	Mean 29.772	Mean 61.0	Mean 52.6	Mean 55.5	Mean 59.8	Mean 50.3	Mean 61.6	Sum 2.704?		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. ………… 9 A.M. 29.709 3 P.M. 29.702

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge…………………… =83 feet 2¼ in.

…………………………………………… above the mean level of the Sea (presumed about) ………… =95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House …………………… =79 feet 0 in.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR OCTOBER, 1829.

1829. October.	9 o'clock, A.M.		3 o'clock P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
	Barom.	Attach. Therm.	Barom.	Attach. Therm.		Fahrenheit.		Self-registering.				
						9 A.M.	3 P.M.	Lowest.	Highest.			
♃ 1	30.252	52.0	30.220	58.8	51	53.0	59.6	43.6	59.7		ENE	Fine—light brisk wind.
♀ 2	30.040	56.7	29.966	58.6	53	57.4	57.8	51.9	58.8		E	Overcast—showery. Brisk wind A.M.
♄ 3	29.840	58.9	29.788	58.6	55	58.8	56.8	54.3	58.8	0.131	SSW	Overcast. Heavy showers P.M.
☉ 4	29.968	53.6	29.961	58.4	52	53.0	57.7	47.7	58.3	0.444	W	A.M. Fine. P.M. overcast.
☽ 5	29.571	56.7	29.570	59.1	54	54.8	56.8	52.7	59.4	0.056	W	A.M. Rain. P.M. Fine.
♂ 6	29.670	51.4	29.675	54.2	46	50·2	52.5	44.6	53.4	0.072	NW	Fine—light haze.
☿ 7	29.509	49.0	29.443	40.2	38	46.5	37.4	42.3	46.5		W	Overcast. Rain and snow P.M.
♃ 8	29.808	43.8	29.921	46.9	27	39.5	47.5	33.3	47.5	0.606	W	Fine and cloudless. A.M. Strong wind early.
♀ 9	30.203	41.8	30.229	47.5	31	40.3	47.0	34.3	47.0		NW	Fine and cloudless—hazy.
♄ 10	30.432	44.0	30.428	50.4	41	43.7	54.7	35.7	56.0		WSW	Overcast and hazy.
☉ 11	30.317	51.7	30.211	55.1	49	52.6	57.7	43.0	57.7		W	Cloudy—light wind.
○ ☽ 12	30.152	54.7	30.138	56.2	48	54.7	56.2	52.2	56.3		NW	A.M. Cloudy. P.M. Fine.
♂ 13	30.033	55.2	29.873	57.2	49	55.4	56.8	50.3	57.5		SW	Lightly cloudy.
☿ 14	29.420	56.7	29.578	51.0	51	52.8	48.8	52.3	52.8	0.078	NNW	Cloudy—light rain.
♃ 15	30.227	43.7	30.259	48.8	34	41.7	48.5	37.3	47.7	0.025	N var	Fine and cloudless—hazy.
♀ 16	30.075	48.6	29.944	51.4	39	47.3	51.0	39.5	54.1		SSW	Overcast.
♄ 17	30.047	52.3	30.119	55.5	45	51.3	55.4	46.8	55.7		NW	Fine—light breeze.
☉ 18	30.127	55.3	30.129	59.0	54	56.0	61.1	50.8	61.4		WSW	A.M. Cloudy. P.M. Cloudless—light wind.
☽ 19	30.080	57.7	30.050	60.8	57	58.2	59.8	53.3	60.5		SW	A.M. Cloudy. P.M. Clear—light clouds.
♂ 20	29.873	58.8	29.808	60.5	56	57.5	59.4	55.5	60.2		SW	Clear—light clouds.
☿ 21	29.805	57.0	29.703	58.6	53	53.8	57.0	48.3	57.7	0.017	S	A.M. Clear & cloudless. P.M. Cloudy. Evening rainy.
♃ 22	29.658	54.4	29.728	55.1	42	51.4	53.0	48.6	53.7	0.100	W	A.M. Cloudy. P.M. Fine.
♀ 23	29.955	49.7	29.934	51.6	41	44.8	49.8	41.7	49.8	0.019	NNW	Fine—hazy.
♄ 24	29.957	46.7	29.913	51.5	43	45.7	51.2	39.8	51.2		NNE	Fine—light clouds.
☉ 25	30.141	45.4	30.197	52.8	42	44.3	51.8	40.2	51.8		NNW	Fine—hazy.
☽ 26	30.350	50.0	30.344	51.5	48	48.7	51.0	43.7	50.7		NNE	Hazy.
● ♂ 27	30.397	46.3	30.322	49.5	41	41.7	49.5	40.2	49.7		NNW	A.M. Foggy. P.M. Cloudless—hazy.
☿ 28	30.245	50.8	30.278	51.2	48	48.6	48.6	41.3	49.7		NNE	Clear—light clouds and wind.
♃ 29	30.347	43.7	30.275	47.5	36	42.1	45.4	39.2	45.4		NNE	Fine—hazy.
♀ 30	30.092	46.7	30.499	53.0	43	46.3	52.2	39.9	53.1		WSW	Lightly cloudy—hazy. Rain P.M.
♄ 31	29.917	51.7	29.990	45.7	44	46.9	49.8	45.4	49.8		NNE	A.M. Overcast. P.M. Clear—light clouds.
	Mean 30.016	Mean 51.1	Mean 30.016	Mean 53.4	Mean 45.5	Mean 49.6	Mean 53.0	Mean 44.8	Mean 53.9	Sum 1.548		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. .... 9 A.M. 29.971 3 P.M. 29.965

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge .... =83 feet 2½ in.

.... above the mean level of the Sea (presumed about) .... =95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House .... =79 feet 0 in.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR NOVEMBER, 1829.

1829. Novem.	9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
	Barom.	Attach. Therm.	Barom.	Attach. Therm.		Fahrenheit.		Self-registering.				
						9 A.M.	3 P.M.	Lowest.	Highest.			
☉ 1	30.157	43.8	30.171	44.5	27	35.7	41.8	31.7	41.8	0.025	NW	Cloudless—hazy.
☽ 2	30.196	41.7	30.212	46.8	36	39.3	47.2	32.8	47.2		WSW	Fine—hazy.
♂ 3	30.209	45.9	30.132	47.6	38	43.6	47.2	38.3	47.3		WSW	Lightly cloudy—hazy.
☿ 4	29.803	48.7	29.647	51.0	45	48.2	50.8	43.0	51.3		S	Overcast and foggy. Rain P.M.
♃ 5	29.967	48.8	30.267	51.5	43	43.2	50.4	39.7	50.4	0.131	W	Nearly cloudless.
♀ 6	29.978	50.0	29.877	53.6	45	48.3	52.0	42.3	52.7		WSW	Lightly cloudy—hazy.
♄ 7	29.908	49.3	29.884	51.2	42	45.2	48.8	42.8	48.8	0.011	WNW	Nearly cloudless—hazy.
☉ 8	30.017	42.5	30.015	49.6	38	38.5	47.0	34.6	46.6		WSW	Fine—lightly cloudy—light wind.
☽ 9	30.046	47.7	30.081	49.6	43	43.8	48.4	37.4	48.4		W	Fine—lightly cloudy—light wind.
♂ 10	30.007	48.4	29.866	51.2	43	43.3	49.5	43.3	52.4		SSW	Lightly cloudy.
○ ☿ 11	29.964	48.6	30.018	51.4	42	42.4	48.8	40.3	52.2		W	Strong fog.
♃ 12	29.884	51.6	29.839	55.6	52	52.7	56.4	41.2	56.4	0.278	W	Overcast—foggy.
♀ 13	30.052	54.6	30.160	52.4	50	50.4	47.0	49.8	50.4	0.006	E	Overcast—foggy.
♄ 14	30.185	51.4	30.117	51.8	43	44.1	46.8	41.4	47.8	0.006	E	Overcast—foggy.
☉ 15	29.890	51.8	29.822	53.6	48	48.5	51.8	42.7	51.8		WSW	Lightly cloudy. Light rain.
☽ 16	30.169	46.3	30.247	48.0	31	38.4	41.2	35.4	41.4	0.011	N	Clear and cloudless—light wind.
♂ 17	30.377	42.6	30.314	45.8	34	34.8	41.4	31.3	41.4		NNE	A.M. Cloudless. P.M. Light clouds.
☿ 18	30.387	43.7	30.388	46.2	38	38.3	41.8	33.8	41.8		NNE	Light clouds and haze.
♃ 19	30.428	39.7	30.364	40.5	32	32.3	35.5	30.3	35.5		NNE	Strong fog.
♀ 20	30.352	39.2	30.300	40.2	30	30.3	36.4	27.5	36.4		NW	Very dense fog. Hoar frost A.M.
♄ 21	30.286	37.9	30.204	39.8	32	32.4	38.6	26.8	38.6		NW	A.M. Strong fog. P.M. Fine.
☉ 22	29.662	39.2	29.477	41.8	38	38.3	42.0	31.3	42.0		SSW	Cloudy—light rain.
☽ 23	29.603	40.7	29.103	42.5	38	40.3	40.8	36.2	41.7	0.064	NNE	Light clouds. Rain P.M.
♂ 24	29.662	40.6	29.016	40.6	33	39.2	38.0	36.4	38.8	0.019	NE	Overcast—light brisk wind.
☿ 25	29.526	37.2	30.109	37.2	32	31.9	33.0	29.7	36.3		NE	Snow.
● ♃ 26	29.858	38.6	29.796	40.5	36	37.8	39.2	30.3	39.2	0.128	E	A.M. Overcast. P.M. Fine.
♀ 27	29.779	41.9	29.720	42.2	38	38.4	40.4	36.2	40.4	0.014	N	A.M. Strong fog. P.M. Light deposition.
♄ 28	29.751	42.3	29.237	43.7	40	40.0	42.5	36.6	42.5		NNE	Foggy.
☉ 29	29.860	44.0	29.843	45.3	41	41.7	43.7	39.3	43.7		NNE	Foggy.
☽ 30	29.858	43.7	29.815	44.2	39	40.7	40.0	38.9	40.7		ESE	Foggy—light wind.
	Mean 29.994	Mean 44.7	Mean 29·935	Mean 46.7	Mean 38.9	Mean 40.7	Mean 44.3	Mean 36.7	Mean 44.9	Sum 0.693		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. .... { 9 A.M. 29.966 3 P.M. 29.902 }

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge .... =83 feet 2¼ in.

.... above the mean level of the Sea (presumed about) =95 feet.

The external Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House .... =79 feet 0 in.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR DECEMBER, 1829.

1829. Decem.	9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
	Barom.	Attach. Therm.	Barom.	Attach. Therm.		Fahrenheit. 9 A.M.	Fahrenheit. 3 P.M.	Self-registering. Lowest.	Self-registering. Highest.			
♂ 1	29.817	43.3	29.843	40.4	40	40.1	39.8	38.9	40.7		ESE	Light clouds and wind.
☿ 2	29.753	42.7	29.714	44.2	40	40.6	43.4	38.3	43.4		E	Foggy.
♃ 3	29.799	43.8	29.995	44.5	41	41.5	42.8	39.8	42.8		ESE	Foggy—light wind.
♀ 4	29.801	44.7	29.842	46.0	44	43.6	44.6	40.4	44.6		ESE	Light fog.
♄ 5	30.272	46.6	30.349	48.2	45	45.5	48.5	42.7	48.5		ESE	Foggy.
☉ 6	30.522	47.3	30.490	46.3	42	42.0	39.7	41.3	42.0		ENE	Overcast—light wind.
☽ 7	30.374	39.7	30.273	40.1	29	32.2	35.2	28.4	35.4		NE	Foggy. Hoar frost A.M.
♂ 8	30.313	39.8	30.341	39.5	29	35.0	35.4	30.9	35.4		WSW	Overcast—light wind.
☿ 9	30.379	39.3	30.291	38.2	29	33.4	36.0	31.4	36.0		NNE	A.M. Foggy. P.M. Fine.
○ ♃ 10	30.139	38.2	30.100	38.6	33	35.7	36.2	32.3	36.2		ENE	Overcast.
♀ 11	30.163	40.0	30.151	41.0	31	37.6	39.2	31.7	39.2		SSE	A.M. Overcast. P.M. Fine.
♄ 12	30.180	40.6	30.172	42.0	35	38.7	41.2	35.0	42.6		SSW	Light clouds and wind.
☉ 13	30.230	43.2	30.213	46.4	39	39.7	46.0	37.7	46.0		S	Clear and cloudless—light wind.
☽ 14	30.304	43.3	30.306	43.8	37	37.0	39.6	35.2	39.6		N	Strong fog.
♂ 15	30.341	40.0	30.306	40.8	33	33.7	39.6	31.6	39.6		N	Fog.
☿ 16	30.268	41.3	30.191	41.2	37	37.0	39.5	32.7	39.5		N	Lightly cloudy. Evening, drizzling rain.
♃ 17	29.985	39.6	29.884	40.6	35	35.3	37.5	34.4	37.5	0.017	NNE	Overcast.
♀ 18	29.628	40.3	29.672	41.4	35	36.0	38.0	31.8	38.3		NE	Cloudy and foggy. A.M. light snow.
♄ 19	29.868	39.3	29.828	40.4	34	35.6	36.5	32.8	37.0		N	Light clouds and wind.
☉ 20	29.852	37.6	29.877	37.2	26	32.3	33.1	30.7	33.1		N	Overcast—light wind. A.M. Fall of snow, early.
☽ 21	29.918	34.7	29.830	36.0	25	28.3	33.0	26.3	33.0		WSW	Foggy. Snow P.M.
♂ 22	29.721	34.7	29.716	36.8	30	32.3	34.8	27.0	34.8	0.033	E	Overcast.
☿ 23	29.747	34.6	29.678	32.5	27	31.7	29.0	29.7	31.7	0.044	NNE	Overcast—brisk wind.
♃ 24	29.714	30.8	29.728	31.8	25	27.7	28.9	24.6	30.3	0.008	N	Overcast. Snow A.M.
♀ 25	30.026	32.9	30.067	33.8	25	29.4	30.6	26.5	30.6		NE	Light clouds and wind.
● ♄ 26	30.307	30.5	30.330	30.5	22	27.2	28.0	25.6	28.0		N	Fine—light clouds.
☉ 27	30.403	29.2	30.380	30.8	24	25.8	28.5	25.5	28.5		NE	A.M. Fine. P.M. Overcast.
☽ 28	30.329	26.5	30.230	28.6	20	20.5	29.8	18.2	29.8		NW	Overcast.
♂ 29	30.302	28.0	30.313	28.2	21	24.5	26.5	20.2	27.2		NE	Overcast—light fog.
☿ 30	30.413	28.8	30.419	30.2	20	27.5	30.0	23.8	30.0		S	Overcast—light wind.
♃ 31	30.500	30.5	30.539	32.4	25	31.0	31.8	27.5	31.8		E	A.M. Overcast. P.M. Fine.
	Mean 30.109	Mean 37.8	Mean 30.099	Mean 38.5	Mean 31.5	Mean 34.1	Mean 36.2	Mean 31.4	Mean 36.6	Sum 0.102		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. .... { 9 A.M. 30.098 | 3 P.M. 30.086 }

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge........................ =83 feet 2¼ in.

....above the mean level of the Sea (presumed about) .... =95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House.......................... =79 feet 0 in.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXXX.

PART II.

LONDON:

PRINTED BY RICHARD TAYLOR, RED LION COURT, FLEET STREET.

MDCCCXXX.

ADVERTISEMENT.

SINCE the appearance of the Papers relating to the Paramatta Observations, which were published at the request and at the expense of the Colonial Office, and appended to the Philosophical Transactions, it has appeared that some of the Observations communicated by Mr. RUMKER were made by that gentleman while he was the paid assistant of Sir THOMAS BRISBANE, at an Observatory founded by Sir THOMAS BRISBANE, and with his instruments; and that some others were actually made by Sir THOMAS BRISBANE himself.

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Meteorological Journal, kept at the Apartments of the Royal Society, by order of the President and Council.

ERRATA in Part I.

Page 74, *dele* the last paragraph but one.
—— 142, in the second formula, line 4th, *for* φ, *read* 1.

Part II.

Page 260, last line, *for* fire *read* ice.
—— 282, line 8, *for* heated *read* luted.

A List of Public Institutions and Individuals, entitled to receive a copy of the Philosophical Transactions of each year, on making application for the same directly or through their respective agents, within five years of the date of publication.

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The Royal Observatory at Altona.

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The Royal Academy of Sciences at Thoulouse.
The "Ecole des Mines" at Paris.
The Geographical Society at Paris.

Germany.

The University at Göttingen.
The Cæsarean Academy of Naturalists at Bonn.

Italy.

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The Royal Academy of Sciences at Turin.

The Netherlands.

The Imperial Academy of Sciences at Brussels.

Spain.

The Royal Observatory at Cadiz.

Portugal.

The Royal Academy of Sciences at Lisbon.

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The Imperial Academy of Sciences at St. Petersburgh.

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The Royal Academy of Sciences at Stockholm.

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The New York Philosophical Society.
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The University of St. Andrews.
The University of Dublin.
The University of Edinburgh.
The University of Glasgow.
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The Royal Institution.
The President of the Royal Society.
The Lowndes' and Plumian Professors of Astronomy, Cambridge.
The Lord Bishop of Cloyne.
Francis Baily, Esq.
Thomas Henderson, Esq.
Captain W. H. Smyth, R.N.
Sir James South.
Edward Troughton, Esq.

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The Royal Academy of Sciences at Paris.
The Imperial Academy of Sciences at St. Petersburgh.
The Royal Academy of Sciences at Stockholm.
The Royal Academy of Sciences at Upsal.
The Board of Longitude at Paris.
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The Observatory at Wilna.
Professor Bessel.
Dr. William Olbers.

PHILOSOPHICAL TRANSACTIONS.

XVII. *Memoir on the occurrence of Iodine and Bromine in certain Mineral Waters of South Britain.* By CHARLES DAUBENY, *M.D. F.R.S. Professor of Chemistry in the University of Oxford.*

Read May 6, 1830.

THE discovery in sea-water of iodine and bromine, two principles which, although in minute proportions, are said to be generally diffused throughout the present ocean, naturally suggested the inquiry, as to whether these same ingredients might not be found to exist in springs occurring in inland situations when containing a similar saline impregnation. This accordingly has been already determined by STROMEYER, LIEBIG, and others, to be the case in many of the brine-springs of Germany, France, and Italy; but at the time my attention was first directed to the subject, I was unacquainted with any trials of the kind having been instituted with reference to those of this country, except by Professor TURNER of the London University, regarding the presence of iodine in the mineral waters of Scotland; in only one of which, that of Bonnington near Leith, he appears to have detected it. I was therefore induced in the course of last spring and summer to undertake a pretty extensive survey of such English springs as are known to contain a considerable proportion of common salt; and having succeeded in detecting in several of them traces of one or both the substances alluded to, I inserted a brief account of the results obtained, in the Philosophical Magazine and Annals of Philosophy for September last.

An article that has appeared in a subsequent number of the same periodical work has, however, been the means of drawing my attention to a little work by Mr. Murray, entitled "Experiments on Chemical Philosophy," which had not before fallen in my way; and from this it is clear, that the detection of iodine in the Gloucester Spa water had been made by that gentleman some time before I had engaged in the inquiry. I am unable, however, to discover in his publication, although it bears so late a date as 1828, any thing that can substantiate the assertion which its author has made in the number of the Philosophical Magazine and Annals referred to, as to his having anticipated me in the discovery of iodine in the springs at Cheltenham*, or in that of bromine in those of Ingestrie. I consider myself, therefore, still warranted in claiming as my own the first public announcement of the existence of bromine in our English springs; but I am far from attaching importance to a discovery which had been previously made in so many similar situations abroad, and would wish it to be understood, that my only pretence for offering to the Royal Society the present communication, is the circumstance of my having examined on the spot most of the mineral springs hereafter enumerated, and having undertaken, wherever it appeared practicable, to obtain an approximation at least to the proportion which these principles bore to the other ingredients present, and to estimate their comparative frequency and abundance in the several rock-formations.

To the geologist, the results of such an inquiry may be of interest, as tending to identify the products of the ancient seas in their most minute particulars with those of the present ocean: and to the physician it may be an object of curiosity, to speculate how far the unexplained virtues attributed to certain mineral waters depend on the presence of these ingredients, the energy of whose action may perhaps compensate for the minute quantity in which they are found. I confess, indeed, that with regard to the former of them, Iodine, we ought to be sceptical as to any medicinal agency that can be exerted by so small a quantity as a single grain diffused through ten gallons of water, the largest proportion in which I have ever detected it. But with respect to the second, Bromine, after considering the statements of its discoverer M. Balard, as to its highly poisonous operation upon animals, which my own experience of the irritating effects of its vapour tends fully to confirm, I cannot view it as absurd to trace the medical virtues ascribed to such waters as those of Ashby-de-la-Zouch, to the presence of even so small a quantity as a grain of hydrobromate of magnesia (if such be the combination) in each pint of the water; and that the proportion would not fall far short of that, my experiments on this particular spring seem to warrant me in concluding. It is curious at least, that almost the only two brine-springs, properly so called, which have acquired any reputation as medicinal agents, that of Kreutznach in the Palatinate, and that of Ashby-de-la-Zouch in Leicestershire, both should contain a larger proportion than common of this new principle; and that in either instance that reputation should have been enjoyed, long before any suspicion as to their peculiar nature could have been entertained.

---

* Mr. Ainsworth, however, one of the editors of the Edinburgh Journal of Natural and Geographical Science, states, that he had communicated the fact of the existence of iodine in the Cheltenham waters previously to my announcement of it.

The objects I had in view in this inquiry being what are above stated, I have chosen to classify the springs noticed in the accompanying Table according to the geological position of the strata from which they issue; and under the head of each have set down the total amount of their saline ingredients; the nature and proportion of them as ascertained by former chemists, or, whenever I could not depend upon the results, by myself; and the proportion which the iodine and bromine, where either of these principles existed, bore to the quantity of water, and likewise to that of the chlorine which the solid ingredients of the spring might contain. The latter statement has been introduced in order to remove an impression which may have been created in consequence of the detection of iodine, as it is said, even in common pump-water*, when very large quantities of it were evaporated; from which circumstance it might be inferred, that this principle is not only a constant accompaniment of common salt, but that its quantity bears a pretty regular ratio to that of the latter ingredient. Although I have myself evaporated no less than forty-eight gallons of the Oxford pump-water without finding the slightest trace of iodine in the last portions, I shall not dispute the truth of the former position, which might possibly have been borne out, had still larger quantities been operated upon†; but that the latter opinion is untenable, will be readily seen from the accompanying Table, which shows that the proportion of iodine to chlorine varies in every possible degree, and that the springs most strongly impregnated with common salt are in some instances those in which I have evaporated the largest quantity without detecting any trace of iodine. The same remark will equally apply to bromine; so that the general inference seems to be, that although these two principles may perhaps be never entirely absent where the muriates occur, yet that their distribution is certainly very unequal, and therefore forms a proper subject of scientific research. The quantity in which the former of these ingredients occurs in mineral waters is commonly so inconsiderable, that I have been unable to determine it by direct analysis, and have been therefore obliged to content myself with obtaining an approximation to its real amount. In the case of one of the Leamington springs, indeed, I employed the agency of nitrate of silver to precipitate the iodine from the concentrated water, and afterwards separated by means of ammonia the chloride from the iodide of silver obtained. I have reason, however, to believe, from some comparative experiments, that where the proportion which this latter ingredient bears to the former one is extremely small, it may be taken up either wholly or in part by ammonia; and I therefore contented myself in other instances with evaporating the water until it began to produce the characteristic blue or violet tinge with starch and sulphuric acid. This was then compared with the colour imparted by the same test to a solution of hydriodate of potass of known strength; and the latter, if not of the same shade already, was brought to it by dilution with a measured quantity of water. Having thus noted the proportion of iodine in the test liquor with which the concentrated solution corresponded, it was easy to calculate what it must have been in the mineral water itself, by knowing the number of times its original quantity had been reduced by evaporation previously to the employment of this re-agent.

---

* Mr. Cuff, a chemist at Bath, has also detected it in the hot springs of that place, by evaporating about thirty gallons of the water.

† I am also loth to question the fact (stated on good authority) of the existence of a minute pro-

The sulphates and muriates present in brine-springs do not appear to interfere with the delicacy of this test; but where bromine was also present, I have

portion of iodine in sea-water, although I have reduced ten gallons of it, taken from the English Channel near Cowes, to less than half an ounce, without being able to detect any in the residuum. There seems reason, however, to infer, from what is stated in the next page, that the starch test cannot be relied upon to detect very minute quantities of iodine, when a comparatively large proportion of bromine is present in the same solution.

seen the liquor, either at the time or shortly after the operation of the re-agent, assume a pinkish hue, owing, as I suppose, to the reddish tinge of the bromine given out mixing with the blue colour of the iodide of starch. In stating therefore, as I have done in the Table, the proportions of iodine in several of the waters, I am far from pretending to offer more than an approximation to the relative quantity in which it occurs, and am fully aware of the necessity of more precise experiments, conducted on a different principle, before the points in question can be considered as satisfactorily determined.

The starch test I find will readily indicate a quantity of iodine not exceeding one grain to 7 gallons of water, or one 450,000dth part; but as in no case that has occurred to me the proportion exceeded one grain to 10 or 12 gallons, and in many appeared scarcely to amount to one-10th of that quantity, I despaired of arriving at more accurate results, by adopting any other method that aspired to greater precision than the one already stated.

In every case in which I have noted that no iodine could be detected, the water had been concentrated at least as far as to one-30th of its original quantity without effect; so that the proportion of this principle, supposing after all any of it to exist, could not well amount to a grain in 200 gallons. In some cases, indeed, where the spring was one of weak impregnation, I have carried the concentration much further, as may be seen in the Llandrindod waters, where no traces of iodine appeared, until they had been reduced to nearly one-50th of their original volume.

In my trials for bromine, I have in great measure conformed to the directions of Balard; first boiling down the water to about a fourth of its original quantity with a portion of quicklime to prevent the bromine from being dissipated by the heat; and then, after filtering the residuum, introducing chlorine as long as any sensible yellowness was caused by its addition. The water was then strongly agitated with ether, which collects on the surface, carrying with it the bromine with which it had combined, and was then poured off into a separate vessel. [The bromine, immediately upon being thus removed from the water, was treated with a quantity of a concentrated solution of pure soda sufficient to render the ether containing it colourless; the latter alkali being employed for this purpose in preference to the vegetable one, as I found that bromine formed with sodium a salt more soluble in alcohol than it did with potassium.]

Unfortunately, however, the salts which are contained in or deposited from the ethereal solution after the addition of the soda, appear to be of a very mixed description, consisting not only of the hydrobromate and bromate of soda, but also of the muriate and chlorate, together with a little uncombined alkali, if the proportion of the sodium to the bromine is not very nicely adjusted. I therefore began by heating the whole product sufficiently to convert the bromate of soda into the bromide, and the chlorate into the chloride, of sodium; and afterwards, in order to ensure the union of any alkali which may have been in excess with carbonic acid, I dissolved the whole in water impregnated with that gas. The solution was then brought to dryness, and strong alcohol added to separate the bromide of sodium as much as possible from the other ingredients; after which, the alcoholic solution, having been evaporated, was re-dissolved in water, and nitrate of silver added to it in order to form the insoluble bromide of silver, the weight of which, when dried and melted, would determine that of the bromine present, every 100 grains according to M. BALARD indicating 41.1 of this principle.

From the weight of the precipitate, however, I felt myself obliged to make a large deduction, in proportion to the quantity of alcohol employed, for the chloride of sodium at the same time taken up; having ascertained by a previous experiment how much common salt a given quantity of this menstruum could dissolve. The latter part of the process, however, being liable to some uncertainty, I should have preferred, had my engagements permitted, re-examining the waters on the spot, and operating on such quantities of them as would have enabled me to extract appreciable quantities of bromine. This indeed I have done in the case of the Middlewich water, but not with sufficient attention to the quantities employed and obtained, to enable me to calculate in this manner the exact proportions between them: with regard to the other springs, the quantity of water which I could conveniently transport to my laboratory was not such as to enable me to pursue with much hope of success this particular method. It is therefore with diffidence that I offer provisionally the statements given in the Table, as an approximation to the relative quantities of bromine existing in some of our English springs, calculated according to the scheme of analysis above stated; and shall hope at some future and not very distant period to obtain results more worthy of reliance, should my further labours on this subject not be rendered in the mean time unnecessary by the investigations of some other chemist. In cases where the quantity of this principle appeared to be less considerable, as in the Leamington, Cheltenham, and Gloucester waters, I have contented myself with guessing at its proportion by concentrating the water until it assumed a decidedly yellow tinge with chlorine, noting what proportion of bromine in water produced a colour of equal intensity.

The earliest of the rock-formations in this country that come under our consideration with reference to the present inquiry, is the greywacke slate of North Wales, which in the neighbourhood of Bualt in Radnorshire gives out springs containing a notable proportion of common salt. Those of Llandrindod have long enjoyed some reputation as medicinal agents, but their composition does not appear to have been correctly ascertained; for the most modern analysis I have seen* assigns to them a considerable proportion of muriate of magnesia, of which I find scarcely a trace. The more newly discovered springs at Bualt itself, though less celebrated, are similar in point of constitution; and being double the strength of those of Llandrindod, ought to possess superior medical virtues. At both places are waters which differ from the rest in containing an unimportant impregnation of sulphuretted hydrogen, but in other respects correspond.

Many of our coal-pits emit streams of salt water; but the most remarkable spring of the kind is that already noticed, of Ashby-de-la-Zouch in Leicestershire, which for the last few years, especially since the erection of the baths, which are now so great an ornament to the spot, has acquired a certain local reputation in the cure of diseases. Previously to the discovery of bromine, of which I detected in this water an appreciable quantity, Dr. THOMSON of Glasgow had examined its composition; and I have therefore been satisfied with adhering to the results of his analysis, which is stated in the Table.

The most important, however, of the salt-springs that we meet with in this country are those in the new red sandstone formation of Cheshire; for an analysis of which I may refer to a paper of Dr. HENRY's, published in the Transactions of this Society. In this instance, also, I have adopted the statements of another, merely making a proportionate deduction from the amount of the ingredients given by that able chemist, in consequence of the weaker impregnation of the samples of water I employed, than of those on which he appears to have operated.

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* Analysis of the Llandrindod Waters; by Mr. WILLIAMS, Surgeon, 1819.

It will be seen by reference to the Table, that all the brine-springs of that district contain bromine, and most of them iodine; indeed it is probable that if I had had time to concentrate larger quantities of the water, the latter would have been detected throughout. It may be remarked, however, that the rock-salt of Northwich in Cheshire contains no trace of either principle; a circumstance explicable from the more deliquescent nature of the hydriodic and hydrobromic salts, which would cause them, together with the earthy muriates, to remain in the mother-liquor after the common salt had crystallized, and thus to become distributed through the substance of the marly beds afterwards formed over the rock-salt, from which the brine-springs appear to derive their saline impregnation.

There is a blue variety of rock-salt met with at Ischel near Saltzburg, which, from the resemblance between its colouring matter and that of the compound of starch and iodine, might be suspected to contain this latter principle united with some kind of vegetable matter. I have been unable to obtain a specimen deeply enough tinged with the colouring matter alluded to, to set the question completely at rest; but on dissolving a portion of the blue salt, which I obtained through the kindness of Mr. Heuland, in water, not the slightest tinge appeared to be communicated to the solution, neither did any blueness appear on re-crystallizing the salt. The specimen alluded to gave no indications of iodine when tested with starch in the usual manner, and was nearly pure from admixture with foreign ingredients, although it appeared to contain a trace of sulphuric acid and of lime. At present, therefore, I am inclined to attribute the colour rather to some peculiar arrangement of the particles of the common salt itself, than to the presence of any other ingredient.

The springs containing purgative salts, which arise from the lias clay in various places along its whole range from Leamington to Gloucester, appear to be derived from the same source as the brine-springs of Cheshire and Worcestershire above alluded to; but their saline contents have been modified by the sulphuric acid generated by the decomposition of the sulphuret of iron present in the stratum from which they immediately proceed. Hence the proportion of earthy muriates is usually greater in them than in the brine-springs properly so called; because the muriatic acid disengaged by the action of the sulphuric acid upon the common salt has dissolved a fresh quantity of lime or magnesia from the surrounding materials of the rock.

If such be the origin of the sulphates of soda and magnesia which impart to these waters their aperient quality, it would be natural to expect that they should be found in greater abundance on the first discovery of the spring than after it has been long drawn upon; and hence, perhaps, the remarkable discrepancy between the results of my examination of the Gloucester and some other waters, and those given on the authority of former chemists, may be explained, without impeaching the accuracy of either.

It will be seen by reference to the Table, that I have represented the ingredients of the Leamington waters on the authority of Dr. THOMSON, as stated by Dr. LOUDON in his Practical Dissertation* on these springs; and those of Cheltenham, with the exception of one lately discovered at Pittville, on that of Dr. SCUDAMORE. The springs of Tewkesbury and Gloucester I have myself examined; there being of the former no analysis at all, and of the latter only one by Mr. ACCUM, which I had reason to believe, what I in fact found, quite inapplicable to its present composition. The spring which goes by the name of the Chalybeate Saline is at present destitute of iron, which I am assured it formerly possessed, whilst the Sulphureous contains no trace of sulphuretted hydrogen. These two springs, which at present appear almost identical, are the ones most strongly impregnated with purgative salts, and therefore approximate more nearly to the character of that analysed by Mr. ACCUM, according to his representation, than either of those termed "the pure saline," which he professed to have examined. Many of these springs it will be seen contain traces of bromine and iodine; but they seem to be less common in the aperient waters which are occasionally met with in the chalk and tertiary districts of this country; for I have examined three—those, namely, of Epsom, of Chad's Well in Gray's-Inn Lane, and of St. Leonard's Hill near Windsor, without discovering traces of iodine in any one. In the Epsom water alone a slight trace of bromine was perceptible.

With regard to the state of combination in which these principles occur, I have only to observe, that they are no doubt combined with hydrogen, forming the hydriodic and hydrobromic acids, and neutralized in all probability by magnesia, both forming with this basis salts decomposable at a low temperature, which seems to be the case with the compounds of both bromine and iodine existing in the waters I have examined. Even long continued boiling, there is reason to believe, diminishes the quantity of bromine originally present; and hence it seems advisable, when the object is to estimate the whole of this principle which a mineral water may contain, to combine the hydrobromic acid with lime, in the manner which I have recommended to be done when speaking of the mode of separating bromine from its combination.

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* Leamington, 1828.

I may conclude by observing, that there is little question as to the possibility of procuring a sufficient supply of bromine from our English brine-springs, should a demand be created for this new substance, either for medical purposes or for the arts of life; for, from a few rough trials of its comparative abundance in the Middlewich and Ashby springs, and in those of Kreutznach in the Palatinate, which affords, it is said, the principal supply for present consumption, I should regard our own quite as highly charged: neither can it be doubted but that the proportion of bromine present in many brine-springs exceeds considerably that contained in the present ocean, which, from experiments recently made by myself on water taken from the English Channel a short distance from Cowes, I have stated in the Table as existing in the proportion of one grain to the gallon.

Table comprehending a List of certain Springs in South Britain, which contain Common Salt in considerable quantity.

TABLE comprehending a List of certain Springs in South Britain, which contain com- the other Ingredients

Geological Position.	Locality of the Spring.	Name of the Spring.	Total of its Saline Contents in the Pint.	Iodine. Its proportion to	
				The Water.	The Chlorine.
TRANSITION SLATES.	Llandrindod, Radnorshire.	No. 1. The Pure Saline.	31.35 Gr.	Seems not to exceed 1 Grain to 343 Gallons.	As 1 to 50.000
	Ditto	No. 2. The Chalybeate Saline.	Nearly as No. 1.	None detected.	———
	Bualt, Radnorshire.	No. 1. The Saline.	77.6 Gr.	A trace, nearly the same as that exhibited by No. 1. Llandrindod.	As 1 to 120.000
	Ditto	No. 2. Sulphuretted Saline.	Nearly as No. 1.	None detected.	———
	Ditto	No. 3. The Chalybeate Saline.	Nearly as No. 1.	None detected.	———
COAL FORMATION.	Ashby-de-la-Zouch, Leicestershire.	The Moira Brine Spring.	179.88 Gr.	None detected.	———
	Near Newcastle-upon-Tyne.	The Walker Colliery Brine Spring.	192.0 Gr.	None detected.	———
	Kingswood, near Bristol.	The Soundwell Colliery Brine Spring.	64.0 Gr.	None detected.	———
	Northwich, Cheshire.	Brine Spring.	1696 Gr.	None detected.	———
	Middlewich, Cheshire.	Ditto	1824 Gr.	A trace, probably not exceeding 1 Grain to 343 Gallons.	As 1 to 2.650.000
	Nantwich, Cheshire.	Ditto	1760 Gr.	1 Grain to about 12 Gallons.	As 1 to 96.000
	Wheelock, Cheshire.	Ditto	1440 Gr.	A trace, apparently not greater than that in the Middlewich.	Nearly as 1 to 2.000.000
	Droitwich, Worcestershire.	Ditto	1746 Gr.	None detected.	———
	Shirleywich, Staffordshire.	Ditto	1552 Gr.	None detected.	———
LIAS CLAY.	Leamington, Warwickshire.	No. 1. Robbins's Well.	95.948 Gr.	1 Grain to about 10 Gallons.	As 1 to 3440
	Ditto	No. 2. Royal Pump Saline Spring.	134.749	Rather less than in No. 1.	———
	Ditto	No. 3. Smith's Pump.	109.992	A trace, but apparently not more than a Grain to 192 Gallons.	As 1 to about 48.000
	Ditto	No. 4. Wise's Pump.	107.396	Nearly as the preceding.	———

mon Salt in considerable quantity; together with a Statement of the Proportion of this and present in a Pint of each.

Bromine. Its proportion to		Chlorides of			Sulphates of			Per-oxide of Iron.	Carbonates or other Salts.	Authority on which the Statement of the Saline Ingredients is given.
The Water.	The Chlorine.	Calcium.	Magnesium.	Sodium.	Lime.	Magnesia.	Soda.			
Not estimated, but a distinct trace.	Not estimated.	6.6	24.75							Daubeny.
As No. 1.	———	Ingredients as No. 1. except that it contains						0.6		Daubeny.
A trace, not estimated.	———	11.2	A trace	66.4						Daubeny.
A trace, not estimated.	———	Ingredients nearly as No. 1.								Daubeny.
A trace, not estimated.	———	Ingredients as No. 1. except that it contains						A trace		Daubeny.
1 Gallon seems to contain 4.68 Grains.	As 1 to 180	36.4	3.72	133.0	4.24		2.52			Thomson.
A trace, not estimated.	———	2.8	2.8	186.0						Daubeny.
None detected.	———	2.5		58.5			3.0			Daubeny.
1 Gallon seems to contain 1.2 Grains.	As 1 to 6600	0.42	1.27	1667.0	25.5				Insoluble matter 1.696	Henry.
9.36 Grains of Brome in 1 Gallon.	As 1 to 860	0.5	1.37	1793	27.35				Insoluble matter 1.3	Henry.
6.32 Grains of Brome in 1 Gallon.	As 1 to 1275	0.45	1.32	1730	26.5				Insoluble matter 1.76	Henry.
A trace, not estimated.	———	0.36	1.1	1415	22.0				Insoluble matter 1.44	Henry.
None detected.	———		A trace	1691			40.25			Daubeny.
4.32 Grains of Brome to 1 Gallon.	As 1 to 1720	38.0	22.0	1490						Daubeny.
1 Grain to about 10 Quarts.	As 1 to 430	23.5	8.468	35.35			28.619	A trace		Thomson.
Nearly as strong as No. 1.	———	20.902	12.365	67.78			32.744	0.956		Thomson.
A trace.	———	19.772	2.121	47.865			40.234	A trace		Thomson.
A trace.	———	18.777	22.592	26.61			39.457	A trace		Thomson.

Geological Position.	Locality of the Spring.	Name of the Spring.	Total of its Saline Contents in the Pint.	Iodine. Its proportion to	
				The Water.	The Chlorine.
LIAS CLAY.	Leamington, Warwickshire.	No. 5. Smart's Saline.	92.589	Nearly as the two preceding.	—
	Ditto	No. 6. Lord Aylesford's.	113.995	None detected.	—
	Ditto	No. 7. Reid's Sulphureous.	79.142	None detected.	—
	Ditto*	No. 8. Reid's Saline.	102.597	None detected.	—
	Gloucester.	No. 1. Sulphureous Saline.	84.2	About 1 Grain to 50 Gallons.	As 1 to 12.000
	Ditto	No. 2. Chalybeate Saline.	In all respects agrees with No. 1 in point of com		
	Ditto	No. 3. Strong Saline.	76.5	About 1 Grain to 96 Gallons.	As 1 to 33.000
	Ditto	No. 4. Weak Saline.	75.22	Nearly as No. 3.	—
	Tewksbury.	The Walton Spring.	46.1	About 1 Grain to 36 Gallons.	As 1 to 6690
	Cheltenham.	Pittville, No. 1. "The Pure Saline."	45.8	None detected.	—
	Ditto	Sherborne, No. 4.	84.44	About 1 Grain to 90 Gallons.	As 1 to 33.000
	Ditto	Thomson's, No. 4.	80.13	About 1 Grain to 30 Gallons.	As 1 to 3600
	Ditto	Old Well, No. 1.	81.51	About 1 Grain to 60 Gallons.	As 1 to 19.000
	Ditto*	Thomson's, No. 2.	52.29	None detected.	—
OOLITIC STRATA.	Melksham, Wiltshire.	The Saline Spring.	107.42	None detected.	—
CHALK FORMATION.	Epsom, Surry.	The Saline Spring.	33.2	None detected.	—
TERTIARY ROCKS.	Windsor.	St. Leonard's Hill Spring.		None detected.	—
	Gray's Inn Lane, London.	Chad's Well.		None detected.	—
PRESENT OCEAN.	Off Portsmouth.			None detected.	—

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* In none of the remaining Cheltenham or Leamington Springs could I satisfy myself of the existence of either iodine or bromine.

Bromine. Its Proportion to		Chlorides of			Sulphates of			Per-oxide of Iron.	Carbonates or other Salts.	Authority on which the Statement of the Saline Ingredients is given.
The Water.	The Chlorine.	Calcium.	Magnesium.	Sodium.	Lime.	Magnesia.	Soda.			
A trace.	——	17.570	26.05	14.534			34.435	A trace		THOMSON.
A trace.	——	20.561	3.266	40.77			40.398	A trace		THOMSON.
A trace.	——	15.777	9.695	25.60			28.065	A trace		THOMSON.
A trace.	——	17.987	10.813	42.92			30.61	0.265		THOMSON.
About 1 Grain to 10 Quarts.	As 1 to 600			50.41	1.2		10.35		Carbonate of Lime 0.2	DAUBENY.
position, and very nearly in the proportion of its ingredients.										DAUBENY.
Rather less than in No. 1.	As 1 to 860			71.5	2.0		1.6			DAUBENY.
Nearly as No. 3.	——			69.2	2.38		1.15			DAUBENY.
None detected.	——	0.3	1.8	37.5			5.6		Carbonate of Lime 1.0	DAUBENY.
About 1 Grain to 6 Gallons.	As 1 to 768		A trace	27.16			17.55		Carbonate of Lime 0.2	DAUBENY.
None detected.	——	4.29	0.59	72.8			6.76			SCUDAMORE.
None detected.	——	3.07	2.02	46.4			28.64			SCUDAMORE.
None detected.	——	6.21	2.54	58.2			14.56			SCUDAMORE.
A trace, about as much as in the Spring at Pittville.	——	3.31	1.52	25.7			21.76			SCUDAMORE.
A trace.	——	12.0	0.42	90.0						DAUBENY.
A trace.	——			6.0	18.3		3.9		Carbonate of Lime 5.0	DAUBENY.
None detected.	——									
None detected.	——									
1 Grain to 1 Gallon.	As 1 to 840									

XVIII.—*Experiments to determine the difference in the Number of Vibrations made by an Invariable Pendulum in the Royal Observatories of Greenwich and Altona. By Captain* Edward Sabine *of the Royal Artillery, Secretary to the Royal Society.*

Read March 25th, 1830.

THE invariable pendulum No. 12, with which these experiments were made, was vibrated in the Royal Observatory at Greenwich in July 1828; in the Royal Observatory at Altona in September and October of the same year; and again at the Royal Observatory at Greenwich in August 1829. The mean of the results obtained at Greenwich, in July 1828 and in August 1829, give the rate of this pendulum at Greenwich, to be compared with its rate obtained at Altona.

The experiments in July 1828 have been already printed in the Phil. Trans. for 1829, Part I. pp. 100-102; the result was as follows:

Therm.	Barom.	Vibrations in Vacuo at 61°.5.
61.50 . . .	29.446 . . .	85970.00

In the experiments in August 1829 (Table A), the same barometer, thermometer, planes, and fixed support were used as in the preceding experiments. The rate of Graham's clock employed in observing coincidences was supplied by Mr. Thomas Glanville Taylor by daily comparison with the Greenwich transit clock. The following is an abstract of the results in Table A.

Greenwich, August 1829; Experiments with Pendulum 12.

	Therm.	Barom.		Vibrations.	Corrected to 63.53.
Aug. 8	63.87	30.102	62.5	85959.22	85959.37
8	65.53	30.072	64.5	85958.41	85959.29
9	62.50	29.988	62.0	85959.76	85959.31
9	64.45	29.951	65.0	85959.24	85959.64
9	64.95	29.908	66.5	85959.04	85959.66
10	62.90	29.806	62.0	85959.63	85959.35
10	63.55	29.829	63.5	85959.29	85959.30
11	61.50	30.020	60.0	85960.27	85959.38
11	62.50	30.047	62.0	85959.79	85959.34
	63.53	29.970	63.0		85959.404
		— .090 red^{n}. to 32°.		Red^{n}. for 29.9 in. of air at 63°.5	+ 9.926
		+ .019 capillary.			
		29.900 at 32°.		Mean	85969.33

The two results then at Greenwich are as follows:

		Vibrations.		Vibrations.
July 1828	61.50	85970.00	Corrected to 62.5 =	85969.56
August 1829	63.53	85969.33	Corrected to 62.5 =	85969.78
	62.5		Mean	85969.67

In Altona the pendulum is vibrated in an apartment on the ground floor of M. Schumacher's house, appropriated to pendulum experiments. The door of this apartment is double, and the windows are double sashed, for the purpose of preserving an uniform temperature: a strong mahogany plank is fixed securely to the wall. The agate planes were screwed on it, and the telescope for observing coincidences was stationed in an aperture made for the purpose in the wall opposite to the pendulum, so that the coincidences were observed without entering the room. The clock for observing coincidences, with which M. Schumacher was so kind as to supply me, was Breguet, No. 3405, the admirable going of which during five centuries has recently been published. The rate of this clock during the experiments was furnished by Lieut. NEHUS of the Royal Danish Engineers, assistant to M. SCHUMACHER in the Trigonometrical Survey of Denmark; who for that purpose compared it daily with the transit clock of the Altona Observatory at those hours which were not suitable for giving its rate whilst the pendulum was in vibration. The heights of the barometer were furnished by M. PETERSON, assistant in the Observatory, from M. SCHUMACHER's standard barometer. The height of the pendulum above the level of the sea in M. SCHUMACHER's house was 15.1 toises.

Having an opportunity on this occasion of comparing the thermometer graduated in 1823 by Mr. DANIELL and myself, and employed in the present experiments, as well as in all the former determinations which I have made with invariable pendulums, with a standard thermometer presented by M. BESSEL to M. SCHUMACHER, I was anxious to avail myself of it; and M. SCHUMACHER with great kindness undertook himself to make the comparison. The thermometers were compared by immersing the bulbs to the same level in water warmed very gradually in a chronometer stove by means of a small night lamp, and suffered again to cool; the thermometers being compared both in the ascending and descending temperatures. I copy the following particulars of the comparison from M. SCHUMACHER's memorandum.

"B. . . . . are the readings of M. BESSEL's thermometer.
E. B. . . are the equations of M. BESSEL's thermometer.
R. T. . . are the real temperatures.
S. . . . . are the readings of M. SABINE's thermometer.
E. S. . . are the equations of M. SABINE's thermometer.

1828. Oct. 20.

h	m	S.	B.	E. B.	R. T.	E. S.
1	38 P.M.	59.6	60.0	−0.20	59.80	+0.20
1	43	60.0	60.5	−0.20	60.30	+0.30
2	33	65.0	65.3	−0.23	65.07	+0.07
3	24	70.2	70.6	−0.26	70.34	+0.14
5	0	78.7	81.1	−0.32	80.78	
7	0	86.35	86.7	−0.38	86.32	−0.02
9	25	91.75	92.1	−0.44	91.66	−0.09
11	47	94.60	95.3	−0.47	94.83	+0.23

Oct. 21.

h	m	S.	B.	E. B.	R. T.	E. S.
7	18 A.M.	71.1	71.4	−0.27	71.13	+0.03
8	35	68.9	69.1	−0.25	68.85	−0.05
10	23	66.0	66.3	−0.24	66.06	+0.06
11	5	65.25	65.5	−0.23	65.27	+0.02
1	10 P.M.	63.1	63.4	−0.22	63.18	+0.08
3	47	61.1	61.3	−0.21	61.09	−0.01
4	03	61.0	61.2	−0.21	60.99	−0.01
4	22	60.75	61.0	−0.21	60.79	+0.04

Oct. 22.

h	m	S.	B.	E. B.	R. T.	E. S.
6	38 A.M.	56.15	56.3	−0.18	56.12	−0.03
7	53	55.8	56.1	−0.18	55.92	+0.12
8	20	55.7	56.0	−0.18	55.82	+0.12
10	40	55.5	55.7	−0.18	55.52	+0.02
11	13	55.4	55.6	−0.18	55.42	+0.02
3	55 P.M.	56.25	56.55	−0.18	55.37	+0.12

The freezing point was found quite correct in pounded melting ice."

On examining the particulars of this comparison, it may be fairly inferred that between the limits compared, that is, between the temperatures of 55° and 90°, the indications of the two thermometers agree every where to less than a tenth of a degree; this agreement is the more satisfactory as it includes a part of the scale of my thermometer, in which the graduation by Mr. Daniell and myself differed a whole degree from the original graduation of the maker. It is probable that M. Bessel's thermometer indicates temperatures more often higher than mine, by a few hundredths of a degree, than lower by the same small quantities; hundredths of a degree are, however, quantities to be spoken of with much confidence, and may safely be neglected on the present occasion.

The experiments with the pendulum at Altona are given in Table B; and the following is an abstract of the results.

Altona; September and October 1828. Experiments with Pendulum 12.

	Therm.	Barom.	Vibrations.	Corrected to 58.32.
Sept. 20	60.93	30.267	85968.46	85969.61
20	60.95	30.267	85968.47	85969.63
20	60.52	30.243	85968.54	85969.51
20	60.42	30.219	85968.54	85969.46
21	59.63	30.139	85968.91	85969.49
21	60.10	30.135	85968.74	85969.52
21	60.33	30.115	85968.81	85969.69
22	60.55	29.855	85968.64	85969.62
22	60.60	29.873	85968.67	85969.67
23	60.71	29.994	85968.73	85969.78
Oct. 1	60.17	29.566	85969.16	85969.97
1	60.30	29.578	85969.17	85970.04
2	60.35	29.688	85969.18	85970.07
2	60.02	29.688	85969.29	85970.04
3	59.40	29.976	85969.45	85969.93
4	58.55	29.913	85969.62	85969.72
8	57.50	29.381	85970.19	85969.83
8	56.70	29.433	85970.49	85969.78
15	55.05	30.030	85971.25	85969.81
15	54.60	30.030	85971.35	85969.71
21	53.70	30.195	85971.89	85969.86
22	54.03	30.041	85971.74	85969.86
23	55.02	30.041	85971.29	85969.84
23	55.40	29.835	85971.10	85969.82
24	55.40	29.982	85971.20	85969.92
24	55.43	29.982	85971.20	85969.93
	58.32	29.941	85969.77	85969.77
Red^{n}. for 29.94 inches of air at 58.32				+ 10.07
				85979.84

The rate at Greenwich, with which this rate at Altona is to be compared, is 85969.67 vibrations at 62°.5. Reducing both these rates to a mean temperature of 60°, in the proportion of 0.44 vibration to one degree of FAHRENHEIT, they become respectively 85979.77 at Greenwich, and 85979.10 at Altona: whence we have an acceleration at Altona of 8.33 vibrations per diem.

TABLE A.—Vibrations of Pendulum 12 at Greenwich, August 1829; on the fixed iron support in the Pendulum room. The Barometer employed was the standard barometer of the Royal Observatory; the Thermometer was Captain SABINE's standard thermometer; the Arc was the same that had been used at Greenwich in July 1828; and at Altona in September and October 1828.

No. of Coincid.	Therm.	Times of			Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours.
		Disapp.	Reapp.	Coincidence.					
EXP. 1. August 8th. Clock making 86309.45 Vibrations. Barom. $\left\{\begin{matrix}30.119\\30.085\end{matrix}\right\}$ 30.102 in. 62°.5.									
	°	m s	m s	h m s	°	°	s	s	
1	61.3	27 19	27 23	7 27 21	0.85	63.87	492.46	+0.30	85959.22
33	66.0	....	....	12 0 0					
45	64.3	28 22	28 36	1 28 29	0.12				
EXP. 2. August 8th. Clock making 86309.73 Vibrations. Barom. $\left\{\begin{matrix}30.085\\30.060\end{matrix}\right\}$ 30.072 in. 64°.5.									
	°	m s	m s	h m s	°	°	s	s	
1	64.4	37 11	37 17	1 37 14	0.86	65.53	490.89	+0.31	85958.41
	67.1	....	....	3 0 0					
41	65.1	4 14	4 45	7 4 29.5	0.14				
EXP. 3. August 9th. Clock making 86309.73 Vibrations. Barom. $\left\{\begin{matrix}30.004\\29.973\end{matrix}\right\}$ 29.988 in. 62°.									
	°	m s	m s	h m s	°	°	s	s	
1	61.7	4 51	4 57	8 4 54	0.72	62.5	492.84	+0.30	85959.76
29	63.3	54 47	55 00	11 54 53.5	0.20				
EXP. 4. August 9th. Clock making 86309.70 Vibrations. Barom. $\left\{\begin{matrix}29.973\\29.930\end{matrix}\right\}$ 29.951 in. 65°.									
	°	m s	m s	h m s	°	°	s	s	
1	63.4	3 25	3 30	12 3 27.5	0.80	64.45	490.0	+0.39	85959.24
27	65.5	36 34	36 45	3 36 39.5	0.23				

TABLE A. (Continued.)

Exp. 5. August 9th. Clock making 86309.70 Vibrations. Barom. {29.930 29.886} 29.908 in. 66°.5.									
No. of Coincid.	Therm.	Times of			Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours.
		Disapp.	Reapp.	Coincidence.					
	°	m s	m s	h m s	°	°	s	s	
1	65.4	44 57	45 2	3 44 59.5	0.82	64.95	491.67	+0.43	85959.04
31	64.5	50 41	50 58	7 50 49.5	0.22				
Exp. 6. August 10th. Clock making 86309.70 Vibrations. Barom. {29.804 29.808} 29.806 in. 62°.									
	°	m s	m s	h m s	°	°	s	s	
1	62.8	37 17	37 24	7 37 20.5	0.91	62.9	492.59	+0.38	85959.63
39	63.0	49 12	49 26	12 49 19	0.18				
Exp. 7. August 10th. Clock making 86309.75 Vibrations. Barom. {29.808 29.850} 29.829 in. 63°.5.									
	°	m s	m s	h m s	°	°	s	s	
1	63.2	57 49	57 53	12 57 51	0.84	63.55	492.15	+0.29	85959.29
44	63.9	50 19	50 48	6 50 33.5	0.12				
Exp. 8. August 11th. Clock making 86309.75 Vibrations. Barom. {30.000 30.040} 30.020 in. 60°.									
	°	m s	m s	h m s	°	°	s	s	
1	61.0	21 51	21 55	8 21 53	0.97	61.5	493.29	+0.47	85960.27
36	62.0	9 31	9 45	1 9 38	0.19				
Exp. 9. August 11th. Clock making 86309.75 Vibrations. Barom. {30.040 30.053} 30.047 in. 62°.									
	°	m s	m s	h m s	°	°	s	s	
1	62.4	18 19	18 25	1 18 22	0.84	62.5	492.88	+0.29	85959.79
42	62.6	54 52	55 28	6 55 10	0.12				

Table B.—Vibrations of Pendulum 12 at Altona; September 1828.

Exp. 1, 2, 3, 4. September 20th. Clock making 86436.48 Vibrations.

No. of Coincid.	Therm.	Times of			Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 Hours.	Barom.
		Disapp.	Reapp.	Coincidence.						
	°	m s	m s	h m s	°	°	s	s		°
1	60.5	43 43	43 49							
2	......	49 52	49 58	9 49 55.33	0.515	60.93	369.17	+0.24	85968.46	30.267 61
3	60.9	56 2	56 5							
	61.0	......	......	11						
24	61.1	5 13	5 21	12 5 17	0.250	60.95	369.32	+0.05	85968.47	30.267 61
	61.1	......	......	1 .						
50	60.8	45 4	45 35							
51	......	51 13	51 45	2 51 28.67	0.100					
52	60.8	57 21	57 54							
	Fresh impulse.									
1	60.7	10 25	10 29							
2	......	16 33	16 39	3 16 37.17	0.560	60.52	369.25	+0.22	85968.54	30.243 61
3	60.8	22 42	22 49							
27	60.3	50 12	50 43	5 50 27.5	0.210					
	Fresh impulse.									
1	60.8	46 55	47 5							
2	......	53 5	53 14	7 53 9.17	0.575	60.42	369.25	+0.20	85968.54	30.219 60
3	60.8	59 14	59 22							
38	60.0	34 32	34 52							
39	......	40 40	41 1	11 40 50.83	0.165					
40	60.1	46 49	47 11							
Exp. 5, 6, 7. September 21st. Clock making 86436.75 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		°
1	59.3	15 21	15 28	7 15 24.5	0.570	59.63	369.275	+0.29	85968.91	30.139 61
26	59.8	18 22	18 38	9 18 30.5	0.290	60.10	369.306	+0.08	85968.74	30.135 61
39	60.4	9 11	9 24	11 9 17.5	0.160					
	Fresh impulse.									
1	60.4	33 38	33 42	11 33 40	0.510	60.33	369.292	+0.17	85968.81	30.115 61
	60.4	......	......	1 0 0						
37	60.2	15 4	15 25	3 15 14.5	0.160					
Exp. 8 and 9. September 22nd. Clock making 86436.66 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		°
1	59.5	40 41	40 47							
2	......	46 50	46 56	9 46 53	0.535	60.55	369.216	+0.18	85968.64	29.855 60
3	60.0	52 59	53 5							
	60.9	......	......	11 14						
38	61.0	28 18	28 31							
39	......	34 26	34 43	1 34 34	0.160					
40	61.0	40 35	40 51							
	Fresh impulse.									
1	61.0	46 36	46 40							
2	......	52 45	52 50	1 52 47.33	0.595	60.60	369.246	+0.17	85968.67	29.873 60
3	......	58 54	58 59							
	60.6	......	......	3 50						
45	......	17 13	17 35							
46	......	23 27	23 41	6 23 34.17	0.120					
47	60.2	29 30	29 59							

TABLE B. (Continued.)

EXP. 10. September 23rd. Clock making 86436.85 Vibrations.

No. of Coincid.	Therm.	Times of Disapp.	Times of Reapp.	Times of Coincidence.	Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 Hours.	Barom.
	°	m s	m s	h m s	°	°	s	s		
1	......	21 46	21 48	9 27 55.83	0.645	60.71	369.142	+0.19	85968.73	29.994 60°
2	60.25	27 54	27 57							
3	......	34 3	34 7							
	60.9	......	......	11						
	60.9	......	......	1						
48	......	10 44	11 9	2 17 5.5	0.115					
49	60.8	16 54	17 16							
50	......	23 5	23 25							

EXP. 11 and 12. October 1st. Clock making 86437.45 Vibrations.

No. of Coincid.	Therm.	Times of Disapp.	Times of Reapp.	Times of Coincidence.	Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 Hours.	Barom.
	°	m s	m s	h m s	°	°	s	s		°
1	59.95	51 7	51 11	9 51 9	0.60	60.17	368.97	+0.24	85969.16	29.566 61
31	......	55 28	55 49	1 1 47	0.20					
32	60.4	1 36	1 57							
33	......	7 46	8 6							
	Fresh impulse.									
1	60.4	14 25	14 31	1 14 28	0.590	60.3	368.98	+0.23	85969.17	29.578 61
32	......	24 57	25 16	4 31 15.33	0.200					
33	60.2	31 3	31 27							
34	......	37 14	37 35							

EXP. 13 and 14. October 2nd. Clock making 86437.70 Vibrations.

No. of Coincid.	Therm.	Times of Disapp.	Times of Reapp.	Times of Coincidence.	Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 Hours.	Barom.
		m s	m s	h m s	°	°	s	s		
1	......	40 52	40 57	9 47 3	0.510	60.35	368.83	+0.19	85969.18	29.688
2	60.05	47 0	47 5							
3	......	53 9	53 15							
	60.70	......	......	11						
31	......	45 7	45 31	12 51 28	0.190					
32	60.30	51 19	51 37							
33	......	57 30	57 44							
	Fresh impulse.									
1	60.20	4 38	4 44	1 4 41	0.550	60.025	368.91	+0.21	85969.29	29.688
33	......	21 15	21 36	4 27 35	0.190					
34	59.85	27 23	27 47							
35	......	33 32	33 57							

EXP. 15. October 3rd. Clock making 86437.58 Vibrations.

No. of Coincid.	Therm.	Times of Disapp.	Times of Reapp.	Times of Coincidence.	Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 Hours.	Barom.
	°	m s	m s	h m s	°	°	s	s		
1	59.25	58 20	58 23	10 4 30.33	0.610	59.40	369.089	+0.25	85969.45	29.976
2	......	4 29	4 32							
3	59.45	10 37	10 41							
	59.70	......	......	11 . 0						
31	59.30	2 46	3 2	1 9 3	0.210					
32	......	8 55	9 11							
33	59.30	15 4	15 20							

TABLE B. (Continued.)

No. of Coincid.	Therm.	Times of			Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 Hours.	Barom.
		Disapp.	Reapp.	Coincidence.						
Exp. 16. October 4th. Clock making 86437.43 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		
1	58.55	38 55	38 59	10 38 57	0.670	58.55	369.32	+0.28	85969.62	29.913
36	......	14 12	14 52							
37	58.55	20 22	20 43	2 20 32.5	0.200					
38	......	26 32	26 54							
Exp. 17. October 8th. Clock making 86437.51 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		
1	57.6	44 44	44 50	2 44 47	0.580	57.5	369.75	+0.20	85970.19	29.381
39	......	38 48	39 7							
40	57.4	44 57	45 17	6 45 7.17	0.160					
41	......	51 7	51 27							
Exp. 18. October 9th. Clock making 86437.51 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		
1	56.8	54 14	54 19	9 54 16.5	0.535	56.7	370.014	+0.19	85970.49	29.433
34	......	17 40	17 54							
35	56.6	23 50	24 5	1 23 57	0.180					
36	......	30 1	30 12							
Exp. 19 and 20. October 15th. Clock making 86437.67 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		
1	......	12 13	12 16							
2	55.2	18 23	18 27	1 18 24.67	0.60	55.05	370.44	+0.26	85971.25	30.030
3	......	24 33	24 36							
28	54.9	58 49	59 3	3 58 56	0.23					
	Fresh impulse.									
1	54.9	23 45	23 49	4 23 47	0.58	54.6	370.57	+0.21	85971.35	30.030
35	......	53 38	53 55							
36	54.3	59 47	0 7	7 59 56.83	0.19					
37	......	5 56	6 18							
Exp. 21. October 21st. Clock making 86437.62 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		
1	54.0	31 3	31 8	12 31 5.5	0.59	53.7	371.04	+0.21	85971.89	30.195
36	53.4	7 20	7 44	4 7 32	0.18					
Exp. 22. October 22nd. Clock making 86437.56 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		
1	53.9	29 15	29 17	11 29 16	0.63	54.03	370.94	+0.24	85971.74	30.041
	54.0	......	......	12 30						
39	......	24 0	24 22							
40	54.2	30 13	30 34	3 30 22.67	0.18					
41	......	36 22	36 45							

TABLE B. (Continued.)

No. of Coincid.	Therm.	Times of Disapp.	Reapp.	Coincidence.	Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 Hours.	Barom.
Exp. 23 and 24. October 23rd. Clock making 86437.34 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		
1	54.4	39 51	39 56	9 39 53.5	0.60	55.025	370.76	+0.23	85971.29	30.041
	55.1	......	......	10 40						
	55.3	......	......	11 50						
35	55.3	9 50	10 9	1 9 59.5	0.19					
	Fresh impulse.									
1	55.3	16 28	16 32	1 16 30	0.61	55.4	370.50	+0.34	85971.10	29.835
19	55.5	7 34	7 44	3 7 39	0.32					
Exp. 25 and 26. October 24th. Clock making 86437.45 Vibrations.										
	°	m s	m s	h m s	°	°	s	s		
1	55.4	25 12	25 16	10 25 14	0.60	55.4	370.59	+0.24	85971.20	29.982
33	55.4	42 45	43 1	1 42 53	0.20					
	Fresh impulse.									
1	55.4	49 50	49 52	1 49 51	0.61	55.43	370.57	+0.26	85971.20	29.982
	55.5	......	......	4						
28	55.4	36 25	36 48	4 36 36.5	0.22					

XIX. *Experiments to ascertain the Correction for Variations of Temperature, within the limits of the natural temperature of the Climate of the South of England, of the Invariable Pendulum recently employed by British observers. By Captain* EDWARD SABINE *of the Royal Artillery, Secretary to the Royal Society.*

Read March 25th, 1830.

HAVING obtained the rate of the invariable pendulum, No. 12, in the Royal Observatory at Greenwich at the temperature of 63°.53 in the month of August 1829, as given in the preceding paper, the pendulum was laid aside until the cold weather of the winter season had set in, when the observations detailed in Table C were made with it, at the mean temperature of 31°.15; the results of which are exhibited in the following abstract:

Greenwich, December 1829, and January 1830: Experiments with Pendulum 12.

	Therm.	Barom.		Vibrations.	Corrected to 31.15.
Dec. 29	28.50	30.212	27.0	85973.99	85972.82
29	28.55	30.227	27.0	85973.84	85972.70
30	29.30	30.307	28.0	85973.66	85972.85
30	29.65	30.311	28.5	85973.52	85972.86
Jan. 1	31.35	30.497	30.5	85972.82	85972.91
1	32.00	30.472	31.0	85972.33	85972.70
2	31.80	30.371	31.0	85972.46	85972.75
3	33.85	30.285	33.0	85971.78	85972.97
5	35.40	30.149	34.5	85970.94	85972.81
		30.315	30		85972.81
		−.002 Red^{n}. to 32°		Red^{n}. for 30.33 in. of air at 31.15	+10.77
	31.15	+.019 Capillary			
		30.332 at 32°			85983.58 at 31.15

Comparing this result with 85969.33 vibrations at the temperature of 63°.53 obtained in August 1829, we have 14.25 vibrations per diem corresponding to 32°.38 of Fahrenheit; which gives a correction of 0.44 of a vibration per diem for each degree of Fahrenheit between 30° and 60°.

The experiments which I formerly made with two pendulums similar to the present, in a chamber artificially heated to between 80° and 90°, gave for the correction for each degree of Fahrenheit, respectively for the two pendulums, 0.432 and 0.430, corresponding to that part of the thermometer scale which is included between 45° and 85°. Those results accord well with each other, and are somewhat different from that which is now obtained, and which correspond to the part of the scale comprised between 30° and 60°. But in the experiments in the chamber artificially heated, the fluctuations of temperature, in spite of every precaution, were considerable, and rendered the determination of the mean temperature more difficult, and probably less exact than in the natural temperatures: hence it would be unsafe to conclude in favour of the inference to which these facts would otherwise lead, that the correction at high temperatures is less than at low temperatures, or that the metal expands a smaller proportion of its length for one degree between 85° and 45°, than for one degree between 60° and 30°.

The experiments at Greenwich were made in those extremes of natural temperature afforded by the climate, in which a tolerably uniform temperature is maintained for several days; a condition requisite in such delicate determinations. The clock by Graham was going well on both occasions, and its rate was assigned by Mr. Taylor from comparisons with the transit clock of the Observatory, with probably as much accuracy as the rates of clocks are ever obtained. The thermometer is entitled, by its comparison with those of Mr. Daniell and of M. Bessel, to be regarded as one of good authority, having, in the absence of a standard thermometer in this country, received the sanction of what must be considered the best existing authorities. Every precaution was adopted, which the experience I have had in obtaining the rates of pendulums has suggested, for maintaining a uniform temperature in the apartment. The examination of the partial observations will best show the success of these precautions. Viewing all these particulars, I regard 0.44 of a vibration per diem for each degree of Fahrenheit as a result obtained under circumstances of a very favourable nature, and as not likely to be surpassed in the confidence which may be due to it, until by a better command of artificial temperatures the experiments can be made to include, with tolerable certainty of determination, a greater difference of temperature than is afforded by the natural climate of this country.

As the many invariable pendulums which have been employed of late years by British observers have all been made of the same kind of brass, it is probable that the same correction for temperature will apply equally to all.

When, as in ordinary cases, the differences of temperature between observations designed to be comparative amount only to a few degrees, the probable error which may be incurred by employing the correction 0.44 for each degree as now determined, can only be very inconsiderable: but when the differences of temperature amount to 50°, which is a case of actual experience in pendulum observations, the question of whether 0.43 or 0.44, for example, be the more correct value, involves an uncertainty in the ultimate result of no less than half a vibration a day. It seems therefore desirable, for the sake of experiments, which are becoming greatly multiplied, and which are daily increasing in accuracy, that means should be devised of obtaining the rates of pendulums in artificial temperatures, embracing a wider range than the natural temperatures, but capable of being determined with equal accuracy.

TABLE C.—Vibrations of Pendulum 12 at Greenwich, December 1829, and January 1830, on the fixed iron support in the Pendulum room. The Barometer employed was the standard barometer of the Royal Observatory; the Thermometer was Captain SABINE's standard thermometer; the Arc was distant 60 inches from the point of suspension, and was divided into degrees, each of 0.8 of an inch in length; the registered arc therefore multiplied by 0.764, gives degrees of the true arc of vibration.

EXP. 1. Dec. 29th. Clock making 86552.75 Vibrations. Barom. $\left\{\begin{matrix}30.205\\30.220\end{matrix}\right\}$ 30.212 27°.

No. of Coincid.	Therm.	Times of			Registered Arc.	True Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours Mean Solar Time.
		Reapp.	Disapp.	Coincidence.						
	°	m s	m s	h m s	°	°	°	s	s	
1	28.6	15 37	15 42	1 15 39.5	0.84	0.64	28.5	298.97	+0.24	85973.99
53	28.4	34 38	34 54	5 34 46	0.24	0.18				

TABLE C. (Continued.)

EXP. 2. Dec. 29th. Clock making 86552.75 Vibrations. Barom. $\left\{\begin{matrix}30.220\\30.234\end{matrix}\right\}$ 30.227 27°.

No. of Coincid.	Therm.	Times of Reapp.	Times of Disapp.	Times of Coincidence.	Registered Arc.	True Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours Mean Solar Time.
	°	m s	m s	h m s	°	°	°	s	s	
1	28.5	40 3	40 9	5 40 6	0.78	0.59	28.55	298.92	+0.20	85973.84
55	28.6	9 1	9 15	10 9 8	0.19	0.15				

EXP. 3. Dec. 30th. Clock making 86552.75 Vibrations. Barom. $\left\{\begin{matrix}30.314\\30.300\end{matrix}\right\}$ 30.307 28°.

No. of Coincid.	Therm.	Times of Reapp.	Times of Disapp.	Times of Coincidence.	Registered Arc.	True Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours Mean Solar Time.
	°	m s	m s	h m s	°	°	°	s	s	
1	29.4	42 27	42 32	10 42 29.5	0.90	0.67	29.30	298.82	+0.22	85973.66
63	29.2	51 9	51 24	3 51 16.5	0.18	0.14				

EXP. 4. Dec. 30th. Clock making 86552.75 Vibrations. Barom. $\left\{\begin{matrix}30.300\\30.322\end{matrix}\right\}$ 30.311 28°.5.

No. of Coincid.	Therm.	Times of Reapp.	Times of Disapp.	Times of Coincidence.	Registered Arc.	True Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours Mean Solar Time.
	°	m s	m s	h m s	°	°	°	s	s	
1	29.5	0 0	0 5	4 0 2.5	0.90	0.67	29.65	298.77	+0.18	85973.52
78	29.8	23 15	23 40	10 23 27.5	0.12	0.09				

EXP. 5. Jan. 1st. Clock making 86552.75 Vibrations. Barom. $\left\{\begin{matrix}30.515\\30.480\end{matrix}\right\}$ 30.497 30°.5.

No. of Coincid.	Therm.	Times of Reapp.	Times of Disapp.	Times of Coincidence.	Registered Arc.	True Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours Mean Solar Time.
	°	m s	m s	h m s	°	°	°	s	s	
1	31.0	17 37	17 40	11 20 7.5	0.98	0.75	31.35	298.36	+0.26	85972.82
2	......	22 34	22 39							
62	......	20 52	21 4	4 23 27.25	0.18	0.14				
63	31.7	25 51	26 2							

EXP. 6. Jan. 1st. Clock making 86552.75 Vibrations. Barom. $\left\{\begin{matrix}30.480\\30.465\end{matrix}\right\}$ 30472 31°.

No. of Coincid.	Therm.	Times of Reapp.	Times of Disapp.	Times of Coincidence.	Registered Arc.	True Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours Mean Solar Time.
	°	m s	m s	h m s	°	°	°	s	s	
1	32.0	40 50	40 53	4 40 51.5	0.86	0.64	32.0	298.14	+0.17	85972.33
74	32.0	43 29	43 43	10 43 36	0.13	0.10				

EXP. 7. Jan. 2nd. Clock making 86552.75 Vibrations. Barom. $\left\{\begin{matrix}30.378\\30.365\end{matrix}\right\}$ 30.371 31°.

No. of Coincid.	Therm.	Times of Reapp.	Times of Disapp.	Times of Coincidence.	Registered Arc.	True Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours Mean Solar Time.
	°	m s	m s	h m s	°	°	°	s	s	
1	31.6	37 24	37 27	11 37 25.5	1.01	0.77	31.8	298.17	+0.24	85972.46
76	32.0	50 1	50 15	5 50 8	0.13	0.10				

TABLE C. (Continued.)

EXP. 8. Jan. 3rd. Clock making 86552.75 Vibrations. Barom. $\left\{\begin{matrix}30.300\\30.270\end{matrix}\right\}$ 30.285 33°

No. of Coincid.	Therm.	Times of			Registered Arc.	True Arc.	Mean Therm.	Mean Interval.	Correction for Arc.	Vibrations in 24 hours Mean Solar Time.
		Reapp.	Disapp.	Coincidence.						
	°	m s	m s	h m s	°	°	°	s	s	
1	33.8	13 27	13 35	2 13 31	0.92	0.68	33.85	297.86	+0.18	85971.78
78	33.9	35 38	35 54	8 35 46	0.11	0.08				
EXP. 9. Jan. 5th. Clock making 86552.75 Vibrations. Barom. $\left\{\begin{matrix}30.110\\30.188\end{matrix}\right\}$ 30.149 34°.5										
	°	m s	m s	h m s	°	°	°	s	s	
1	35.4	16 25	16 31	2 16 28	0.98	0.75	35.4	297.42	+0.20	85970.94
81	35.4	52 52	53 11	8 53 1.5	.12	0.08				

XX. *On a new Register-Pyrometer, for measuring the Expansions of Solids, and determining the higher Degrees of Temperature upon the common thermometric scale.* *By* J. FREDERIC DANIELL, *Esq.*, *F.R.S.*

Read June 17, 1830.

IN the year 1821 I published in the Journal of the Royal Institution* an account of a new pyrometer, and the results of some experiments with it, which were the means of correcting the highly erroneous notions which had, up to that time, been generally entertained of the degrees of temperature beyond the boiling point of mercury. The instrument was capable of affording correct determinations, connected in an unexceptionable manner with the scale of the mercurial thermometer; but, although applicable to scientific investigation in careful hands, it could be inserted only into experimental furnaces of a particular construction, which greatly limited its use. The great desideratum still remained of a pyrometer, which might universally be applied to the higher degrees of heat, as the thermometer has long been to the lower; and which, in addition to its use in delicate researches, might effect for the potter, the smelter, the enameler and others, in the routine of their business, what the latter daily performs for the brewer, the distiller, the sugar-refiner, and the chemist.

I shall now have the honour of laying before the Royal Society a description of a contrivance which, I trust, will be found to answer all the desired purposes; and which, while simple enough to be intrusted to the hands of common workmen in every variety of fire-place, I hope to prove, by the results of my experiments, to be sufficiently delicate to extend considerably our knowledge of the expansion of metals, upon which so much labour has been bestowed by some of the first philosophers.

I was not aware, at the time when I wrote the account above referred to, that the subject had been previously investigated by M. GUYTON DE MORVEAU, and that he had proposed to apply the expansion of platinum as a measure of high temperature, and more particularly to the purpose of connecting the indications of Wedgwood's pyrometer with the mercurial scale and verifying its regularity. I have since carefully studied his laborious papers in the Annales de Chimie *, and the Mémoires de l'Institut †, which appear to have been but very little known in this country; and previously to entering upon the more particular object of the present paper, I must claim indulgence for a few remarks upon the general state of the inquiry at the time when its pursuit was abandoned by that able philosopher.

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* Vol. xi. p. 309.

M. Guyton's pyrometer consisted of a small bar or plate of platinum 45 millimetres (1.77 inch) long, 5 millimetres (about 0.2 inch) broad, and 2 millimetres (about 0.08 inch) thick, placed in a groove formed in a piece of highly baked porcelain. One extremity of this bar rested upon the solid end, which terminated the groove, and the other pressed upon the short arm of a bent lever, the longer arm of which terminated in a point and moved on a pivot over the graduated arc of a circle; indicating by its motion any lengthening of the bar by increase of temperature. The short arm of the lever was 2.5 millimetres and the long arm 50 millimetres in length, and the latter carried a nonius by which the tenths of a degree might be read off. The whole was constructed of platinum; and a plate of the same metal was made to press, in the manner of a spring, upon the extremity of the index, to prevent any displacement when withdrawing it from the fire. The description of this instrument in the first Essay, published in the year 1803, was not accompanied by any explanatory figure; and the notice in the Annales terminates by announcing that the inventor had at that time only begun "a series of experiments to determine its march, to compare it with the pyrometer pieces of Wedgwood, and to ascertain the degree of confidence which might be placed in the indications of the latter." The second Essay did not appear till the year 1808, and in it M. Guyton observes that "many persons had expressed a wish to be made acquainted with the improvements which he had made in the instrument since its first construction; and that he had determined in consequence to give a fresh description of it accompanied by drawings, which might enable artists who undertook its construction to render it comparable. He, however, thought it right to give a previous account of the labours of others in this branch of science, and

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* Tome xlvi. p. 276.

† 1808, Second Semestre, tome ix.—1811, ibid. tome xii.

to remove certain errors which had prevailed up to that time concerning the pyrometer then most in use (WEDGWOOD's), and which might possibly prove most commodious, and consequently most useful, if once the degree of exactitude could be determined of which it was susceptible." The remainder of the paper is taken up with an account of the most accurate experiments upon the expansion of the metals from the time of NEWTON.

The third and last Essay was delayed till the year 1811; and in it no further description of the platinum pyrometer is to be found; but a laborious comparison,

1st, of the indications of the platinum pyrometer with those of the mercurial thermometer;

2nd, of the same pyrometer with that of WEDGWOOD; and

3rd, of the degrees determined by these instruments with those previously known of the expansion, ebullition and fusion of various substances; in a range of temperature comprising the highest degrees of the thermometric scale and the lowest of WEDGWOOD's.

Now it is very remarkable that all M. GUYTON's efforts in this paper are directed to the valuation of the degrees determined by Mr. WEDGWOOD's clay pieces; but that he carries the comparison of the platinum pyrometer by actual experiment no higher than the melting point of antimony. He clearly establishes a great error in Mr. WEDGWOOD's original estimation of his degrees to that point; and, by calculation upon this basis, continues the correction to the melting point of iron, "en admettant toujours une progression uniforme jusque dans les plus hautes températures." The experimental comparison was obviously stopped by some practical difficulty at higher temperatures; and it is easy to perceive in what this must have consisted. Platinum at a red heat becomes very soft and ductile; and the lever against which the pyrometric bar pressed, being of such very slender dimensions, would obviously be liable to bend and thus frustrate the experiment: in addition to which, I can speak from my own experience, the platinum spring plate and the centre pin would be liable to a change of texture which would impede the motion of the lever and it would finally become welded to the index; for a very moderate pressure at a high temperature would produce this effect.

The conclusion, indeed, of these Essays seems to admit that the author did not expect that the platinum pyrometer could ever come into general use: "enfin, ces corrections ne peuvent manquer d'ajouter à l'utilité du pyromètre d'argile, soit dans les travaux chimiques, soit dans les arts; quand même le pyromètre de platine, plus exact mais moins usuel, serait réservé pour en assurer la marche, et pour servir à des recherches plus importantes."

M. GUYTON, however, although he abundantly proves the incorrectness of Mr. WEDGWOOD's estimate of the higher degrees of temperature, is very far indeed from establishing the point at which he so earnestly laboured, namely, the regularity of the contraction of the clay pieces; or from substituting a more correct value of the degrees throughout the whole range of the gauge than the one which he so completely overturned. His comparative experiments with the platinum pyrometer, at the boiling points of mercury and linseed oil, and the melting point of antimony, led him to reduce the equivalent of each degree from 130° FAHR. to 62°.5. The zero point of the clay pyrometer was thus carried back to 517° instead of 1077°; but it seems to have escaped his notice that this zero point was declared to be a red heat visible in the day-light,—a description which cannot be mistaken, and which clearly could not be below the temperature of boiling oil, melting lead, or boiling mercury; all of which are, however, placed above it in M. GUYTON's table. M. GUYTON also places the melting point of silver at the 22nd degree of Mr. WEDGWOOD's scale instead of the 28th, which was, according to his own determination, a correction first suggested by Sir JAMES HALL in the 9th volume of NICHOLSON's Journal. Taking the value of each degree at 62°.5 FAHR., it fixes this point at 1892° FAHR., which agrees very nearly with my own experiment in the paper before alluded to; but continuing the calculation up to the melting point of iron, upon the supposition of an uniform progression, the 130th degree corresponds with 8696° FAHR., which, although only about half the amount 17977° assigned by Mr. WEDGWOOD, is very far removed from the result of my calculation 3479°.

Nevertheless, it is a curious fact, that M. GUYTON's Essay contains proof that his determination is erroneous, and that mine is a near approximation to the truth. As a collateral means of verifying the indications of instruments intended to measure high degrees of temperature, he refers to the calorimeter as capable of affording the necessary data by a calculation from the amount of heat communicated to known quantities of fire or water by bodies in a state of incandescence; and he quotes the very exact experiments of MM. Clement and Desormes, who had in this manner determined the following points:

	By the Liquefaction of Ice. Fahr.	By the Heat communicated to Water. Fahr.
Temperature of soft iron melted	3988°	3902°
Cast iron just on the point of melting	3164°	——
Red hot iron	2732°	——
White hot ditto	3282°	——
Iron just ceasing to be luminous in day-light	——	1272°
Melted copper	——	2294°

My own determinations of the melting point of cast iron, 3479°, of that of copper, 2548°, and of a red heat, about 1000°, agree, very closely and satisfactorily with these results, with which I was unacquainted at the time of my experiments. M. Guyton's remark upon the latter is: "Il suffit de jeter un coup-d'œil sur les résultats, pour recueillir de nouvelles preuves univoques de la nécessité de réduire les valeurs données par Wedgwood aux degrés de son pyromètre. Mais je ne crains pas de dire que ces réductions sont ici portées trop loin, ainsi qu'on peut en juger en les rapprochant de celles auxquelles j'ai été conduit par l'ensemble des expériences rapportées dans cet essai. Ce n'est pas que je veuille répandre des doutes sur l'exactitude des observations dont je dois la communication aux deux habiles chimistes ci-dessus cités; mais il est aisé de faire voir que la différence des résultats est due, pour la plus grande partie, à la différence des procédés; de sorte que les évaluations qu'ils ont données aux degrés de l'échelle de Wedgwood, peuvent, en dernière analyse, et en prenant les termes moyens dans la latitude que comportent des opérations aussi délicates, servir plutôt à confirmer qu'à détruire le systême de correction que j'ai établi."

It is worthy of observation, that had the degrees of Wedgwood's pyrometer been valued from this determination of the fusing point of iron, the result would have better corresponded with the whole series of phenomena. Instead of 130° Fahr. as fixed by the inventor, or 62°.5 as corrected by M. Guyton, they would have been estimated at about 20° Fahr.; and taking Mr. Wedgwood's original determination of the fusing point of silver at 28° of his scale and the zero point at 1077°, the former would come out about 1650°. By raising the zero point a little, (and it is much more probable that the temperature of a red heat fully visible in the day-light is above 1077° than below it,) we arrive at something like an approximation to the truth. These wide discrepancies, and the practical disuse of both Mr. Wedgwood's and M. Guyton's pyrometers for a long time past, prove the expediency of further investigating a subject of so much interest and importance.

The pyrometer, which I shall now proceed to submit to the judgement of the Society, consists of two distinct parts, which I shall designate as the Register and the Scale.

The first is a solid bar of black-lead earthenware, eight inches long, seven-tenths of an inch wide, and of the same thickness, cut out of a common black-lead crucible. In this a hole is drilled three-tenths of an inch in diameter, and 7½ inches deep. At the upper end of this bar and on one of its sides about six-tenths of an inch in length of its substance is cut away to the depth of half the diameter of the bore. When a bar of any metal 6½ inches long is dropped into this cavity, it rests against its solid end; and a cylindrical piece of porcelain about 1½ inch long, which I shall call the index, is placed upon the top of it, which projecting into and beyond the open part, is firmly confined to its place by a ring, or strap of platinum; which passing round the black-lead bar and over the piece of porcelain, is made to press upon the latter with any required degree of tension by means of a small wedge of porcelain inserted between the bar and the strap on the side of the former. It is obvious that when such an arrangement is exposed to a high temperature, the metallic bar will force the index forward to the amount of the excess of its expansion over that of the black-lead, and that when again cooled, it will be left at the point of greatest elongation. It may also be observed, that the exact indication of this amount is not in the slightest degree interfered with by any permanent contraction which the black-lead may undergo at high degrees of heat; as any such contraction will take place at the moment of the greatest expansion of the metal, and the index will still mark its point of furthest extension upon this contracted basis.

The problem now consists in the accurate measurement of the distance which the index has been thrust forward from its original position; and although the amount can in any case be but small, there is no reason why it may not be determined with the same precision as is now commonly attained in similar quantities in astronomical and geodetical operations. For this purpose the

Fig. 1.

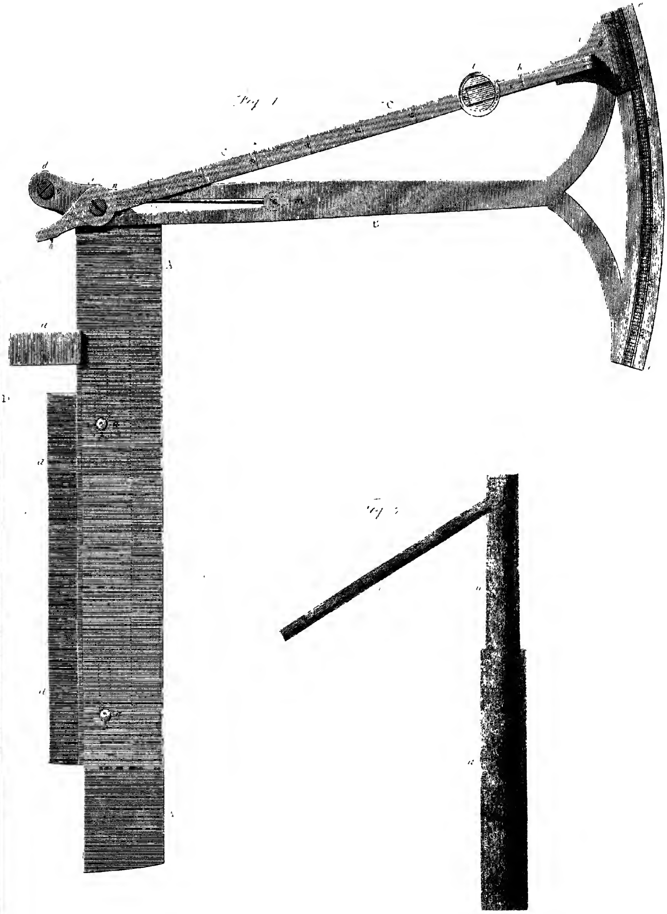
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Fig. 2.

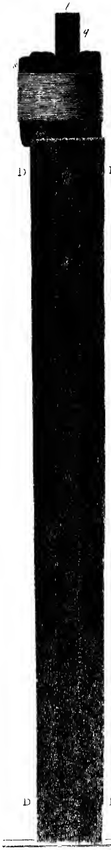

Fig. 3.

Mr Daniell's Register Pyrometer.

scale is constructed of two rules of brass, accurately joined together at a right angle by their edges, and fitting square upon two sides of the black-lead bar, and of about half its length. At one end of this double rule a small plate of brass projects at a right angle, which plate, when the two sides of the former are applied to the two sides of the register, is brought down upon the shoulder formed by the notch cut away at its upper end, and the whole may be thus firmly adjusted to the black-lead bar by three planes of contact.

On the outside of this frame another brass rule is firmly screwed down, which projecting beyond it, and bending a little so as to bring its end opposite to the cavity in the black-lead bar when applied to it, supports a moveable arm exactly 5½ inches long, turning at its fixed extremity upon a centre, and at its other carrying an arc of a circle, accurately divided into degrees and thirds of a degree, whose radius is exactly 5 inches. At the centre of this circle upon the arm, and of course at the distance of half an inch from the centre of motion, another lighter arm is made to turn, one end of which, being the exact radius of the circular arc, carries a nonius with it, which moves upon the face of the arc and subdivides the former graduation into minutes. The other end crosses the centre; and at the exact distance of one-tenth of the radius, or the distance between the two centres of motion, terminates in an obtuse steel point turned inwards at a right angle. These graduations and distances are laid down with the greatest precision by Mr. Troughton's dividing engine. This part of the apparatus may be regarded as a pair of proportional compasses attached to the end of the brass rule and frame, whose longer legs carrying the arc and nonius are to its shorter as ten to one; and the opening of the latter being regarded as a chord of a small circle, is magnified in the same proportion by the former, and measured upon the scale. A small steel spring let into the larger arm is made to press upon the smaller, so as to adjust the nonius to the commencement of the graduation; and when forced back it tends to restore it to its original position.

The annexed figures, in which all the parts are drawn of their real dimensions, will assist the comprehension of the preceding description. Plate X. fig. 1. represents the scale. A A is the principal brass rule, upon the under side of which the frame *a a a a a a'* is adjusted by the screws *b b*, and which supports upon its bent extremity *c*, the arm B moving upon the centre *d*, and terminating in the arc of the circle *e e*.

C C is the lighter arm moving upon the centre *f* upon the arm B, and carrying at one end the nonius *g*, and at the other the steel point *h*, the distance of which from the centre *f* is exactly half an inch or one-tenth of the radius *f g*, and equal to the distance of the two centres *f d*. *i* is a small lens represented as lying down, but which may be raised by the centres *k* and *l* perpendicularly above the nonius to facilitate the reading. *m m* is the steel spring, which being fixed in a cavity cut out of the arm B, presses upon a small pin *n* on the arm C, and throws the radius back to the commencement of the arc.

Fig. 2. represents the register. D D D D is the black-lead bar, with its cavity *o o*. At *p p p p* it is cut away to the depth of half the bore. *q q* is the porcelain index, which is placed upon the top of the metallic bar, and confined to its place by the pressure of the platinum strap *r* acting by the force of the small porcelain wedge *s*.

When an observation is to be made, the metallic bar is placed in the cavity of the register, the index is to be pressed down upon it and firmly fixed in its place by the platinum strap and porcelain wedge. The scale is then to be applied by carefully adjusting the brass rules to the sides of the black-lead bar, and fixing it by pressing the cross piece (a') upon the shoulder: holding the whole together steadily in the left hand, the moveable arm should be so placed that the steel point (*h*) of the other leg of the compasses may rest upon the edge of the porcelain index, against which it will be pressed with some force by the spring: then moving the arm gently forward with the right hand, the point will slide along the end of the index till it drops into a small cavity (*t*) formed for its reception, and which exactly coincides with the axis of the metallic bar in the register, and the centre of motion of the compasses on the brass rule. The minute of the degree must then be noted, which the nonius indicates upon the arc. A similar observation must be made after the register has been exposed to an increased temperature and again cooled; and the number of degrees or minutes which the nonius will then mark will, by a simple calculation from the known length of the radii and angle, give the length of the chord comprised between the original position of the compasses and the point to which they have moved, or the distance which the index has been forced forward. Such an operation appears complex in the description, but is in fact extremely simple after a little practice, and does not require more than a few seconds for its performance. The scale of this pyrometer being completely detached from the part which is exposed to the fire, obviates one important objection which has always been made to other contrivances of the same nature, from the uncertain degree of heat and expansion to which they are liable; while the simplicity of that part of the arrangement which alone is subjected to great heats, renders it little liable to injury; and together with the cheapness of the materials of which it is constructed, occasions but a very trifling expense for replacing it when injured.

The calculation of the absolute expansion of the bar indicated by the scale may be performed as follows:—As radius to double the sine of half the arc read off, and found in a table of natural sines, so will the radius B be to the chord of the same arc; and this divided by ten (the radius of B being ten times the length of the radius *f h*) will give the length required. Suppose the arc read off upon the scale to be 4°,

then $\overset{\text{Radius.}}{1.0000000} : \overset{\text{Sine of } 2^\circ.}{0348995} \times 2 :: \overset{\text{Inches.}}{5} : \overset{\text{Inch.}}{.3489950} \div 10 = .0348995.$

Now in working out this proportion it will be observed, that the multiplication by 2 and by 5 being both constant may, in conjunction with the division by 1.0, be omitted; and leaving out also the final division by 10, the case resolves itself into seeking the sine of half the arc, read off upon the scale, in a table of natural sines, and reading it as the decimal of an inch.

Moreover, the chords of small arcs are so nearly proportional to their arcs that, the number of degrees measured upon the scale never exceeding 10, they may be considered without sensible error as denoting equal increments of expansion. The following short Table of the value of a degree, and minutes of a degree, may therefore be useful in practice.

TABLE I.

°	′		Inch.
1	0	=	.00872
0	30	=	.00436
0	20	=	.00290
0	15	=	.00218
0	10	=	.00145
0	5	=	.00072
0	2	=	.00029
0	1	=	.00014

The chord of ten degrees derived from this Table by multiplying .00872 by 10 would therefore be .0872, whereas it is more accurately .0871; but the difference being only $\frac{1}{10,000}$dth of an inch may, in most cases, be disregarded.

I shall now proceed to show the degree of confidence which may be placed in this new pyrometer, by comparing the result of its indications with those of the best experiments upon the expansion of metals. Those of MM. DULONG and PETIT* are well adapted to this purpose. These able philosophers, in their celebrated prize Memoir on the Measure of Temperatures, and on the Laws of the Communication of Heat, have given, from experiment, the expansion of rods of platinum and iron at different intervals between the freezing point of water and the boiling of mercury. Their mode of experimenting was unexceptionable; but it is to be regretted that they have not corrected their final results for an error of calculation which has been pointed out by Mr. CRICHTON†, which is by no means unimportant to the reasoning which they have founded upon them. The error, however, affecting the amount of expansion in volume, is reduced to one-third in the linear expansion, which is the subject of the present investigation, and may therefore be disregarded.

The following Table of the expansion of iron and platinum is extracted from their work.

TABLE II.

Temperature deduced from the dilatation of Air.	Mean absolute dilatation of Iron for 180 degrees.	Mean absolute dilatation of Platinum for 180 degrees.
From 32° to 212°	$\frac{1}{28200}$	$\frac{1}{37700}$
From 360° to 572°	$\frac{1}{22700}$	$\frac{1}{36300}$

Whence we deduce the linear expansion of platinum for 180° FAHRENHEIT, from 32° to 212° .00088420: and for 180°, from 360° to 572° .00091827: and of iron, from 32° to 212° .00118203: from 360° to 572° .00146842, showing an increasing dilatation in each when referred to an air-thermometer.

The bars of the different metals used in the following experiments were all exactly 6.5 inches in length.

EXP. 1. A square bar of platinum $\frac{2}{10}$ths of an inch thick, was carefully arranged in the black-lead register, which was placed in the apparatus represented, upon a diminished scale, at fig. 3. *a* is an iron tube about two inches diameter, and closed at the bottom: *b* is a black-lead tube closed at the top, and fitted to the mouth of the former by grinding: *c* is a smaller black-lead tube projecting from the side of the latter near its upper end, and likewise fitted to its place by grinding. The whole forms a kind of alembic, which may be readily put together, and in which mercury may be easily boiled on a common fire, and the vapours collected without loss or annoyance to the operator. The register was fixed in its place by a wire, so that when mercury was poured into the iron bottle it was prevented from floating. The mercury in this experiment rose a little above half the length of the register. The whole apparatus was then placed upon a fire, and in ten minutes the mercury began to boil: in ten minutes more it freely distilled over; and in ten minutes further the apparatus was removed, the register taken out and allowed to cool. The arc measured upon the scale was in this instance 1° 17′.

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* Ann. de Chimie et Physique, vii. 113.

† Annals of Philosophy, New Series, vol. vii. p. 241.

The experiment was repeated, merely having the head of the alembic off, and suffering the mercury to boil freely in the iron bottle for a quarter of an hour. The arc measured was 1° 23′.

The register was next allowed to float upon the mercury, so that when the head of the alembic was adjusted and the mercury made to boil, it was not immersed in the metal, but surrounded by its vapour: the reading was 1° 16′. A repetition of this arrangement gave 1° 23′.

In another repetition of the experiment, the time was extended to twenty minutes from the first boiling of the mercury; the reading of the scale was 1° 20′.

Again; the time was reduced to ten minutes, and the measurement was 1° 23′.

In the various repetitions of this experiment the mercury freely distilled over, and the temperature was such, that every part of the black-lead tubes, in which the vapour circulated, would just scorch, but not blacken, a piece of writing paper held against them.

The following Table collects these results into one view, and exhibits the expansion denoted by each reading, and the mean result.

TABLE III.

1° 17′ = .01119
1 23 = .01206
1 16 = .01105
1 23 = .01206
1 20 = .01163
1 23 = .01206
Mean 1 20 = .01163

The temperature of the atmosphere was about 64° during these observations.

EXP. 2. A bar of soft iron, of the same dimensions as that of platinum, was substituted for the latter in the register. The experiment was repeated five times; twice with the register immersed in the mercury, and three times exposed only to the vapour. The time of exposure varied from twenty minutes to ten, from the first moment when the metal began to boil.

The following Table exhibits the several readings and the appropriate expansions.

TABLE IV.

2° 13′ = .01933
2 33 = .02224
2 10 = .01890
2 23 = .02079
2 20 = .02036
Mean 2 20 = .02036

The greatest variation from the mean was therefore only $\frac{6}{10,000}$dths of an inch in the platinum experiment, and $\frac{13}{10,000}$dths in the iron.

We shall now compare these results with the preceding determinations of MM. DULONG and PETIT.

The expansion of Platinum.

Length of Bar.

From 32° to 212° = .00088420 × 6.5 . . . = .005747300

From 360° to 572° = .00091827 × 6.5 . . . = .005968755

.011716055

From 212° to 360° = Mean of the above . . . = .005858027

Total expansion from 32° to 572° . . . . = .017574082

Add for the expansion from 572° to 660°, the temperature of boiling mercury, calculated at the highest rate:—

180° : .005968755 : : 88° : .002918058 . . =.002918058

.020492140

Deduct expansion for 32°, the experiment with the pyrometer having been made at 64° . . . . = .001021742

Calculated at the lowest rate:—

180° : .005747300 : : 32° : .001021742

Real expansion of the bar by Dulong and Petit = .019470398

If from the real expansion thus obtained 01947

We deduct the apparent expansion obtained by the pyrometer .01163

The remainder .00784

will be the expansion of the black-lead.

The expansion of Iron.

Length of Bar.

From 32° to 212° = .00118203 × 6.5 . . . = .007683195

From 360° to 572° = .00146842 × 6.5 . . . = .009544730

.017227925

From 212° to 360° = Mean of the above . . . = .008613962

Total expansion from 32° to 572° . . . . . . = .025841887

Add for the expansion from 572° to 660°, the temperature of boiling mercury, calculated at the highest rate:—

180° : .009544730 : : 88° : .004666311 . = .004666311

.030508198

Deduct expansion for 32°, the experiment with the pyrometer having commenced at 64° . . . = .001365901

Calculated at the lowest rate:—

180° : .007683195 : : 32° : .001365901

Real expansion of the bar by Dulong and Petit . .029142297

From the real expansion 02914
Deduct the apparent expansion obtained by the pyrometer .02036

The remainder .00878

is again the expansion of the black-lead as obtained by this series of experiments.

Expansion of 6.5 inches of Black-lead.

From 64° to 660° by platinum bar 00784
by iron bar 00878

Mean .00831

either determination differing from the mean by less than $\frac{5}{10,000}$dths of an inch.

This close agreement in results from two metals whose expansions differ so much from each other is highly satisfactory; but the great delicacy of the instrument may be still better appreciated from the following experiment of the expansion of nine different metals from the temperature of 62° (the temperature of the air at the time of observation) to 212°.

Exp. 3. Bars of the following metals were successively placed in the register and immersed in hot water, which was gradually heated to the boiling point, and kept boiling for ten minutes in each instance. The following Table exhibits the readings of the scale and the appropriate expansions.

Table V.

	°	′			
Platinum . .	0	19	=	.00276	from 60° to 212°
Iron (soft) . .	0	35	=	.00508	———
Copper . . .	0	47	=	.00683	———
Tin (grain). .	0	56	=	.00814	———
Zinc	1	40	=	.01454	———
Lead	1	25	=	.01223	———
Brass . . .	0	55	=	.00799	———
Gold (fine) . .	0	36	=	.00552	———
Silver (fine) .	0	56	=	.00814	———

In the subsequent Table I have given the absolute expansions of the same metals from 32° to 212° from the best authorities; and for the sake of comparison have added from calculation their expansion from 62° to 212°, by reducing the former in the proportion of 180 : 150.

TABLE VI.

	Length of Bar.		From 32° to 212°.		From 62° to 212°.	Authorities.
Platinum	.00088420 × 6.5	=	.005747300	=	.004789416	DULONG & PETIT.
Iron .	.00118203 × 6.5	=	.007683195	=	.006402662	DULONG & PETIT.
Copper	.00171821 × 6.5	=	.011168365	=	.009306970	DULONG & PETIT.
Tin .	.00217298 × 6.5	=	.014124370	=	.011770308	LAVOISIER & LAPLACE.
Zinc .	.00294200 × 6.5	=	.019123000	=	.015935833	SMEATON.
Lead .	.00284836 × 6.5	=	.018514340	=	.015428616	LAVOISIER & LAPLACE.
Brass .	.00193000 × 6.5	=	.012545000	=	.010454166	SMEATON.
Gold .	.00146606 × 6.5	=	.009529390	=	.007941158	LAVOISIER & LAPLACE.
Silver .	.00190974 × 6.5	=	.012413310	=	.010344424	LAVOISIER & LAPLACE.

Upon deducting from the amount of these several absolute enpansions the apparent expansions in the black-lead register, we shall obtain the expansion of the latter from 62° to 212°, as derived from the several metals. The results are comprised in the following Table.

TABLE VII.

	Expansion of the Metal Bars.	Expansion of Black-lead Register.	Difference from Mean.
Platinum . .	absolute .00478		
	apparent .00276	=.00202	−.00032
Iron	absolute .00640		
	apparent .00508	=.00132	−.00102
Copper . . .	absolute .00930		
	apparent .00683	=.00247	+.00013
Tin	absolute .01177		
	apparent .00814	=.00363	+.00129
Zinc	absolute .01593		
	apparent .01454	=.00139	−.00095

TABLE VII. (Continued.)

	Expansion of the Metal Bars.	Expansion of Black-lead Register.	Difference from Mean.
Lead	absolute .01542		
	apparent .01223	=.00319	+.00085
Brass	absolute .01045		
	apparent .00799	=.00246	+.00012
Gold	absolute .00794		
	apparent .00552	=.00242	+.00008
Silver	absolute .01034		
	apparent .00814	=.00220	−.00014
		Mean .00234	

In five instances out of these nine, the difference of the expansion of the black-lead from the mean does not exceed $\frac{32}{100,000}$dths of an inch, two being in deficiency, and three in excess: and it is worthy of observation that they are the metals whose dilatations have always been considered the most regular, and concerning which there is the least difference of authorities, viz. gold, silver, platinum, copper, and brass. The greatest difference is in the tin, which amounts to nearly $\frac{13}{10,000}$dths of an inch in excess; and it is more than probable that the absolute expansion of this metal has not hitherto been obtained with sufficient precision, and that it even varies in different states. I shall return to this subject in the second part of this Paper, which I reserve for a future communication; in which I hope to be able to lay before the Society observations and tables of the dilatations of metals to their melting points. It is my intention in this first part to touch no further upon the subject of expansion than is sufficient to establish confidence in the pyrometer as a measure of heat.

Another confirmation of the precision of these observations may be derived by calculating the expansion of the black-lead register for the 150°, from the greater expansion previously determined by the boiling point of mercury for

$$596^\circ : .00831 :: 150^\circ : .00209$$

which only differs $\frac{25}{100,000}$dths of an inch from the above mean.

Exp. 4. It was a principal object to ascertain whether any and what difference existed in the expansion of different specimens of the black-lead earthenware: two or three registers which I had cut out of the same crucible gave me almost identical results by exposure to boiling mercury. I then selected another specimen by a different manufacturer. Its grain was very fine, and its texture more close and compact than the former. It was twice exposed with the platinum bar to boiling mercury. The first time it was boiled for a quarter of an hour, and the arc measured was 1° 45′. The second time the boiling was continued for only ten minutes, and the reading was precisely the same. The expansion was therefore .01526.

Absolute expansion as before	.01947
Apparent expansion	.01526
Expansion of black-lead	.00421

Exp. 5. The same register of the fine-grained black-lead was exposed for a quarter of an hour with the iron bar to boiling mercury: the arc measured on the scale was 2° 49′ = expansion .02457.

Absolute expansion as before	.02914
Apparent expansion	.02457
Expansion of black-lead	.00457

Fine-grained black-lead by platinum . .	.00421
by iron . . .	.00457
Mean .	.00439

The two experiments differing from the mean by less than $\frac{2}{10,000}$dths of an inch. This shows that the fine-grained ware expands less than the coarser, and proves the necessity of ascertaining the expansion of each register for itself by boiling in mercury; at least till some means be taken to insure their uniform composition. Every register should also be marked with a reference to its proper expansion; and I would recommend all those who may use the instrument for delicate researches, to verify this point for themselves; as they may easily do with the apparatus before described.

Exp. 6. The expansion of the last specimen of black-lead ware being nearly the least which has fallen under my observation, I repeated with it the experiment of the dilatation of six of the former list of metals to the boiling point of water; as the accuracy of these observations is a point which it is of the greatest importance to establish.

The subjoined Table contains the results.

Table VIII.

Platinum . .	0° 22′	= .00319	from 60° to 212°
Iron	0 39	= .00566	———
Copper . . .	0 54	= .00785	———
Brass . . .	0 59	= .00857	———
Gold	0 41	= .00595	———
Silver . . .	0 58	= .00843	———

The differences of the observed expansions and the real are also subjoined and ranged by the side of those obtained by the first series of observations.

Table IX.

	Expansion of the Metal Bars.	Expansion of Black-lead. Second Series.	Differ. from Mean.	First Series.	Differ. from Mean.
Platinum	absolute .00478 apparent .00319	=.00159	.00000	. . .00202	−.00032
Iron . .	absolute .00640 apparent .00566	=.00074	−.00085	. . .00132	−.00102
Copper .	absolute .00930 apparent .00785	=.00145	−.00014	. . .00247	+.00013
Brass . .	absolute .01045 apparent .00857	=.00188	+.00029	. . .00246	+.00012
Gold . .	absolute .00794 apparent .00595	=.00199	+.00040	. . .00242	+.00008
Silver .	absolute .01034 apparent .00843	=.00191	+.00032	. . .00220	−.00014
	Mean .	.00159	. . . . . .	.00234	

The agreement of this second series with each other is quite as close as that of the first; and it is worthy of remark, that the greatest variation from the mean is in both cases with the iron in deficiency, and nearly to the same amount of one half. It is not unlikely, therefore, that there may be some error in estimating the absolute dilatation of this metal, which is probably something greater than we have assumed.

If we estimate the expansion for these 150° to the boiling point of water from the result obtained by the boiling of mercury, we shall have the following proportion:— 596° : .00439 : : 150° : .00110

which does not differ quite $\frac{5}{10,000}$dths of an inch from the foregoing mean.

Having thus, I trust satisfactorily, established the accuracy of the pyrometer, and the degree to which confidence may be placed in its indications, I shall conclude this part of my subject with the details of some experiments upon the fusing points of different metals. I shall designate the registers of coarse and fine-grained black-lead respectively by the letters A and B.

Exp. 7. About 30lbs. of the clippings of thin sheet copper were very gradually melted in a crucible in the blast furnace of the Royal Institution. The platinum bar was adjusted in the register B, and when the metal was about half run down, it was placed perpendicularly with the index upwards in the crucible, and held down with a pair of tongs. The crucible was then gradually fed with the clippings till the melted metal covered about two-thirds of the register. In this situation it was kept ten minutes, and when it was lifted out some of the metal remained unmelted. A crust of oxide, mixed with metal, had also affixed itself to the upper part of the black-lead. This was partially dissolved away and loosened by immersing the register with great care, when cold, in a diluted mixture of sulphuric and nitric acids. The whole was thus easily removed, and the black-lead exhibited a perfectly clean surface. The arc measured upon the scale was 5° 49′, denoting an expansion of .0508. The temperature of the laboratory was about 65°.

I am indebted to the kindness of Mr. Mathison for unexceptionable opportunities of taking the melting points of gold and silver at the Royal Mint, who also most obligingly assisted me in the operations. Two new registers were prepared, which I shall designate as II and III: their rates of expansion were not determined till after the experiments.

Exp. 8. The register II was carefully adjusted with the platinum bar.

About 90lbs. of fine gold were weighed, and one of the ingots was cut into ten pieces for the purpose of gradually feeding the crucible, and keeping the temperature down to the true melting point during the observation. The remainder was melted in a black-lead crucible in a wind-furnace. When just fused, one of the pieces was thrown in, and the melted metal immediately congealed upon the surface. The register, which had been slowly heated in another crucible to a dull red, was then taken up with a pair of tongs and plunged perpendicularly into the gold about two-thirds of its height. In this situation it was kept ten minutes, and during the time two more lumps of the metal were thrown in. It was then carefully lifted out and set apart to cool. Its surface was perfectly clean, only a few small globules adhering to it, which were easily removed. I may here remark that stirrers of the black-lead earthenware are constantly used at the Mint for agitating the melted gold. The arc measured from this experiment was 6° 10′, equivalent to an expansion of .0537. Temperature of the air about 65°.

Exp. 9. The register III was fitted with the iron bar, and also heated to a dull red. The temperature of the melted gold was prevented from rising by constant feeding with the pieces; the crucible being never left without some portion unmelted. It was then plunged beneath the surface of the metal as in the preceding experiment, and held in that situation for ten minutes. The arc measured was 9° 2′, indicating an expansion of .0787.

Exp. 10. The rates of expansion of the two last registers were determined by boiling them for ten minutes each in mercury. The results were as follow:

	Arc.		Expansion.
II with the platinum bar	1°.50	=	.0159
III with the iron bar	2°.38	=	.0229

Exp. 11. About 50lbs. of pure silver were melted in a black-lead pot: a little scum floated upon the surface, which appeared at first like drops of oil upon a basin of water. I was afterwards informed that the metal had been refined with nitre, and the dross was owing to the action of a little remaining potash upon the crucible. Two registers had been prepared for the platinum and iron bars; but the observations were lost from the same action upon their substance. They were so deeply corroded in a line which corresponded with the level of the fluid metal, as to render it impossible to apply the scale, with any certainty, to their surfaces.

Exp. 12. Two new registers were selected, whose rate of expansion was found by boiling in mercury to be equal; the arc in both cases being with the platinum bar 1° 20′ = .0116. They were marked IV and V.

IV was adjusted with the platinum bar. An ingot of silver, which had been refined by cupellation, weighing about 35lbs., was placed in a black-lead crucible in a wind-furnace. When somewhat more than three-fourths were melted, the register, previously heated to a dull red, was plunged into it as before, and held down for ten minutes. When lifted out its surface was found perfectly good, and the few adhering globules of metal were easily removed. When cool the scale was applied, and the arc found to be 4° 10′ = expansion .0363. Temperature of the air 65°.

Exp. 13. The iron bar was placed in the register V, and having been previously heated was plunged into the same pot of metal. The silver at first set about the black-lead and adhered to it in a large lump. At the expiration of ten minutes this was just melted off, and the instrument was raised out of the crucible in a perfectly clean state. When cool the arc measured was 7° 24′ = expansion .0645.

Exp. 14. I made several attempts at the Royal Institution to ascertain the melting point of cast iron; but owing to the large quantity of the metal necessary; to the difficulty of keeping the temperature steady by constant feeding; and to the failure of crucibles,—I did not succeed. I am under obligation to Mr. Parker of Argyle Street, for the readiness with which he afforded me every facility of performing the experiment at his foundry.

I selected a new register for the occasion, which was marked I. Its rate of expansion was not determined till after the experiment. A crucible was prepared capable of containing about 35lbs. of the metal. It was filled with pieces of the best grey iron, and placed in a powerful wind-furnace, which admitted of the operator standing immediately above the crucible with complete command over it. When the metal was melted, the crucible was lifted from the furnace, and the dross skimmed off its surface. It was then replaced: a lump of the same iron was thrown into it, and the register, previously heated red hot, was immersed in the fluid to about the same depth as in the former experiments. It was kept in this situation by means of a pair of tongs for ten minutes, and afterwards gently lifted out and laid upon hot sand. A thin scale of iron adhered to the black lead, which when cold was easily removed, and retained the form of the bar like a sharp cast, and left the surface of the register perfectly clean and bright. The arc measured after the experiment was 6° 16′ = expansion .0546. Part of the lump of metal remained unmelted.

Exp. 15. Another register, which had been prepared with the iron bar, was immediately immersed in the fluid metal. The fire, however, had been allowed to fall, and the iron almost instantly congealed; and in attempting to lift the register out, it was found to be set fast and broke. The experiment was so far instructive, that it proved how nearly the exact melting point had been attained in the preceding experiment. The iron bar was removed uninjured.

Exp. 16. The register I with the platinum bar was boiled in mercury for ten minutes: the arc afterwards measured was 1° 20′ = expansion .0116.

Exp. 17. About 30lbs. of zinc were carefully melted in a crucible set in a common fire, assisted with the bellows. The register A was prepared with the iron bar and held down in the metal, which was supplied from time to time so as to insure its very gradual fusion, and some portion always remaining in the solid state. In ten minutes time it was removed, and when cold the arc measured was 2° 45′, equivalent to an expansion of .0239.

A dry stick of deal plunged into the melted metal for a few seconds caused a violent ebullition, and was deeply charred. The zinc in this state did not appear red in the light.

Exp. 18. About 12lbs. of zinc were melted in a smaller crucible: the register B prepared with the iron bar was immersed in it; but instead of being gradually supplied, the heat was allowed to increase after fusion till it began to burn: at this point there was an evident blush of red upon its surface. The arc measured upon this occasion was 4° 7′ = expansion .0358.

I shall now collect together the results of the preceding experiments, for the purpose of showing what conclusions may be derived from them with regard to the degrees of temperature which they indicate when referred to the common thermometric scale. I shall make the calculations first upon the supposition that equal amounts of expansion denote equal increments of temperature; and I shall thus be enabled to compare the present series with that which I formerly obtained with my first pyrometer, and to offer a few remarks upon the differences of the two.

I shall adopt the corrected temperature of 662° (350° Centigrade) for the boiling point of mercury, as proposed by MM. Dulong and Petit; which agrees very closely with the amount employed in my first calculations, and which, deducting 62° for the mean temperature at which my experiments commenced, gives 600° for the interval for which the several expansions were determined.

The first column of the following Table refers to the number of the experiment; the second to the mark of the register and the bar which was employed; and the third to the amount of expansion in the same, occasioned by boiling mercury or 600° of temperature upon Fahrenheit's scale. The fourth column exhibits the arc measured upon the scale; and the fifth the equivalent expansion. The sixth contains the corresponding temperature; the seventh records the state of the metal, which was the object of the experiment; and in the eighth I have recapitulated the corresponding results of my former Essay.

Table X.

No. of Experiment.	Mark of Register and Bar.	Expansion for 600°.	Arc measured on Scale.	Expansion.	Temperature.		Metals observed.	Temperature by former Pyrometer.
7	B Platinum	.0152	5° 49′	.0508	2005+65	2070°	Copper, fusing point.	2548°
8	II Platinum	.0159	6 10	.0537	2026+65	2091	Gold, fusing point.	2590
9	III Iron	.0229	9 2	.0787	2061+65	2126	ditto ditto	
12	IV Platinum	.0116	4 10	.0363	1877+65	1942	Silver, fusing point.	2233
13	V Iron	.0203	7 24	.0645	1906+65	1971	ditto ditto	
14	I Platinum	.0116	6 16	.0546	2824+65	2889	Iron, fusing point.	3479
17	A Iron	.0203	2 45	.0239	708+65	773	Zinc, fusing point.	648
18	B Iron	.0245	4 7	.0358	876+65	941	Zinc, inflaming.	

The most remarkable fact displayed by the preceding comparison is the beautiful accordance of the results obtained from two metals whose expansions are so different as those of platinum and iron. The temperature indicated by the latter exceeds that by the former in the instance of the fusing point of gold 35°, and in that of silver only 29°; and this excess is in accordance with the conclusion of MM. DULONG and PETIT, exhibited in Table II., that the expansion of iron increases in the higher degrees in a greater proportion than that of platinum.

The discrepancy between the temperatures derived from the observations with my first pyrometer and the present are considerable, but may be sufficiently accounted for by the differences in the circumstances of the experiments, without imputing inaccuracy to the instrument. In the paper to which I have before alluded, I stated that "I did not offer the results as positive and accurate determinations of the different degrees, but only as nearer approximations than any that had yet been furnished from actual observation. The only method which I had it in my power to adopt for the purpose, I do not consider to be susceptible of absolute accuracy. The arrangement made consisted of a muffle of black-lead placed in an excellent draught-furnace. This muffle was furnished with a door, through a round hole in which the stem of the pyrometer was passed up to the shoulder. Another door, which could be stopped at pleasure, admitted a full view of the interior. The metal to be tried was placed in a small black-lead receptacle, of the same thickness as the pyrometer tube, in the middle of the muffle. Now it is evident that the pyrometer so situated would indicate the mean heat of the whole of the muffle; which heat might, and did, vary in different parts. Of two pieces of silver of the same size placed within an inch of each other, one fused some time before the other." I also suggested that "means might be contrived to surround the instrument with the metal in a state of fusion; but that it required particular opportunities, which it was to be hoped that those would avail themselves of who had them in their power."

That the latter method is the only one which can admit of accuracy will be evident from a few reflections. Setting aside the inequality of the heat of different parts of the same heated muffle, which however is a consideration of the utmost importance, it is obvious that its temperature must considerably exceed the true melting point of the metal exposed to its influence. Just as a piece of ice would never melt in a chamber of the temperature of 32°, but would require a considerably higher heat in proportion to its mass to supply the caloric which becomes latent during the process,—a mass of iron would exhibit but little signs of liquidity till subjected to a heat much above its true point of fusion. When once in a liquid state, both would rapidly rise to the temperature of the medium to which they were exposed. When metals are melted for the purposes of the arts, they of course require to be heated very far beyond their fusing points, that they may flow into the minutest fissures of the moulds in which they are cast, notwithstanding the cooling influences to which they are suddenly exposed. In some of the finer castings of brass, the perfection of the work depends upon the intensity to which the metal is heated, which in some cases is urged even beyond the melting point of iron. With a fire whose power in all cases must so greatly exceed the temperature required, it is necessary to bestow great care in supplying the metal gradually, as we have before described; as it is inconceivable with what rapidity it rises after the solid pieces are completely dissolved. Evidence of the same fact may be derived from the experiments of MM. Clement and Desormes, which I have before quoted. They calculated the heat of melted iron at 3988°, and of iron just on the point of melting at 3164°,—a difference of 800°. And it is clear from the circumstances of the experiment, that the former must have considerably exceeded the true melting point, or it never could have been transported in a liquid state from the crucible to the apparatus in which the water was heated or the ice melted. It is probable that the process which they employed, of the calorimeter, was not susceptible of great accuracy; but the discrepancy of the results from those which I obtained from the metal in analogous circumstances is not great.

Iron just melting	3164°	by the former
	2889	by Pyrometer
	——	275° difference.
Iron melted at a high heat .	3988	by the former
	3479	by Pyrometer
	——	509° difference.

A similar excess also appears in their determination of the heat of melted copper, and obviously admits of the same explanation.

After performing these experiments upon the melting points of the metals, I was desirous of ascertaining the effects of the most intense heat which it was possible to produce in a furnace; and to measure the utmost limits of expansion in a platinum bar. For this purpose I made use of an excellent wind-furnace in the Royal Institution, in which upon former occasions hob-nails had been completely fused into a button.

Exp. 19. The register I, which had not been the least injured by the previous experiments, was fitted with a new bar of platinum, which had been drawn as a wire, was $\frac{5}{20}$ths of an inch in diameter and very ductile. The iron bar was also adjusted to a new register, and both were placed upright in a well heated crucible. About half an inch of powdered charcoal was strewed upon the bottom to prevent any adhesion; and two soft iron nails, and a piece of unglazed Wedgwood's porcelain, were thrown in for the purpose of affording some indication of the degree of heat attained. The crucible was then set in the furnace, another smaller crucible inverted upon it, covered with coke, and the heat urged to the utmost for two hours. The fire was suffered to burn out, and the crucible with its contents removed for examination. It was sound, but the luting had been completely fused. The nails were found melted into two complete buttons, and the porcelain was partially fused upon the surface.

The register I appeared to be uninjured, but the platinum ring and wedge were loose, evidently from a contraction having taken place in the substance of the black-lead. This was no doubt owing to the heat having exceeded that at which it had been originally baked. The amount of expansion consequently could not be measured. The platinum ring, both of this and the other register, exhibited a remarkable change of texture; they had become very rough and crystalline, and were perfectly brittle, breaking easily between the fingers. The platinum bar also, which there was some difficulty in removing from the cavity, presented a very extraordinary appearance. It was apparently embossed with crystals, and was evidently larger at the lower end than at the top: it was also something contracted in length. Upon examination with a lens no regular facets could be detected, but it had the appearance of a bar constructed of plates of native platinum loosely welded together.

The register which contained the iron bar was considerably bent, and had several transverse clefts in its substance, owing possibly to its having become inclined in the crucible. Partial fusion had taken place upon the surface of the bar, which had run down and formed a knot at its lower extremity. About an inch of the same end was found to have been converted into steel, but all the rest retained the character of soft iron.

Exp. 20. I repeated the last experiment with the same platinum bar in the register I. The arrangement was precisely the same, with the exception of the second register with the iron bar, and the fire was maintained with equal intensity for an equal time.

The iron nails were found perfectly melted, and the porcelain superficially fused as before. The ring and wedge, however, were fixed in their places, and the index undisturbed, but the measure was unfortunately lost from an accident. The texture of the platinum ring was changed, as in the previous experiment, and the bar tightly fixed in the cavity. By frequent gentle concussions it was removed without injury to the black-lead, which had some slight marks of fusion upon its surface, but was in a perfectly good condition. The bar was in a still rougher state than before, highly crystalline, and exhibited several large longitudinal clefts in its substance. It was found, by measurement with callipers, to be $\frac{1}{20}$th of an inch larger in diameter at its lower than at its upper end, and seemed to be approaching a state of complete disintegration. It was, however, perfectly hard and inflexible. My intention was to have again exposed it for several hours to the same degree of heat, with the expectation that the disintegration would have been complete, and that it would actually have fallen in pieces during the operation: in the mean time I chanced to make it red hot upon a common charcoal fire; and upon attempting to lay hold of it with a pair of tongs the two ends dropped off, and I only withdrew the small portion which I had grasped, and which was flattened and fractured by even this slight compression. The two ends were afterwards carefully, but with difficulty, raised from the fire, and when cold were perfectly hard and inflexible. I again heated a portion of the bar to a dull red, and it crumbled to powder from a slight blow with a hammer.

Exp. 21. It being a point of the greatest interest to ascertain the maximum of expansion which took place in the platinum previous to this remarkable change of structure, I adjusted the original platinum bar, with which the greater part of my experiments had been made, and which presented a perfectly smooth surface, and was very soft and ductile, in the register I. A crucible was placed in the same wind-furnace, containing only a little charcoal powder, with the iron nail and fragments of porcelain as test pieces. The fire was urged to the utmost; and when it had been continued two hours the cover was removed, and the register, previously made red hot, was carefully introduced, the cover replaced, and the ignited fuel heaped upon it. At the expiration of a quarter of an hour it was lifted out and cautiously cooled. An excellent measure was obtained, and the arc determined to be 7° 24′ = expansion .0645.

The test pieces were found in the same state as in the previous experiments. The platinum bar was loose in the cavity, and had not altered its form; but its surface had assumed a slightly crystalline texture, and it had become very hard and inflexible.

The expansion registered would, upon the hypotheses before assumed of equal amounts of expansion denoting equal increments of temperature, indicate a heat of 3336°; or, adding the initial temperature 65°, = 3401°. But it must be remembered that this is probably rather the temperature at which the change in the structure of the platinum took place, than the utmost heat of the furnace. The latter may possibly exceed the degree at which the expansion of the metal ceases, and at which its particles evidently form a new arrangement; but this point cannot at present be determined. The coincidence of this result with that obtained in the former series of my experiments, is very remarkable. The temperature at which I obtained the fusion of cast iron at that time was calculated at 3479°, and was produced by the utmost energy of an excellent wind-furnace; and this, it will be observed, is within 80° of the present maximum.

Exp. 22. Being desirous of ascertaining whether the register and platinum bar had undergone any change in their rates of expansion by the intense heat to which they had been exposed, I again adjusted the latter in the register I, which had now been once immersed in melted iron, and three times subjected to the action of the wind-furnace, and boiled them for ten minutes in mercury. The arc measured was 1° 19′ = expansion .01148: the difference of 1′ may safely be ascribed to the uncertainty of the reading.

The temperatures thus determined will require correction, if we adopt the conclusion derived from the experiments of MM. Dulong and Petit,—that the dilatability of solids, referred to an air-thermometer, increases with the heat.

The amount of this correction will be as the rate of increase; and according to those gentlemen is 11°.6 of the Centigrade thermometer, or 20°.8 Fahr. from 32° to 572°, or the calculated temperature is to the true as .00091827 : .00088420. Supposing the increase of dilatability to continue the same for equal intervals of temperature, which however has not yet been proved, the following Table will exhibit the corrected temperatures derived from the preceding experiments with the platinum bar.

Table XI.

	Observed.	Corrected.
Melting point of Silver	1942	1873[*]
———— Copper	2070	1996
———— Gold	2091	2016
———— Iron	2889	2786
Temperature of the maximum of expansion of Platinum	3401	3280

If we reason in the same way from the increase of the dilatation of iron, as laid down by the same authors, the discrepancy between the temperature derived from the platinum and iron is very considerable; the melting point of silver coming out 1682°, and that of gold 1815° by the latter: but I conceive that the determination of this point in the iron is open to objections which do not apply to the platinum, and my suspicion is confirmed by the anomalous expansion of the iron exhibited in Tables V. and IX., and to which I shall recur upon a future occasion.

The general utility of the pyrometer, however, will in no way be affected by any uncertainty in these corrections. The indications which it is capable of affording will always be positive determinations, which it will be easy to modify by calculation, as our theories may improve. For all common purposes (and I must own that I look forward with hope that this instrument will prove eminently useful in many of the common processes of the arts) it will not even be necessary to note the expansion indicated by the arc measured; but each minute of the degree may at once be valued in degrees of Fahrenheit's scale at the time of taking its rate of expansion by the boiling of mercury: and a

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[*] Mr. Prinsep, from a laborious series of experiments upon the expansion of air confined in a bulb of gold, determines the melting point of silver to be 1830°.—Phil. Trans. 1828. p. 94.

Table of such values should be furnished for each register by the maker of the instrument. The following, for example, would be the proper Table for register I, which has been so often referred to, in which the arc for the boiling of mercury or 600° (without adding the initial temperature) was 1° 20′.

TABLE XII.

°	′		Expansion.		Temperature.
1	0	=	.00872	=	450°
0	30	=	.00436	=	225
0	20	=	.00290	=	150
0	15	=	.00218	=	112
0	10	=	.00145	=	75
0	5	=	.00072	=	37
0	2	=	.00029	=	15
0	1	=	.00014	=	7.5

With such a Table an intelligent workman could employ the instrument without any material error. Those who might object to the expense of a platinum bar may substitute an iron one for ordinary purposes, and the cost of the black-lead register can never be an obstacle to its general use. Other substances might obviously be employed in its construction, but the facility with which it can be worked, its small expansion, its infusibility, and the impunity with which it bears the most sudden changes of temperature (as when red hot it may even be quenched in water without injury), will probably always give the black-lead ware the preference. The only precaution to be taken with it is to expose it previously, out of the contact of air, to a heat at least as great as that in which it is intended to employ the instrument.

XXI. *On the Phenomena and Laws of Elliptic Polarization, as exhibited in the Action of Metals upon Light.* *By* DAVID BREWSTER, *LL.D. F.R.S. Lond. & Edin.*

Read April 22, 1830.

FROM the first dawn of the science of polarization, the action of metals upon light has presented a troublesome anomaly. MALUS at first announced that they produced no effect whatever; but by employing a different method of observation, I found that the light reflected by metallic surfaces was so far modified as to produce, when transmitted through thin crystallized plates, the complementary colours of polarized light. From a second series of experiments made previous to mine, MALUS came to the conclusion, that the difference between transparent and metallic bodies consisted in this: that the former refract all the light which they polarize in one plane, and reflect all the light which they polarize in another; while metallic bodies reflect what they polarize in both planes.

Having discovered the property of transparent bodies to polarize light by successive reflexions at angles at which a single reflexion produced no perceptible effect*, I resolved to apply this method of examination to metals; and on the 7th of February 1815, when I first made the experiment, I discovered the curious property possessed by silver and gold of dividing a polarized ray into complementary colours by successive reflexions. As this subject promised to open a wide field of inquiry, I prepared for the ardent prosecution of it with all the metallic bodies which could be procured; but the pressure of professional business prevented me for about a month from doing any thing very effectual.

On the 6th of March 1815, I received a letter from M. BIOT, requesting some information on a matter of business; and in answering this letter on the same day, I communicated to him an account of the discovery above mentioned*. Immediately after this I received the most perfect plates of silver, one pair polished by friction, and another by hammering; two pair of plates of gold, one of jewellers', and another of fine gold; with plates of steel, platinum, palladium, copper, brass, and speculum metal; and with their help I obtained the general result, that a single reflexion from a metallic surface produces the same effect upon polarized light as a certain thickness of a crystallized body, with many other results, which it is unnecessary here to indicate.

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* Phil. Trans. 1815, p. 142.

As soon as M. Biot had received notice of my discovery, he seems to have devoted himself to the same inquiry; and with all the leisure of an Academician, and the splendid apparatus presented to him by the Institute, he obtained many of the results at which I had arrived, and others to which I have no claim; and on the 29th of March he transmitted to me, through Dr. Wollaston, an open letter containing an abstract of his experiments, and expressing the hope that they would be of use to me in my researches.

Although this expression led me to believe that I should enjoy the privilege of publishing the first account of my own discovery, yet I took the precaution of having all my papers on the subject signed by the Treasurer of the Royal Society of Edinburgh, and I proceeded with new zeal in the further examination of the subject. I soon learned, however, from M. Biot, that he meant to treat the subject in his Traité de Physique; and though I remonstrated against this as a breach of courtesy, I had the mortification to see the discovery, to which I perhaps attached too much importance, published for the first time in a foreign work.

I trust the Society will excuse these details as a necessary apology for having so long delayed to fulfil the promise, more than once made in their Transactions, to communicate to them an account of these experiments†. The reasons which I have assigned were subsequently strengthened by new inquiries which at first threw great doubts over the views which M. Biot and I had taken of the subject, and finally convinced me of the rashness of our generalizations. The study of M. Fresnel's fine discoveries respecting circular polarization enabled me to advance still further in the inquiry; and having more recently resumed the investigation, I trust I shall now be able to present to the Society a satisfactory analysis of the singular phenomena exhibited in the action of metals upon light.

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* It is related in the History of Optics, Edinburgh Encyclopædia, vol. xv. p. 493, note, that I communicated this discovery to M. Biot on the day on which it was made:—this is a mistake, as it was done a month afterwards.

† In a letter to Sir Joseph Banks, dated July 28th, 1815, I communicated an abstract of these and other experiments, with a request that he would permit the MS. to remain in his possession, as an evidence of my claims. Sir Joseph complied with this request: but nearly two years afterwards, happening to see the MS., he thought that it had been intended for publication, and laid it before the Royal Society without my knowledge. It was accordingly read on the 23rd of January 1817, under the title of Abstract of Experiments on Light, and ordered to be printed. When the proof-sheet was sent me for correction, I requested the paper to be cancelled, as it was not intended for publication.

Sect. I. *On the action of metals upon common light.*

When we analyse with a rhomb of calcareous spar a ray of common light, reflected at different angles from a metallic surface, there will be observed in one of the images a defalcation of light, as if a portion of the incident ray was polarized in the plane of reflexion. This effect will be still more distinctly seen if we examine the system of polarized rings formed round the axes of crystals by means of the light reflected from metals. If the light had suffered no modification by reflexion, or if the metal reflected in equal quantities the light polarized in opposite planes, the rings would not be visible at all; but it will be found that they are easily seen in the light reflected by all metals. They are most distinctly visible at an incidence of about 74°, at an average, and become fainter and fainter as the incidence exceeds or falls below that angle. They appear best defined in light reflected from galæna and metallic lead, and with least distinctness in light reflected from silver and gold, as shown in the following Table, in which the metals are arranged in the order in which they exhibit the rings most brightly, and consequently in the order in which they polarize the greatest quantity of light in the plane of reflexion.

Galæna,
Lead,
Gray cobalt,
Arsenical cobalt,
Iron pyrites,
Antimony,
Steel,
Zinc,
Speculum metal,
Platinum,
Bismuth,
Mercury,
Copper,
Tin plate,
Brass,
Grain tin,
Jewellers' gold,
Fine gold,
Common silver,
Pure silver.

If we now take two plates of each of these metals and examine the light which has undergone more than one reflexion, we shall find that the quantity of light which each polarizes in the plane of reflexion increases with every reflexion, and that in several of them the whole incident pencil is completely polarized.

When the luminous object is a wax-candle placed at the distance of ten feet, eight reflexions from a plate of steel at angles between 60° and 80° polarize the whole of the light, while at angles above 80° and below 60° a greater number of reflexions is required. With galæna, lead, cobalt, and antimony, a much smaller number of reflexions polarizes the whole pencil; whereas with pure and highly polished silver a very great number is necessary: the light reflected from the silver becomes redder and redder, indicating an increasing absorption or dispersion of the less refrangible rays.

By the use of common light it would be in vain to attempt to discover the law according to which the polarization of the incident pencil is effected in different metals; but by another mode of analysis we shall be led to the mathematical law for computing the exact proportion of the reflected pencil which is polarized at certain angles when the number of reflexions exceeds one.

SECT. II. *On the action of metals upon polarized light.*

If a pencil of polarized light is received on a polished metallic surface placed so as to have a rotatory motion round the polarized ray, the reflected light will receive no modification (excepting what arises from its property of apparently polarizing a portion of light in the plane of reflexion) when the plane of incidence is inclined 0°, 90°, 180°, and 270° to the plane of primitive polarization; but in every other azimuth of the plane of incidence the reflected pencil will be found to have suffered a remarkable change, which gradually increases as the azimuth of that plane varies from 0° to 45°, from 90° to 135°, from 180° to 225°, and from 270° to 315°. At the azimuths of 45°, 135°, 225°, and 315°, the effect is a maximum, and it gradually diminishes from 45° to 90°, from 135° to 180°, from 225° to 270°, and from 315° to 360°.

In order to investigate the nature of this change, we shall suppose the plane of reflexion from the metal to be inclined −45°, or to the left of the plane of primitive polarization. In this position let a plate of highly polished steel receive the polarized ray of ordinary intensity. At 89°, 88°, and 87° of incidence, almost no change is produced upon it by the action of the metal. We can easily see that the plane of polarization of the ray is turned from right to left, exactly as it would be by a transparent surface. In like manner at all angles of incidence from 0° to about 40° no decided effect is produced, except the change in the plane of polarization. At angles less than 87° the change begins to appear, reaches its maximum at about 75°, and diminishes gradually to 40°. By means of the analysing rhomb, it is easily seen that a great portion of the original pencil has had its plane of polarization changed from +45° to 0°, as the incidence diminishes from 75° to 0°. If, indeed, we measure the rotation of the principal section of the rhomb when the extraordinary pencil is a minimum at different angles of incidence, we shall find it to correspond with $45° - \varphi$, φ being calculated from the formula $\tan \varphi = \frac{\cos(i + i')}{\cos(i - i')}$ in which $\frac{\sin i}{\sin i'} = 3.732$, the index of refraction for steel. The value of φ will be found to be nearly the same at 87° and 40°, which shows why at these two angles the change under our consideration is just beginning to appear with light of ordinary intensity.

The physical effect of the metallic surface being a maximum at 75°, we shall now examine the character of the pencil reflected at that angle.

1. The pencil thus reflected is not polarized light, because it does not vanish during the revolution of the analysing rhomb.

2. It is not common light, because when we reflect it a second time at 75° from another steel surface, it is restored to light polarized in one plane.

In order to discover its nature, let it be transmitted along the axis of calcareous spar. The system of rings is changed into the form shown in Plate X. fig. 1. almost exactly in the same manner as if a thin film of a crystallized body which polarizes the pale blue of the first order had crossed the system. If we substitute for the calcareous spar films of sulphate of lime, which give different tints, we shall find that these tints are increased according as the metallic action coincides with, or opposes that of the crystal.

On the authority of this experiment I was led to believe that metals acted upon light like crystallized plates; and when I found that the colours were not only better developed, but more pure after successive reflexions, it was a natural though a rash generalization, to conclude as I did, and as M. Biot did after me, that each successive reflexion corresponded to an additional thickness of the crystallized film.

In order to show the incorrectness of this deduction, let a ray polarized +45° be reflected twice from steel at angles of 75°. In this case the effect of the second reflexion should be to double the tint produced by the first, if the tints are those of crystallized plates. The result, however, is, that the whole of the light is polarized in one plane, in place of consisting of two pencils polarized in opposite planes. M. Biot got over this embarrassment by regarding the tint produced by two reflexions as the white of the first order, which, in consequence of its complementary tint being black, is the only one where the light is all polarized in one plane: but had he examined the light reflected four times, six times, or eight times at 75°, he would have still found it all polarized in one plane, a result entirely incompatible with the supposition of the tints rising with the number of reflexions. That the tint is not the white of the first order may be more easily proved by making it pass along the axis of calcareous spar; for we shall find that in place of producing an increment of tint, the effect of the second reflexion has been to destroy entirely the effect of the first, and to restore the ray to common polarized light. All this will appear by the perfection of the system of rings seen through the spar. If we examine in a similar manner the light which has undergone any number of reflexions between the plates, we shall easily ascertain that the effect never exceeds that of a quarter of a tint in Newton's scale.

Having thus ascertained that light polarized + 45°, and reflected at the maximum polarizing angle of metals, is neither common light nor polarized light, nor light constituted like that which passes through thin crystallized plates, I conceived the idea of its resembling circularly polarized light—that remarkable species of light which comports itself as if it revolved with a circular motion during its transmission through particular media.

According to Fresnel's beautiful discovery, a ray of light polarized +45° is circularly polarized when it has suffered two total reflexions from glass at an angle of 54½°; and when such a ray is made to suffer other two reflexions at the same angle, it is restored to the state of light polarized −45° to the plane of reflexion, whatever be the azimuth of the second plane of reflexion in relation to the first. In like manner I shall proceed to show that a ray of light polarized +45°, and reflected once at the maximum polarizing angle from metals and certain metallic ores, has an analogous polarization, viz. a polarization hitherto unrecognized, and intermediate between circular and rectilineal polarization.

Let the ray polarized +45° be reflected at 75° from steel, and let a second plate of steel be made to turn round the ray thus reflected. At the azimuths of 45°, 135°, 225°, and 315°, with the plane of primitive polarization, that is, when the planes of the two reflexions are either coincident or rectangular, the first reflected ray will be restored to polarized light at an incidence of 75°. At azimuths of 0° and 180° the restoration will be effected at an incidence of 80°, while at azimuths of 90° and 270° it will take place at an incidence of 70°, and at intermediate azimuths it will take place at intermediate incidences. Hence the ray of light reflected from steel, though it has the general properties of a circularly polarized ray, differs from it in this remarkable particular, that it requires different angles of incidence in different azimuths to restore the polarized light.

In circular polarization, as we have seen, the ray has the same properties in all its sides; and the angles of reflexion at which it is restored to polarized light in different azimuths are all equal, like the radii of a circle described round the ray. Hence, without any theoretical reference, the term circular polarization is from this and other facts experimentally appropriate. In like manner, without referring to the theoretical existence of elliptic vibrations produced by the interference of two rectilineal vibrations of unequal amplitudes, we may give to the new phenomena the name of elliptic polarization, because the angles of reflexion at which this kind of light is restored to polarized light may be represented by the variable radius of an ellipse.

In circular polarization the restored ray has its plane of polarization always inclined −45° to the plane of the second system of reflexions. In elliptic polarization the difference is remarkable. The inclination of the plane of the restored pencil is likewise —, but always less than 45°, as will appear from the following Table, which contains the greater number of metallic bodies.

Names of Metals.	Angles of Restoration.		Names of Metals.	Angles of Restoration.	
	°	′		°	′
Total reflexions	45	0	Bismuth	21	0
Pure silver	39	48	Speculum metal	21	0
Common silver	36	0	Zinc	19	10
Fine gold	35	0	Steel	17	0
Jewellers' gold	33	0	Iron pyrites	14	0
Grain tin	33	0	Antimony	16	15
Brass	32	0	Arsenical cobalt	13	0
Tin plate	31	0	Cobalt	12	30
Copper	29	0	Lead	11	0
Mercury	26	0	Galæna	2	0
Platina	22	0	Specular iron	0	0

The bodies in this Table are obviously in the inverse order according to which they polarize most light in the plane of reflexion.

I have inserted at the top of the Table the inclination of the restored pencil in total reflexions, which is 45°; and at the bottom, that of specular iron, which is 0°; in order to show the transition from elliptic polarization to circular polarization on the one hand, and to rectilineal polarization on the other.

In these experiments the primitive ray was polarized +45° to the plane of reflexion; but when this angle diminishes, the plane of the restored ray approaches to the plane of reflexion, and ultimately coincides with it at 0°; and when this angle increases, the plane of the restored ray recedes from the plane of reflexion, and the two planes form an angle of 180° when the other angle becomes 90°.

The following experiments were made with plates of pure silver, in which the inclination φ was 39° 48′, when the inclination x of the plane of polarization was 45°.

Inclination x of the Plane of primitive Polarization to Plane of Reflexion.	Observed Inclination of the restored Ray to the Plane of Reflexion or φ.	Inclination φ calculated by the Formula.
°	° ′	° ′
+90	−90 0	−90 0
85	84 36	84 0
75	74 10	72 10
65	63 51	60 46
55	52 18	49 57
45	$\theta =$ 39 48	39 48
35	32 23	30 28
25	23 10	21 14
15	13 16	12 35
5	4 40	4 10
0	0 0	0 0

Calling θ the inclination or value of φ at 45°, we may represent these observations by the formula, $\tan \varphi = \tan \theta \tan x$, and the actual change of the plane of polarization, or R, will be $R = x + \varphi$.

When φ is given, $\tan x = \frac{\tan \phi}{\tan \theta}$, and when $\varphi = 45°$, and consequently $\tan \varphi = 1$, we have, $\cot x = \tan \theta$, and $x = 90° - \theta$.

Since light polarized +45° is elliptically polarized by one reflexion from steel at 75°, and is restored to light polarized −17° by a second reflexion at 75°, it is clear that a third reflexion at 75° will again polarize it elliptically, while a fourth reflexion at 75° will again restore it to light polarized $+\varphi$, φ being a quantity less than 17°, and given by the preceding formula. The same effects will be reproduced with different numbers of reflexions, as in the following Table.

No. of Reflexions from Steel at 75° of Incidence.	State of the Light Reflected.	Inclination of the Plane of Polarization. Observed.	Inclination of the Plane of Polarization. Calculated.
1	Elliptically polarized.	° ′	°
2	Restored to light polarized	−17 0	−17

No. of Reflexions from Steel at 75° of Incidence.	State of the Light Reflected.	Inclination of the Plane of Polarization.	
		Observed.	Calculated.
3 . .	Elliptically polarized.		
4 . .	Restored to light polarized . .	+5° 10′ . .	+5° 22′
5 . .	Elliptically polarized.		
6 . .	Restored to light polarized . .	−2 0 . .	−1 38
7 . .	Elliptically polarized.		
8 . .	Restored to light polarized . .	0 0 . .	+0 30
9 . .	Elliptically polarized.		
10 . .	Restored to light polarized . . .	0 0 . .	−0 9
11 . .	Elliptically polarized.		
12 . .	Restored to light polarized . . .	0 0 . .	+0 3

Hence it follows, that at every odd number of reflexions at the maximum polarizing angle the light is elliptically polarized, and at every even number it is restored to a single plane of polarization. In circular polarization the inclination φ of this plane is always $\mp 45°$, even after fifty reflexions, as I have ascertained by direct experiment; but in elliptical polarization the inclination diminishes at every restoration; and in the case of steel it is reduced to near 0° after eight reflexions, when the light is all polarized in the plane of reflexion; that is, the elliptic polarization gradually diminishes and terminates in rectilineal polarization.

The value of φ, as given in the preceding Table, and consequently the number of reflexions when it approaches to 0°, may be deduced from the formula,

$$\tan \varphi = \tan \theta \,.\, \tan x.$$

After the first reflexion $x = +45°$, and φ, or the inclination of the plane of the ray as restored by the second reflexion, is $= -17°$, as given by experiment. Hence the light which suffers the third reflexion, and is thereby elliptically polarized, is not, as originally, polarized $+45°$, but only $-17°$; and consequently, when it is restored after the fourth reflexion, the value of φ must be such as corresponds to an equality in the values of x and θ, both of them being $= 17°$. Hence the formula becomes,

$$\tan \varphi = \tan^2 x, \text{ or } \tan \varphi = \tan^n x;$$

n being the number of pairs of reflexions, or half the number of reflexions which the restored ray has undergone. In this way the last column of the preceding Table has been calculated. The same formula represents also, as it should do, the phenomena at the limits of elliptic polarization. In the case of circular polarization, where the plane of polarization of the restored ray is 45°, we have,

$$x = 45°, \tan x = 1, \text{ and } \tan \varphi = \tan^n x = 1, \text{ or } \varphi = 45°$$

after any number of reflexions however great. In like manner, in rectilineal polarization, where $x = 0°$, we have $\varphi = 0°$, that is, the ray is polarized in the plane of reflexion.

The above formula is suited to any series of reflexions at any angle when the value of φ for the first term of the series is known. The value of φ for two reflexions, the first term of the principal series, can be determined only by experiment, and has been given in a former Table for several metals; but we may determine from it the value of φ for the first term of any other series, provided it is an even number, in the following manner. Making $x =$ the inclination for two reflexions at the maximum polarizing angle, and φ the value of x at any number of reflexions $2n$, we shall have,

$$\tan \varphi = \frac{\tan x + \tan^n x}{2}; \qquad \text{(A)}$$

where $\tan^n x$ is the value of φ at the maximum polarizing angle for $2n$ reflexions; but as no odd number can occur in the principal series, the preceding rule will not apply to such numbers.

The following Table shows the coincidence between the formula and experiment.

SILVER.

Number of Reflexions.	Values of n.	Angle of Incidence.	Inclination of Plane of Polarization. Observed.	Calculated.
		° ′	° ′	° ′
2	1	73 0	39 48	39 48
4	2	82 30	37 45	37 22
6	3	85 6	35 0	35 22

STEEL.

Number of Reflexions.	Values of n.	Angle of Incidence.	Observed.	Calculated.
2	1	75 0	17 0	17 0
4	2	83 30	11 30	11 17
6	3	85 45	9 30	9 30

When the number of reflexions which begin the series is odd or fractional, we must determine, by the preceding formula, the value of φ for the even number immediately above it: and calling ν the number of odd or fractional reflexions, and N the number of even reflexions immediately above ν, φ the inclination for N reflexions as given by the formula (A), and φ the inclination required, we shall have,

$$\tan \varphi' = \tan x - (\nu - 2 \left(\frac{\tan x - \tan \varphi}{\mathrm{N} - 2}\right). \qquad \text{(B)}$$

The truth of this formula will appear from the following Table.

SILVER.

Number of Reflexions.	Angles of Incidence.	Inclination of the Plane of Polarization.	
		Observed.	Calculated.
	° ′	° ′	° ′
3	79 40	38 28	38 33
5	77 13	33 10	33 36
5	84 5	26 0	26 24
		STEEL.	
3	77 37	13 15	14 11
5	84 38	10 30	10 23

The same results will be obtained at the angles of equal phase below the maximum polarizing angle.

This last rule is suited to even as well to odd numbers of reflexions, but it does not give precisely the same results for even numbers as the formula (A). The difference, however, is far within the limits of the errors of observation. The inclination, for example, at 4 reflexions, is by formula (A) 37° 22′ for silver, whereas by formula (B) it is 37° 34′, the difference being only 12 minutes.

In circular polarization, therefore, the plane of polarization of the restored light continues by successive reflexions to oscillate on each side of the plane of reflexion with a never varying amplitude from +45° to −45°; while in elliptical polarization, the same plane oscillates with an amplitude continually diminishing till it is brought to nothing in the plane of reflexion.

In steel, as we have seen, the polarization is highly elliptical, and the amplitude of the oscillations of the plane of restoration is quickly brought to zero;

but in silver, where the polarization approaches nearly to circular, the oscillations diminish very slowly in amplitude, as the following Table shows.

No. of Reflexions from Silver at 73° of Incidence.	State of the Reflected Light.	Inclination of the Plane of Polarization, or φ. Observed.	Inclination of the Plane of Polarization, or φ. Calculated.
1	Elliptically polarized.		
2	Restored to light polarized	−38° 15′	−38° 15′
3	Elliptically polarized.		
4	Restored to light polarized	+31 15	+31 52
5	Elliptically polarized.		
6	Restored to light polarized	−26 0	−26 6
12	Restored to light polarized		+13 30
18	Restored to light polarized		− 6 42
36	Restored to light polarized		+ 0 47

Owing to the high dispersive power of silver, I found it difficult to carry the comparison any further with white light, as the colours closed in upon the points of evanescence, and rendered it impossible to determine with any precision the inclination of the plane of polarization.

The preceding results afford the clearest explanation of the phenomena which steel and silver exhibit in the reflexion of common light. As common light is similar to two equal pencils polarized +45° and −45°, and as steel brings two such pencils into a state of parallelism with the plane of reflexion, common light must therefore be wholly polarized in the plane of reflexion after 8 reflexions. In like manner we see why the same effect is not produced by silver, because after 8 reflexions the two planes of the pencils are inclined 42°, so as to form a partially polarized pencil.

The same results also furnish us with a method of computing the proportion of polarized light in any pencil of common light, reflected from metals at angles at which the restoration of the elliptical polarized pencil is effected. In order to determine this proportion for steel after two reflexions at 75°, we must consider that a pencil polarized +45° is restored by these two reflexions to light polarized −17°, and consequently a pencil polarized −45° to light polarized +17°. Hence a beam of common light will consist after two reflexions of two pencils +17° and −17° of equal intensity, and consequently in the same state of partial polarization as if common light had been reflected either at an angle of 45° or 68° from a surface of glass. Consequently in the formula *

$$Q = 1 - 2 \sin^2 \varphi, \quad \text{we have } \varphi = 17^\circ \text{ and } Q = 0.829.$$

Hitherto we have considered elliptical polarization as produced only at the maximum polarizing angle. It may be produced, however, by a sufficient number of reflexions at any given angle either above or below the maximum polarizing angle, as appears from the following Table, in which the reflexions are made from two parallel plates of steel.

No. of Reflexions from Steel at which Elliptic Polarization is produced.	No. of Reflexions at which the Pencil is restored to a single Plane.	Angles of Incidence.	
		Calculated.	Observed.
		° ′	° ′
3, 9, 15, &c.	6, 12, 18, &c.	85 45	86 0
2½, 7½, 12½, &c.	5, 10, 15, &c.	84 38	84 0
2, 6, 10, &c.	4, 8, 12, &c.	83 30	82 20
1½, 4½, 7½, &c.	3, 6, 9, &c.	79 39	79 0
1, 3, 5, &c.	2, 4, 6, &c.	75 0	75 0
1½, 4½, 7½, &c.	3, 6, 9, &c.	68 53	67 40
2, 6, 10, &c.	4, 8, 12, &c.	60 2	60 20
2½, 7½, 12½, &c.	5, 10, 15, &c.	56 5	56 25
3, 9, 15, &c.	6, 12, 18, &c.	51 24	52 20

The numbers given in the third column are calculated by the following method. The relation of the preceding phenomena to the angle of maximum polarization is obvious; and if we consider the nature of the formula, $\tan \varphi = \frac{\cos(i + i')}{\cos(i - i')}$, we shall see that the angles at which the rectilineal polarization of the primitive pencil is destroyed have a reference to the rotation which the reflecting surface produces in the plane of polarization. The angles indeed in the third column, at which similar effects are produced above and below 75°, are those at which φ has equal values. This is a very important relation, and enables us to determine the phase P of the two inequal portions of oppositely polarized light, by the interference of which the elliptic polarization is produced. It may be expressed by $P = 2\,R.$

But $R = 45^\circ - \varphi,$

Hence $P = 90^\circ - 2\,\varphi,$

$$\tan \varphi = \frac{\cos(i + i')}{\cos(i - i')}.$$

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* See my Paper "On the Law of the Partial Polarization of Light by Reflexion," *supra*, p. 76.

In this manner we obtain the following results.

No. of Reflexions for Elliptic Polarization.	Angle of Incidence on Silver.	Angle of Incidence on Steel.	Inclination of Plane, or φ.	Rotation of Plane, or R.	Phase, or P.
3	85° 6′	85° 45′	30° 0′	15° 0′	30° = $\frac{1}{3}$ of 90°
2½	83 49	84 38	26 15	18 45	37½ = $\frac{5}{12}$ of 90
2	82 30	83 30	22 30	22 30	45 = $\frac{1}{2}$ of 90
1½	78 8	79 39	11 15	33 45	67½ = $\frac{3}{4}$ of 90
1	73 0	75 0	0 0	45 0	90 = $\frac{1}{1}$ of 90
1½	66 25	68 53	11 15	33 45	67½ = $\frac{3}{4}$ of 90
2	57 16	60 2	22 20	22 30	45 = $\frac{1}{2}$ of 90
2½	53 17	56 5	26 15	18 45	37½ = $\frac{5}{12}$ of 90
3	48 38	51 24	30 0	15 0	30 = $\frac{1}{3}$ of 90

In the results of the two preceding Tables, where the number of reflexions is an integer, it is easily understood how an elliptically polarized ray begins to retrace its course, and recover its state of polarization in a single plane, by the same number of reflexions by which it lost it: but it is interesting to observe, when the number of reflexions is 1½, 2½, 3½, that the ray must have acquired its elliptic polarization in the middle of the second and the third reflexion; that is, when it had reached its greatest depth within the metallic surface. It then begins to resume its state of polarization in a single plane, and recovers it at the end of 3, 5, and 7 reflexions. This stationary point at which the retrograde effect commences, may be made to have its position at any depth beneath the surface, by changing the angles of some of the reflexions, or by combining plates of metal of different polarizing powers.

The same curious property is exhibited in total reflexions, as I have found that the circular polarization can be produced by 2½, 3½, &c. reflexions.

Hitherto we have chiefly examined the phenomena when the reflexions are performed either all above or all below the polarizing angle. We shall now proceed to the case when one reflexion is made on one side, and one on the other side of the maximum polarizing angle.

When a ray polarized + 45° has been reflected once from steel at an angle of 85° or of 54°, it has acquired partially the state of elliptic polarization, and to such a degree that three reflexions more at the same angle will complete the effect. But if the ray partially polarized elliptically by one reflexion at 85° suffers a second reflexion at 54°, it does not acquire more elliptic polarization, but it retraces its course, and recovers its state of single polarization. The same phenomenon occurs at the following angles.

Angles of partial Elliptic Polarization.		Values of φ.		Angles at which it recovers its Polarization.		Values of φ.	
1 Reflex. at	87½°	36°	5′	1 Reflex. at	41°	36°	11′
———	85	27	28	———	54	28	0
———	80	12	12	———	68	12	36
———	77	5	24	———	72	5	59
———	75	0	0	———	75	0	0

It is obvious, by comparing these angles with those in the preceding Table, that they correspond, and are those at which equal phases or rotations are produced.

The effect of two reflexions, at angles of equal phase, upon the inclination I of the plane of polarization is shown in the following Table.

				Inclination I of the Plane of Polarization.		
				Observed.	Calculated.	
1 Reflex. at	90°	and 1 at	0°	45°	45°	0′
———	87½	———	41	—	30	0
———	85	———	54	26	26	5
———	80	———	68	19	20	8
———	77	———	72	18	17	2
———	75	———	75	17	17	0

The last column of the table is calculated by the formula

$$I = \tan \varphi \, (45^\circ - i') + i',$$

i being 17°, or the inclination after two reflexions at the maximum polarizing angle.

In the preceding inquiry we have considered only the phenomena when the consecutive reflexions are performed in coincident planes. The investigation becomes more troublesome, and the results more interesting when the plane of the second reflexion is presented in every different azimuth to the ray that is either wholly or partially elliptically polarized by the first reflexion.

Let a pencil be elliptically polarized by one reflexion from steel at 75°, and let the azimuths be reckoned from the plane of this reflexion. We have already seen that a second reflexion at 75° in azim. 0° and 180° restores the pencil to a single plane of polarization; but if we turn the plane of the second reflexion into azim. 45° or 225°, we shall find that the angle of restoration is no longer 75°, but 78°. At azim. 90° and 270° it is again 75°, and in azim. 135° and 315° it is only 68°, having varied from 68° to 78°.

The following Table shows the observed and calculated angles of restoration in different azimuths.

Azimuths from Plane of first Reflexion.	Angles of Restoration from Steel.	Complement of Angles of Restoration, or Elliptical Radii. Observed.	Calculated.
0° and 180°	75°	15°	14°.9
22½ . 202½	77	13	12.7
45 . 225	78	12	12
67½ . 247½	77¾	12¼	12.7
90 . 270	75	15	14.9
112½ . 292½	70	20	19
135 . 315	68	22	22
157½ . 337½	70	20	19
180 . 360	75	15	14.9

The radii in the two last columns are obviously those of a curve approaching to an ellipse whose major and minor axes are situated, the one 45° to the right, and the other 45° to the left of the plane of the first reflexion. The major semiaxis is 22°, and the minor 12°. Hence calling x the variable radius of the ellipse, a the greater and b the lesser semiaxis, and θ the azimuth, reckoned from the lesser axis, in which the radius x is wanted, we shall have

$$x = \frac{a\,b}{\sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta}}.$$

When $\theta = 45°, 135°$, &c. $\sin^2 \theta \cos^2 \vartheta = \frac{1}{2}$ and $x = \frac{a\,b}{\sqrt{\frac{1}{2} a^2 + \frac{1}{2} b^2}}$.

By calculating the values of x corresponding to the azimuths in the Table, we obtain the numbers in the last column, which are so near the observed numbers as to leave no doubt that an ellipse represents the observations.

If we perform the same experiments with a plate of silver at 73°, we shall observe, with surprise, that the angle of restoration is the same in all azimuths, that is, that the ellipse has merged into the circle. There is a slight deviation indeed, just sufficient to show that the circle is slightly oval, but I could not measure the amount of it.

This result arises from the elliptical polarization of silver being very nearly circular. If we call β the angle of restoration after two reflexions, the ratio of *a* to *b*, the major and minor axes of the ellipse, may be thus expressed:

$$a : b = \sin 2\beta : \text{rad}.$$

In steel, where $\beta = 17°$ and $2\beta = 34°$, we have $a : b = 0.559 : 1 = 12 : 21\frac{1}{2}$, differing very little from 12 : 22 the actual ratio.

In silver, where $\beta = 39° 48'$, $a : b = 0.9835 : 1 = 17 : 17\frac{1}{4}$.

In circular polarization, where $\beta = 45°$, $a : b = 1 : 1$, which gives a circle.

In rectilineal polarization, where $\beta = 0$, $a : b = 0 : 1$, which gives a straight line.

It now becomes an interesting subject of inquiry to ascertain the form and position of the ellipse, when the angle of incidence on the first plate exceeds or falls below the maximum polarizing angle.

The following experiments were made with silver at angles of incidence of 80° and 68°, the maximum polarizing angle being 73°.

SILVER.—Angle of Incidence on First Plate 80°.

Azimuth to Right.	Complement of Angle of Restoration by Second Plate.	Azimuth to Left.	Complement of Angle of Restoration by Second Plate.
0°	28° 2′	0°	28° 2′
11¼	26 35	11¼	24 40
22½	25 20	22½	21 0
33¾	21 13	33¾	16 40
45	18 20	45	14 35
56¼	14 20	56¼	11 10
67½	11 32	67½	10 0
78¾	10 15	78¾	10 0
90	10 0	90	10 0

SILVER.—Angle of Incidence on First Plate 68°.

0°	13°	0°	13°
11¼	14	11¼	13
22½	15⅓	22½	13½
33¾	16	33¾	14
45	17	45	14½
56¼	19	56¼	15½
67½	20	67½	16½
78¾	20	78¾	18
90	20	90	20

In the first of these sets of experiments, the semiaxes of the ellipse are as 10°

to 28°, and its major axis is in azim. 0° and 180° or in the plane of the first reflexion.

In the second series the ratio of the semiaxes is as 13° to 20°, and the major axis is in azim. 90° and 270°, or perpendicular to the plane of the first reflexion; but in both series there is a want of symmetry in the curve to the right of azim. 0° where it bulges out, showing that in both series the greater axis is a little to the right of azim. 0°.

Hence it appears that in silver, whose elliptic polarization is nearly circular, the ellipse which regulates the angles of restoration has its greater axis in the plane of the first reflexion for all angles greater than 73°, the maximum polarizing angle; and from a circle it increases in ellipticity till at the limit of 90° the lesser semiaxis is 0°, and the greater 90°, and it becomes a straight line. For angles above 73° the ellipse has its greater axis perpendicular to the plane of reflexion, and gradually increases in ellipticity from the circle till at the limit of 0° its lesser semiaxis is 0°, and its greater 90°, when it becomes a straight line.

The peculiar character of elliptic polarization shows itself in another manner, and with peculiar interest, in the variable position of the ellipses which regulate the angles of restoration upon steel.

We have already seen that the curve which is circular in silver at the maximum polarizing angle, is in steel an ellipse whose semiaxes are as 12° to 22°, the greater axes being inclined 45° to the right of azim. 0°.

The following Table will show how the effect varies at angles of incidences above and below the polarizing angle.

STEEL.—Angle of Incidence 80°.

Azimuth to Right.	Complement of Angle of Restoration by Second Plate.	Azimuth to Left.	Complement of Angle of Restoration by Second Plate.
0°	23°	0°	23°
11¼	25	11¼	20
22½	26	22½	16⅓
33¾	24	33¾	13
45	20½	45	11½
56¼	18	56¼	10
67½	15⅓	67½	9½
78¾	11	78¾	9¾
90	10	90	10

STEEL.—Angle of Incidence 68°.

Azimuth to Right.	Complement of Angle of Restoration by Second Plate.	Azimuth to Left.	Complement of Angle of Restoration by Second Plate.
0°	11°	0°	11°
$11\frac{1}{4}$	24	$11\frac{1}{4}$	10
$22\frac{1}{2}$	$24\frac{1}{2}$	$22\frac{1}{2}$	9
$33\frac{3}{4}$	$25\frac{1}{2}$	$33\frac{3}{4}$	$9\frac{3}{4}$
45	$26\frac{1}{3}$	45	$11\frac{1}{2}$
$56\frac{1}{4}$	$25\frac{1}{3}$	$56\frac{1}{4}$	15
$67\frac{1}{2}$	20	$67\frac{1}{2}$	18
$78\frac{3}{4}$	21	$78\frac{3}{4}$	20
90	22	90	22

By comparing these results with those obtained from steel at 75°, and with the observations already made on the passage of the ellipse into a straight line, the following results may be deduced.

Angle of Incidence on first Steel Plate.	Ratio of Semiaxes of the Ellipse.	Character of the Ellipse.	Position of the greater Axis of the Ellipse.
0°	0° : 90°	Straight line.	Azim. 90° and 270°
68	9 : 26	Ellipse	—— betw. 45° and 56° to R.
75	12 : 22	Ellipse	—— 45° to R.
80	$9\frac{1}{2}$: 26	Ellipse	—— $22\frac{1}{2}$° to R.
90	0 : 90	Straight line.	—— 0

Hence it is obvious that the major axis of the ellipse is $45^\circ \mp \varphi$ R to the right of 0° of azimuth, φ being computed from the formula

$$\tan \varphi = \frac{\cos(i + i')}{\cos(i - i')}.$$

There is a deviation at the incidence of 68° and 80° of some amount, but still it is scarcely without the limits of the errors of observations when common light is used. In strong lights the coincidence will doubtless be more perfect.

The best method of determining the position of the major axis, is to place the second plate at such an angle to the ray received from the first, that it may exceed by two or three degrees the angle of restoration in azim. 0°. Hence if we turn the second plate round the ray into all azimuths from 0° to 90° in the right hand quadrant where the greater axis lies, it must come into two azimuths where the restoration takes place at the same incidence. The complements of these two angles of incidence will be equal radii of the ellipse, and consequently the azimuth which bisects the two azimuths in question, will be that of the major axis of the ellipse. By increasing the angle of incidence on the second plate, other two azimuths containing equal radii of the ellipse will in like manner be found; and we might, if necessary, at last obtain an angle of incidence where the two radii coincided with the greater axis.

The position of the ellipse being thus given, we may determine it for all angles of incidence. Calling x the angle of incidence on the first plate, then we shall have four points in the ellipse as follows. The radii in azim. 90° and 270° are always $90° - x$, and the radius in azim. 0° and 180° is the complement of the angle of incidence at which φ in the last equation has the same value as at the angle x. Hence the form of the ellipse is also given.

In these experiments the polarization of the primitive ray has always been +45°. When this plane varies its position, that of the restored ray also changes, as we have already shown; but it remains to be seen what change takes place in the angles of restoration. At all angles of incidence, a variation in the plane of primitive polarization does not alter the angles of restoration or the corresponding radii of the ellipse in azim. 0°, 90°, 180°, and 270°; but at all intermediate azimuths of the second plate the angles of restoration diminish while the primitive plane varies from 45° to 0°, and increase when it varies from 45° to 90°. The following experiments show the progress of the change when the azimuths of the second reflexion are +45° and −45°.

STEEL.

Inclination of Plane of prim. Polarization.	Azimuth of Second Reflexion +45° to Right.		Azimuth of Second Reflexion −45° to Left.	
	Observed.	Calculated.	Observed.	Calculated.
°	°	° ′	°	° ′
0	0	0 0	1	0 0
5	2	2 4	2	1 0
10	4	3 46	3½	2 3
15	6	5 43	4½	3 7
20	8½	7 50	6	4 14
25	11	9 54	6	5 26
30	13	12 11	7	6 43
35	15	14 40	8	8 6
40	18	17 25	9½	9 41
45	20½	20 30	11½	11 30

These observations are represented by the formula, $\tan \theta = \tan a \,.\, \tan x$; a being the angle of restoration when θ, the inclination of the plane of primitive polarization, is 45°.

I have not given the values of θ from 45° to 90°, because it is difficult to ascertain even in strong lights when the evanescence commences. At 90° the action of the first plate is 0, so that at this limit the angle of restoration is the angle at which the elliptic polarization is no longer visible, from the smallness of the angle of incidence, an angle which varies with the intensity of the light employed.

Hitherto we have attended only to the phenomena produced by two similar metals. When the metals are dissimilar, the one silver and the other steel, I found that at the mean maximum polarizing angle of 74°, the inclination of the plane of the restored ray was 28° 30′. But $28° \, 24' = \frac{39° \, 48' + 17°}{2}$, so that the inclination is an arithmetical mean between that of silver and that of steel. By four reflexions at 74° the inclination was reduced to 14°, while by four reflexions at about 83° and 58° the inclination was $21\frac{1}{2}°$, nearly equal to $\frac{28° \, 30' + 14°}{2}$, according to the formula in page 297. By thus combining dissimilar metals we may produce elliptic polarization of all degress of intensity intermediate between those produced by similar metals.

As the circular polarization of total reflexion is the limiting case of elliptical polarization, it becomes important to establish by experiment their intimate connexion and almost perfect similarity. Upon combining metallic and total reflexions this was at once evident; and I found in general that circular polarization of any intensity, as produced by either one or more reflexions from glass, may always be restored to rectilineal polarization by one or more metallic reflexions, provided the latter are all made at angles less than the maximum polarizing angle, and that the two classes of reflexions are performed in coincident planes.

As this takes place throughout the whole range of total reflexion from 41° to 90°, it follows that total differs from metallic reflexion in its not having two opposite kinds of circular polarization, like the two opposite kinds of elliptical polarization which take place on each side of the maximum polarizing angle of metals. But notwithstanding this, the circular like the elliptic polarization has a maximum at about 50°, declining rapidly to zero at 41°, and on the other side slowly to zero at 90° of incidence.

When one reflexion from steel was combined with two total reflexions from glass at 54½°, the inclination of the plane of the restored ray was 30½°, an arithmetical mean between 45° that of total reflexion, and 17° that of steel, for $\frac{45° + 17°}{2} = 31°$. With silver the inclination was 42½°, and $\frac{45° + 39°\ 48'}{2} = 42°\ 24'$.

If we make the metallic reflector receive the circularly polarized ray in every azimuth, we shall find that in azimuth 90° the circular polarization is compensated by a metallic reflexion above 80°. As the azimuth diminishes to 0°, this angle of compensation diminishes also, passes through 75° in the case of steel, and diminishes to a number depending on the angle of incidence at which the total reflexion is made. We are thus enabled to study the phenomena of circular polarization by the aid of metals, and to obtain results at which it would be exceedingly difficult, if not impossible, to arrive by any other method. This subject, however, presents too wide a field to be treated thus incidentally.

SECT. III.—*On the complementary colours produced by successive reflexions from the polished surfaces of metals.*

I have already given a general account of the phenomena of colour produced by successive reflexions; and I have shown that the tints thus produced are by no means the same as those of crystallized plates, as they do not rise in the scale by successive reflexions.

In my early experiments on total and metallic reflexions, I regarded the two classes of phenomena as exactly the same, mutatis mutandis; and in communicating these results to Dr. YOUNG, I pointed out their coincidence with his theoretical views. Dr. YOUNG noticed these experiments in the following manner[*].

"Dr. BREWSTER has also shown that the total reflexion of light within a denser medium, and the brilliant reflexion at the surfaces of some of the metals, are capable of exhibiting some of the appearances of colour as if the light concerned were divided into two portions, the one partially reflected in the first instance, the other beginning to be refracted, and caused to return by the continued operation of the same power. The original interval appears to be extremely minute, but is capable of being increased by a repetition of similar reflexions as well as obliquity of incidence."

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[*] Art. CHROMATICS, Supp. Encyc. Brit. p. 157.

In a letter which I received from this eminent philosopher, dated March 25th 1816, he thus modifies an objection which he had previously made to my opinion, that the phenomena were owing to the interference of the light which had entered the surface with that which had suffered partial reflexion.

"The light which you suppose to have entered a little way into a reflecting surface, in the case of total reflexion, is singularly circumstanced with regard to the objection I mentioned in my last letter. I did not like the idea of supposing a surface of any kind to contain a finite space: but, in fact, if your theory should be confirmed, this objection might be greatly diminished by the consideration, that the thickness of the surface would still be like an infinitesimal of a different order from the interval corresponding to its apparent effect, being the versed sine of a curve of which that small interval is the arc, and possibly in a circle of curvature not very minute."

In continuing my experiments on this subject, I found that the colours of total reflexion did not rise in the scale by successive reflexions; and as they modified the tints of crystallized bodies by adding to, or subtracting from, them a given portion of a tint, I announced in the end of 1816, in the Journal of the Royal Institution, that I had discovered "a new species of moveable polarization, in which the complementary tints never rise above the white (the blueish white) of the first order, by the successive application of the polarizing influence*." I determined, experimentally, the angles at which this tint was successively produced and destroyed, and thus discovered some of the leading properties of total reflexion, before, I believe, M. Fresnel had made any experiments on the subject. It was he, however, who ascertained that this new species of polarization was circular polarization; and it is impossible to speak too highly of the ingenuity and talent which he exhibited in that difficult inquiry.

This view of the phenomena of total reflexion unsettled the opinions which I had entertained respecting the action of metals, and I was thus led to revise and extend the unpublished experiments which I had made on the subject.

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* Journ. Roy. Inst. vol. iii. p. 213.

In order to ascertain the effect of a single metallic surface, I took a crystallized plate of glass whose central tint was the blueish white of the first order, and positive like sulphate of lime. This tint varied from a quarter of a tint in value down to zero. The primitive ray was polarized +45°, and the plate of steel was horizontal. This ray was received at an incidence near 90°, and the principal section of the analysing prism was in the plane +45°, while the length of the plate of glass was fixed perpendicular to the plane +45°, or to the principal section of the prism, so as to move along with it.

At an incidence of 88° the metallic action destroyed the action of the equivalent crystallized plate when the section of the analysing prism was turned from +45° to +38°.

At an incidence of 83½° the same effect was produced when the same section was turned into the plane +22½°.

And at an angle of 75°, viz. the maximum polarizing angle, the compensation took place when the axis of the crystal had moved round 45°.

In like manner, at an angle of 60° the compensation took place when the axis of the crystal was turned round 45° + 22½°, or − 22½°; and,

At an angle of incidence of 40° the compensation was effected when the axis of the crystal had turned round 45° + 37°, or into the plane − 37°. The same results are obtained when the light falls on the metal before it passes through the crystal.

Hence it follows, that at the maximum polarizing angle the effect of the equivalent crystal placed in azimuth 45° to the plane of primitive polarization, is compensated by the action of the metallic surface, while at greater angles of incidence the compensation is effected in azimuths less than 45°; and, at less angles of incidence, in azimuths greater than 45°.

When the reflexion from the metal is made in a plane perpendicular to the meridian, the opposite effect is produced.

The angles at which the compensation takes place in the preceding experiments are obviously such, that calling R the angle of rotation of the axis of the crystal, it has always to i the angle of incidence the same relation as in the formula, $\tan(45° - R) = \frac{\cos(i + i')}{\cos(i - i')}$.

Hence we are led to the important conclusion, that the pencil which enters the metal follows the changes of polarization of the partially reflected pencil, which is regulated by the same law as in transparent bodies.

It now became interesting to examine the effect produced by the joint action of the metal, and an equivalent crystal, in changing the plane of polarization of the restored ray. The following are the results with different metals at the maximum polarizing angle.

Metals.	Position of the Plane of Polarization.	Rotation effected.
Silver (pure)	+ 42°	3°
Copper	+ 36½	8½
Mercury	+ 35	10
Platina	+ 34	11
Speculum metal	+ 32	13
Steel	+ 30½	14½
Lead	+ 26	19
Galæna	+ 17½	27½

These metals follow the same order in their action upon the plane of polarization that they hold in the Table in page 294, though in reference to the rotation actually produced in both cases the order is inverted.

The preceding Table points out in a very instructive manner the difference between the action of a metallic surface and an equivalent crystallized film. When two metallic surfaces act together, the plane of polarization of the restored ray is invariably thrown beyond the plane of reflexion; whereas in the combination of a crystallized film with a metallic surface, the same plane never reaches the plane of reflexion, the plane having always a negative position in the former case, and a positive one in the latter. Thus in two reflexions from silver at 73°, the primitive ray polarized + 45° has its plane of polarization changed into − 39° 48′, whereas in the combination of one reflexion from silver with the crystallized film, the plane is changed only into +42°.

In order to determine the law of the metallic action at different incidences and with different numbers of reflexions, I interposed between the eye and the metal, which was silver, a plate of calcareous spar, which exhibited its uniaxal system of rings.

Fig. 2

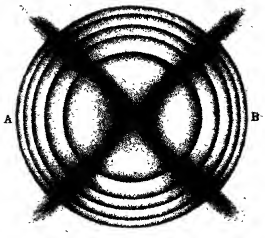


Fig. 1

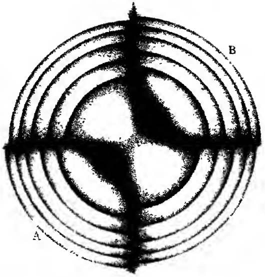
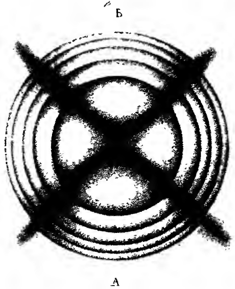


Fig. 3



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The influence of the metal in modifying the rings was a maximum at 73°, as shown in Plate XI. fig. 1., exactly as if they had been crossed by a positive crystalline film which polarized a quarter of a tint, or the pale blueish white of the first order, and whose axis was situated in a plane +45°, or that which bisects the planes of the two pencils oppositely polarized by the metal. The influence of the metal, or the tint which it polarizes, diminishes gradually from 73° to 90°, where it vanishes, and consequently where the rings recover their symmetry and their tints. At this limit the position of the axis of the equivalent film is A B (fig. 2.), a line still bisecting the planes of the two oppositely polarized pencils. In fig. 2. the rings are not represented of their own shape, but just as they are beginning to be invaded by the metallic action as at an incidence of 86° or 87°. At incidences from 73° to 0° the opposite effect takes place, the rings recovering their symmetry at 0°, and the position of the axis of the equivalent film being now vertical, and bisecting the planes of the two oppositely polarized pencils. The form of the rings before they recover their symmetry is shown in fig. 3.

At all intermediate angles of incidence the axis A B has intermediate positions; and calling A the inclination of the axis to the plane of reflexion, we shall have

$$A = \varphi + 45^\circ,$$

φ being positive or + from 90° to 73°, and negative or − from 73° to 0°.

The intensity of the metallic tint, so to speak, or of the positive equivalent plate T, will be

$$T = \tfrac{1}{4}\,\frac{P}{90} = \frac{2\,R}{360} = \left(\frac{45^\circ - \varphi}{180}\right).$$

Hence we see the error of the proposition hitherto maintained, that an increase of incidence, reckoning from the perpendicular, produces the same effect as an increase of thickness in thin crystallized plates.

When the rings are combined with two reflexions at 73° in silver, or 75° in steel, they do not suffer the slightest change, the principal section of the prism being placed in the plane −39° 48′ with silver, and −17° with steel. By two reflexions, however, between 73° or 75° and 90°, an effect is produced on the rings which increases gradually in silver from 73° to 82° 30′, and diminishes from 82° 30′ to 90°. At 82° 30′ the effect is the same as after a single reflexion at 73°; for since four reflexions at 82° 30′ restore the elliptically polarized ray, two reflexions at the same angle must have produced complete elliptical polarization. At angles between 82° 30′ and 90° the pencil is only partially polarized elliptically; whereas from 82° 30′ to 73° the light has been more than elliptically polarized, the restoration of it having been begun during the second reflexion. Hence, in order to determine the phase for any angle between 82° 30′ and 90°, we must take the sum of the phases for each reflexion, or 2 P; whereas between 82° 30′ and 73° we must take the excess of the sum of the two phases above 90° or 90° − 2 P. In both cases the pencil has suffered a partial elliptic polarization;—in the former, from the sum of the actions of the two reflexions, and in the latter, from their unbalanced actions. The very same effects take place between 73° and 57° 16′, the other maximum, as between 73° and 82½°; and between 57° 16′ and 90°, as between 82½° and 90°.

In the case of three reflexions there are two points or nodes of restoration, viz. 78° 8′ and 66° 35′, the maxima being at 85° 6′, 73°, and 48° 38′, at each of which points the phase is 90°. At 73° the second reflexion restores the ray elliptically polarized by the first reflexion, and the third reflexion again produces elliptic polarization. At 85° 6′ and 48° 38′, six reflexions produce a restoration of the pencil, and consequently three reflexions must have polarized the pencil elliptically with a phase of 90°. From 85° 6′ to 90°, and from 48° 38′ to 0°, the pencil has been only partially elliptically polarized, and the phase at any angle between these will be 3 P. At any angle between 48° 38′ and 85° 6′, the phase will be $\overline{2 \times 90} - 3\,\mathrm{P}$.

In general, calling *n* the number of reflexions, the phase between 90° and the nearest maximum, and between 0° and the nearest maximum, will be *n* P, while at all other angles of incidence it will be $(\overline{n - 1} \times 90) - n\,\mathrm{P}$.

In order to give a general view of the number of points of restoration, and of the other phenomena which take place after different numbers of reflexions, I have drawn up the following Tables.

Table I. Showing the numbers of reflexions from silver at which elliptically polarized light is restored to a single plane of polarization, with the corresponding angles of incidence, and the position of the plane of restoration in relation to the plane of reflexion, computed for 20 reflexions.

(For angles less than the maximum polarizing angle.)

No. of Reflexions.		Integer Multiples.	Angle of Restoration.		No. of Reflexions.		Integer Multiples.	Angle of Restoration.	
			°	′				°	′
−2	2	+4−6+8−10+12−14+16−18+20	71	0	$3\frac{4}{5}$	3.8	+19	58	15
$2\frac{1}{9}$	2.111	+19	71	42	−4	4.0	+8−12+16−20	57	16
$2\frac{1}{8}$	2.125	+17	71	32	$4\frac{1}{4}$	4.25	+17	55	54
$2\frac{1}{7}$	2.143	+15	71	23	$4\frac{1}{3}$	4.333	+13	55	29
$2\frac{1}{6}$	2.167	−13	71	8	$4\frac{1}{2}$	4.5	−9+18	54	42
$2\frac{1}{5}$	2.200	+11	70	48	$4\frac{2}{3}$	4.667	−14	53	54
$2\frac{2}{9}$	2.222	−20	70	34	$4\frac{3}{4}$	4.75	−19	53	31
$2\frac{2}{8}$	2.25	−9+18	70	17	+5	5	+10+15+20	52	27
$2\frac{2}{7}$	2.286	−16	69	53	$5\frac{1}{3}$	5.333	−16	51	5
$2\frac{2}{6}$	2.333	+7+14	69	29	$5\frac{1}{2}$	5.5	−11	50	27
$2\frac{3}{8}$	2.375	−19	69	3	$5\frac{2}{3}$	5.667	−17	49	49
$2\frac{2}{5}$	2.4	−12	68	59	−6	6	+12−18	48	38
$2\frac{3}{7}$	2.428	−17	68	33	$6\frac{1}{3}$	6.333	+19	47	23
$2\frac{1}{2}$	2.5	−5+10−15+20	67	54	$6\frac{1}{2}$	6.5	−13	46	57
$2\frac{4}{7}$	2.571	−18	67	14	$6\frac{2}{3}$	6.667	−20	46	32
$2\frac{3}{5}$	2.6	+13	66	58	+7	7	+14	45	35
$2\frac{2}{3}$	2.667	−8+16	66	25	$7\frac{1}{2}$	7.5	−15	44	13
$2\frac{5}{7}$	2.714	+19	66	0	−8	8	+16	43	0
$2\frac{3}{4}$	2.75	−11	65	45	$8\frac{1}{2}$	8.5	+17	41	52
$2\frac{4}{5}$	2.8	−14	65	23	+9	9	+18	40	51
$2\frac{5}{6}$	2.833	+17	65	9	$9\frac{1}{2}$	9.5	−19	39	51
$2\frac{6}{7}$	2.857	−20	65	0	−10	10	+20	39	0
+3	3	+6+9+12+15+18	63	43	+11	11		37	15
$3\frac{1}{6}$	3.167	−19	62	29	−12	12		35	50
$3\frac{1}{5}$	3.2	−16	62	15	+13	13		34	33
$3\frac{1}{4}$	3.25	−13	61	55	−14	14		32	30
$3\frac{1}{3}$	3.33	−10	61	20	+15	15		32	15
$3\frac{2}{5}$	3.4	−17	60	53	−16	16		31	17
$3\frac{1}{2}$	3.5	−7+14	60	15	+17	17		30	30
$3\frac{3}{5}$	3.6	−18	59	38	−18	18		29	42
$3\frac{2}{3}$	3.667	+11	59	13	+19	19		28	56
$3\frac{3}{4}$	3.75	−15	58	42	−20	20		28	10

TABLE II. Showing the numbers of reflexions from silver at which elliptically polarized light is restored to a single plane of polarization, with the corresponding angles of restoration, and the position of the plane of restoration in relation to the plane of reflexion, computed for 20 reflexions.

(For angles greater than the maximum polarizing angle.)

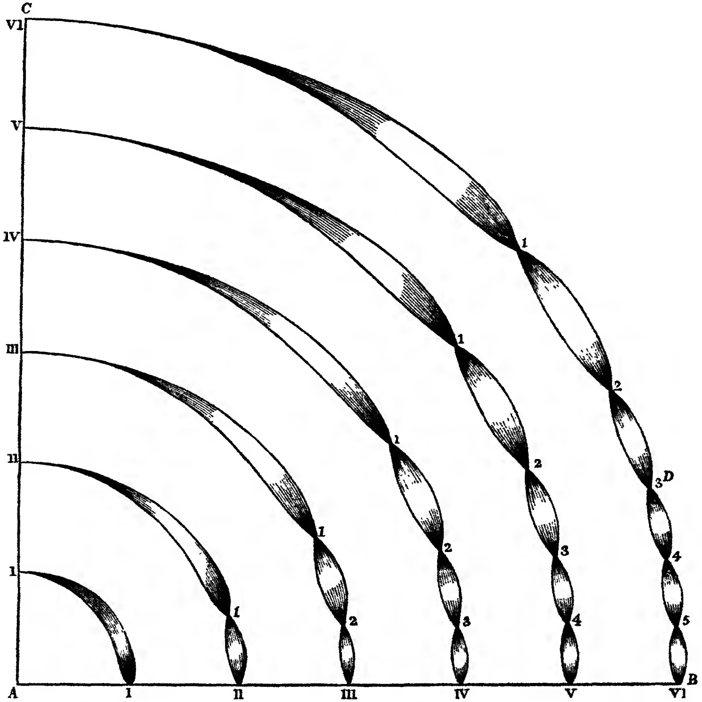
No. of Reflexions.		Integer Multiples.	Angle of Restoration.		No. of Reflexions.		Integer Multiples.	Angle of Restoration.	
			°	′				°	′
−2	2	+4−6+8−10+12−14+16−18−20	73	0	$3\frac{4}{5}$	3.8	−19	82	8
$2\frac{1}{9}$	2.111	−19	74	9	− 4	4	+8−12+16−20	82	30
$2\frac{1}{8}$	2.125	+17	74	18	$4\frac{1}{4}$	4.25	+17	82	58
$2\frac{1}{7}$	2.143	−15	74	28	$4\frac{1}{3}$	4.333	−13	83	16
$2\frac{1}{6}$	2.167	+13	74	44	$4\frac{1}{2}$	4.5	+9+18	83	23
$2\frac{1}{5}$	2.200	−11	75	0	$4\frac{2}{3}$	4.667	−14	83	38
$2\frac{2}{9}$	2.222	−20	75	12	$4\frac{3}{4}$	4.75	+19	83	45
$2\frac{1}{4}$	2.256	+9+18	75	26	− 5	5	+10−15+20	84	5
$2\frac{2}{7}$	2.286	−16	75	43	$5\frac{1}{3}$	5.333	−16	84	27
$2\frac{2}{6}$	2.333	−7+14	76	2	$5\frac{1}{2}$	5.5	+11	84	38
$2\frac{3}{8}$	2.375	+19	76	19	$5\frac{2}{3}$	5.667	−17	84	48
$2\frac{2}{5}$	2.4	−12	76	33	− 6	6	+12−18	85	6
$2\frac{3}{7}$	2.428	+17	76	44	$6\frac{1}{3}$	6.333	−19	85	22
$2\frac{1}{2}$	2.5	+5+10+15+20	77	13	$6\frac{1}{2}$	6.5	+13	85	30
$2\frac{4}{7}$	2.571	−18	77	38	$6\frac{2}{3}$	6.667	−20	85	36
$2\frac{3}{5}$	2.6	−13	77	48	− 7	7	+14	85	49
$2\frac{2}{3}$	2.667	−8+16	78	38	$7\frac{1}{2}$	7.5	+15	86	7
$2\frac{5}{7}$	2.714	−19	78	23	− 8	8	+16	86	21
$2\frac{3}{4}$	2.75	+11	78	33	$8\frac{1}{2}$	8.5	+17	86	35
$2\frac{4}{5}$	2.8	−14	78	47	− 9	9	+18	86	46
$2\frac{5}{6}$	2.833	+17	78	57	$9\frac{1}{2}$	9.5	+19	86	56
$2\frac{6}{7}$	2.857	−20	79	4	−10	10	+20	87	5
−3	3.0	+6−9+12−15+18	79	40	−11	11		87	20
$3\frac{1}{6}$	3.167	+19	80	37	−12	12		87	35
$3\frac{1}{5}$	3.2	−16	80	24	−13	13		87	46
$3\frac{1}{4}$	3.25	+13	80	34	−14	14		87	56
$3\frac{1}{3}$	3.333	−10+20	80	50	−15	15		88	4
$3\frac{2}{5}$	3.4	−17	81	2	−16	16		88	11
$3\frac{1}{2}$	3.5	+7+14	81	19	−17	17		88	18
$3\frac{3}{5}$	3.6	−18	81	35	−18	18		88	24
$3\frac{2}{3}$	3.667	−11	81	45	−19	19		88	28
$3\frac{3}{4}$	3.75	+15	81	57	−20	20		88	33

The first column of the preceding Tables shows the smallest number of reflexions at which a pencil of elliptically polarized light is restored to a single plane of polarization at the angle contained in the fourth column; and consequently the half of these numbers is the number of reflexions at which light is elliptically polarized at the same angle. Thus at three reflexions the ray is restored to a single plane of polarization at 63° 43′, and 79° 40′, and consequently at 1½ reflexion it is elliptically polarized at that angle. This is easily understood when the number of reflexions is an integer; but it requires some explanation when the number is partly fractional. It has been already stated, in page 301, that elliptical polarization may be completed at any fractional part of a reflexion; and since it begins to be restored the instant the polarization is complete, and again begins to be elliptically polarized after every restoration, the points of restoration may take place in the middle of a reflexion; and though we cannot possibly examine what takes place at these points, yet the effect must appear when the fractional number of reflexions in the first column has been repeated so many times as to become a whole number. Thus a ray elliptically polarized by 1⅓ reflexion will be restored to a single plane at 2⅔ reflexions at the same angle. It will also be restored at $2\frac{2}{3} \times 2 = 3\frac{1}{3}$, and at $2\frac{2}{3} \times 3 = 8$, in which case its restoration will be seen at the eighth reflexion at the same angle; and also at the sixteenth and twenty-fourth, &c. In this case the phase P will be $\frac{90^\circ}{1\frac{1}{3}} = 67\frac{1}{2}^\circ$, $R = 33^\circ\ 45'$, and $\varphi = 11^\circ\ 15'$, from which we deduce the angles of incidence to be 63° 43′, and 79° 40′. In order to ascertain the existence of these points of restoration, I made the experiment at five and seven reflexions as multiples of 2½ and 2⅓, and I found the angles to be for five reflexions 68°, and for seven reflexions 70°, in place of 67° 54′, and 69° 29′, as computed from the formula.

The numbers in the third column, with the signs + and −, are the integer multiples of those in the first column, and show the number of reflexions at which the elliptically polarized light is restored, the numbers being carried the length of twenty reflexions. The sign + shows that the plane of the restored ray is to the right, and the sign − that it is to the left of the plane of reflexion. In order to determine the sign of the restored ray, we must consider that in the same quadrant the signs necessarily alternate. Now at 73°, the maximum polarizing angle, the signs are −2, +4, −6, +8, −10, +12, &c.; and I have also found that all the integer numbers in column 1st, Table I. have their signs + or positive, as +3, +5, +7, +9, &c., and all the even numbers their signs − or negative, as −4, −6, −8, −10, &c.; whereas in Table II. all the integer numbers are negative whether odd or even, thus, −3, −4, −5, −6, &c.

By setting out therefore from these points, and attending to the alternation of the signs, it is easy to determine for any number of reflexions its proper signs, whether it is a multiple of an integer or of a mixed number.

In order to illustrate this Table, I have here projected some of its results as far as six reflexions. The concentric arches I I, II II, &c. represent the quadrant



of incidence for one, two, &c. reflexions, B being the point of 90°, and C that of 0° of incidence. The point D or the line A D is the point or line of maximum polarization, viz. 73° for silver; and the figures 1, 2, 3, 4, 5, &c. show the points or nodes, and their distances from C, the angles of restoration. The loops or double curves lying between the points 1, 2, 3, are drawn to give an idea of the intensity of the elliptic polarization, which has its minimum at 1, 2, 3, &c. and its maximum at intermediate points. These points of maximum intensity do not bisect the loops, or are not equidistant from the minima 1, 2, &c.; but such is their relation to them, that the maximum for n reflexions is the minimum for $2n$ reflexions corresponding to the same angle. Thus the maximum for one reflexion, viz. 73°, is the minimum for two reflexions; and the maxima for two reflexions, viz. 82° 30′ and 63° 43′, are the minima for four reflexions. The maximum may be found directly by computing the angle of incidence, which corresponds to a phase intermediate between the two minima, within which the maximum lies.

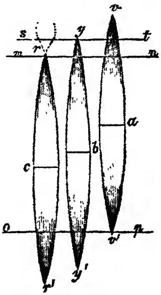
Having thus determined the various points of the quadrant, at which elliptic polarization is produced, and at which it is destroyed, after any number of reflexions; and also the position of the plane of the restored ray, I shall proceed to investigate the cause of those brilliant complementary colours which accompany these phenomena.

As all transparent bodies have different values of their maximum polarizing angle, appropriate to the index of refraction for each colour of the spectrum, it is reasonable to suppose that as elliptic polarization is effected at the maximum polarizing angle, this angle would vary for the differently coloured rays. That this is the case may be easily proved by observing the angles of restoration for homogeneous light after two reflexions. In silver the difference of the angles for red and blue light is about 5° in the sun's rays; so that calling 73° the maximum polarizing angle for the mean yellow ray, the angle will be 70½° for blue, and 75½° for red light. Hence if we examine a pencil of white light twice reflected at 70½°, and place the principal section of the analysing prism in the plane −39° 48′, the blue rays will disappear and the red will remain visible. In like manner, at an angle of 75° 30′ the red will disappear, and the complementary blue will be visible; while at an angle of 73° the yellow will disappear and red and blue will be seen together, one on each side of the place where the yellow has vanished. At angles of incidence greater than 75½° and less than 70½°, and also at intermediate angles, the blue or the red wil still predominate in the pencil, the blue being in excess at all angles greater than 73°, and the red in excess at all angles less than 73°. Such are precisely the phenomena which take place, as will appear from the following Table.

Angle of Incidence of the two Reflexions.	Colours with ordinary Light.
63°	Very pale yellow, growing whiter at less incidences
64	Pale yellow.
65	Pale saffron yellow.
66	Saffron yellow.
67	Paler orange yellow.
68	Orange yellow.
69	Reddish orange.
70	Tile red.
70½	Vermilion red.
71	Scarlet.
72	Bright pink.
73	Dark pink.
74	Deep China blue.
75	Indigo.
75½	Pure bright blue.
76	Paler blue.
77	Whitish blue.
78	Blueish white, growing white at greater angles.

It is obvious from what has been already stated, that with homogeneous yellow light the pencil will not vanish in passing from 73°, where it is evanescent, to 90°, and 0° where it is also evanescent; but the intensity of the extraordinary pencil of the analysing rhomb will increase from 0° to half the reflected light, from 73° to 82½°, and from 73° to 57° 16′, and will decrease from the same points to 90° and 0°. The same is true of the red and blue rays, the former having its maximum intensity at an angle greater than 82½° and greater than 57° 16′, and the latter at an angle less than 82½° and less than 57° 16′.

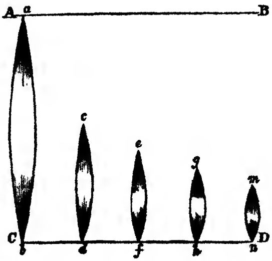
In order to ascertain the phenomena in homogeneous light, let us suppose that polarized yellow light suffers four reflexions from silver, and let us consider what should take place in the loop 2, 3 of the quadrant IV, IV. (See Fig. p.318.) At the node 2, or 73°, the inclination of the restored pencil is +31° 52′, and at the node 3, or 82° 30′, it is −37° 22′, and the point of maximum between 2 and 3 is at 78° 8′. If at 73° we place the principal section of the analysing prism in the plane +31° 52′ the extraordinary ray will vanish, and the light will pass into the ordinary image; and if at 82° 30′ we place it in −37° 22′, the same effect will be produced. At 74° a small portion of light will pass into the extraordinary image, and this portion will gradually increase to 78° 8′, the principal section of the prism having been turned round gradually from +31° 52′ to 0°, as described in page 291. The ordinary and extraordinary images now approach most to equality, and they vary in intensity according to the same law in passing from 78° 8′ to 82° 30′, the axis of the prism having now come into the plane −37° 22′. The very same phenomena take place with red and blue light, only the points of restoration and the maximum occur at different angles of incidence, so that the spaces between the minima have different lengths for the differently coloured rays. These spaces or loops, therefore, will overlap each other, as will be understood from the annexed diagram, where they are shown separately, $r\,r'$ being the red loop, $y\,y'$ the yellow, $v\,v'$ the violet one, the points r, y, v, r', y', v' the minima or nodes, and a, b, c the maxima. When these loops are viewed superposed as when they form white light, then the tint in the extraordinary image will be white, minus the three quantities of light that have disappeared from the extraordinary ray. At the line $m\,n$, passing through the node of the red loop, the red will have vanished, and the mixture of the yellow and the violet which remains will constitute a greenish blue pencil, decreasing in its blue tint towards a, and becoming pink, and then red towards $t\,s$, in consequence of part of the light of the other red loop above r now passing into the extraordinary ray. At v and at v', where the violet disappears, the mixture of the yellow and the red will form an orange pencil, which will be reddest at v and v', and shading off to white at a. At the line $s\,t$ the yellow vanishes, and across the upper part of the luminous disc, there will be light with an excess of red, and across the lower part of it, light with an excess of blue. This takes place with even numbers of reflexions; with odd numbers the blue light is uppermost and the red undermost.



The phenomena of colour, as seen by white light, vary greatly with the number of reflexions, both with respect to the depth of the colours themselves and the rapidity of their changes. In order to investigate the nature of these variations, let us consider what will take place at 2, 4, 6, 8, and 10 reflexions from silver in the loops above and adjacent to 73° the maximum polarizing angle. The following are the numbers which regulate the phenomena.

Fig. 6.	No. of the Reflexions.	Nature of the Reflexions.	Limits of the Loops.	Length of the Loops.	Inclination of the Plane, or φ.
			° ° ′	° ′	° ′
a b	2	First of the series	73 — 90 0	17 0	39 48
c d	4	First of the series	73 — 82 30	9 30	37 22
e f	6	Multiple of 3	73 — 79 40	6 40	32 25
g h	8	Multiple of 2⅔	73 — 78 8	5 8	27 53
m n	10	Multiple of 2½	73 — 77 13	4 13	24 16

This Table may be illustrated by the annexed diagram, where A B passes through the incidence of 90°, and C D through that of 73°, the points *m*, *g*, *e*, *c*, *a* corresponding respectively with the incidences of 77° 13′, 78° 8′, 79° 40′, and 82° 30′, or those at which the ray is restored by 10, 8, 6, and 4 reflexions. The curvilineal spaces *a b*, *c d*, *e f*, *g h*, and *m n*, are the loops already referred to, whose breadths represent the intensity of the extraordinary ray, which is a minimum at the nodes *a*, *c*, *e*, *g*, *m*, and *b*, *d*, *f*, *h*, *n*, and reaches its maximum near the middle of the loops.



If the image reflected from the silver is a circular disc of white light of a given magnitude, then by two reflexions at 73°, or at the point *b* the extraordinary image will be red above and blue below, when the principal section of the analysing prism is in the plane − 39° 48′; but these colours will be very faint, as the disc occupies but a small part of the loop *a b*. The disc indeed may be made so small, that the extraordinary image will entirely disappear in this loop. In this case the ordinary image will be white, as all the reflected light will pass into it. At four reflexions the loop *c d* is little more than one half of *a b*, and consequently the light will vary much more rapidly from *d* to the maximum. When the analysing prism has its principal section in the plane −37° 22′, the extraordinary image at *c* will be coloured with red light above and blue below; and when it is in the plane + 31° 52′, the extraordinary image at *d* will be similarly coloured: The colours will be much brighter than in the case of two reflexions, and consequently the extraordinary image will not vanish. The consequence of this is, that the ordinary image is not white as before, but yellow, because a considerable portion of red and blue light are left in the extraordinary image.

As the number of reflexions increase, and the loops *e f*, *g h*, &c. diminish, the disc will occupy a greater proportion of the whole loop, and the red and blue colours with which it is crossed grow brighter and brighter, and come closer and closer to their line of junction in the middle of the disc. Hence a greater quantity of red and blue light is left out of the ordinary image, which on this account becomes yellower and yellower, and at last of a greenish hue.

In order to determine the position of the principal section of the analysing prism, when the extraordinary image is a minimum for any angle of incidence α, and any number of reflexions, let $\psi, \chi =$ the inclinations of the plane of polarization of the restored ray at the nodes a, b; $m, n =$ the inclinations or values of φ in the formula $\tan \varphi = \frac{\cos (i + i')}{\cos (i - i')}$ suited to the angles of incidence at the nodes; $x =$ the inclination φ suited to the incidence α.

Now it is obvious that at the one node, the position of the principal section of the analysing prism, when the extraordinary image is a minimum, is $+ \psi$, and that it gradually changes to 0° and then passes to $- \chi$, thus undergoing a change equal to $\psi + \chi$, while the inclination φ varies by a quantity equal to $m - n$. Hence calling I the inclination of the principal section to the plane $+ \psi$ at the angle of incidence α, we have $m - n : \psi + \chi = m - x : \mathrm{I}$.

$$\text{Hence } \mathrm{I} = \psi + \chi \left(\frac{m - x}{m - n}\right)$$

$$\text{When } x = n, \ \mathrm{I} = \psi + \chi.$$

$$\text{When } x = \frac{m - n}{2}, \quad \frac{m - x}{m - n} = \tfrac{1}{2}\, . \ \text{and } \mathrm{I} = \frac{\psi + \chi}{2}.$$

When the nodes of the loop are on different sides of the maximum polarizing angle, which happens only in the middle loop of 3, 5, 7, &c. reflexions, then m and n have opposite signs, and consequently their difference is $m + n$, and, as in this case $m = n$, the formula becomes $\mathrm{I} = \psi + \chi \left(\frac{m - x}{2\, m}\right)$.

It is impossible to determine the relative intensities of the ordinary and extraordinary image at any angle α, because this must depend on the relative intensities of the pencils by whose interference the elliptical polarization is produced. In silver these pencils approach to equality, but in steel and other metals they are very unequal.

Having thus shown how to determine the phenomena of elliptic polarization for any angle of incidence, for any number of reflexions, and for homogeneous light of any colour, I shall conclude this paper with some observations on a very remarkable anomaly which has presented itself in the course of this inquiry.

The phenomena which have been described, indicate very clearly that the angle of maximum elliptic polarization for one reflexion, or the angle of restoration after two equiangular reflexions, is the maximum polarizing angle of the metal, and consequently that its tangent is the index of refraction, as shown in the following Table*.

Names of Metals.	Angles of Maximum Polarization.	Index of Refraction.
Grain tin	78° 30′	4.915
Mercury	78 27	4.893
Galæna	78 10	4.773
Iron pyrites	77 30	4.511
Grey cobalt	76 56	4.309
Speculum metal	76 0	4.011
Antimony melted	75 25	3.844
Steel	75 0	3.732
Bismuth	74 50	3.689
Pure silver	73 0	3.271
Zinc	72 30	3.172
Tin plate hammered	70 50	2.879
Jewellers' gold	70 45	2.864

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* This Table completely proves that the refractive index of metals cannot be deduced from their reflective power; for silver, which surpasses them all in reflective power, stands very low in refractive power. Mr. HERSCHEL has noticed the difference between the indices of refraction deduced by these two methods in the case of mercury, which he makes 5.829 as given by its reflective power, and 4.16 as given by its polarizing angle. He makes the index for steel 2.85. When we consider that metals reflect the light that enters their substance, it must be obvious that the quantity of light which

This conclusion is not opposed by any of the phenomena, when we consider merely the mean refrangible ray to which these numbers refer: but when we use homogeneous light, a very strange anomaly occurs. The maximum angle of elliptic polarization for red light in the case of silver is 75° 30′, and for blue light 70° 30′, giving

		Angle. °	′
Index of refraction for red light	3.866 . . .	75	30
———————— mean ray	3.271 . . .	73	0
———————— blue light	2.824 . . .	70	30

the order of the refrangibilities being inverted.

The perfect similarity between the action of metals, and the total reflexion of the second surfaces of transparent bodies, promised to throw light upon this difficulty. I accordingly examined the formula of Fresnel for total reflexion, where the phase P is thus expressed:

$$\text{Cos P} = \frac{2\,m^2 (\sin i)^4 - (m^2 + 1)(\sin i)^2 + 1}{m^2 + 1\,(\sin i)^2 - 1}.$$

From this formula it follows that when $m = 1.51$, and $i = 54° 37'$, P will be 45° for one reflexion, and consequently for two reflexions $2\,\text{P} = 90°$. If m increases as it does for blue light, then the phase will be 45° at an angle of incidence above 54° 37′, that is, the circular polarization of the pencil will take place at a greater angle of incidence for blue than for red light, which is the reverse of what takes place in metals. Upon making the experiment, however, with total reflexion, we shall find that the blue rays are circularly polarized by two reflexions at a less angle than the red rays, thus approximating the two classes of phenomena even with respect to this singular anomaly. Hence in order to accommodate M. Fresnel's formula to homogeneous light of different colours, let m be the index of refraction for the homogeneous ray, and d the difference between it and the mean index, then the formula for the phase P will become

$$\text{Cos P} = \frac{2\,(m \pm d)^2 (\sin i)^4 - ((m \pm d)^2 + 1)(\sin i)^2 + 1}{((m \pm d)^2 + 1)(\sin i)^2 + 1}$$

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they reflect is a function not only of their refractive power, but of their transparency, which will be proportional to the intensity of the reflected pencil that has entered the metal. If this is the case, the transparency will be proportional to the inclination of the plane of the restored ray after two reflexions at the maximum polarizing angle, and the order of the transparencies of the different metals will be that of the Table, p. 294. See Mr. Herschel's Treatise on Light, § 594, 845.

the sign + being used for the red or least refrangible rays, and − for the blue or most refrangible.

For the same reason, in calculating the phases of an elliptically polarized homogeneous ray by means of the formula $\tan \varphi = \frac{\cos (i + i')}{\cos (i - i')}$, we must determine i' from the formula $\sin i' = \frac{\sin i}{m \pm d}$, the sign + being used for the red or least refrangible, and − for the blue or most refrangible rays.

As the theoretical considerations upon which M. Fresnel is said* to have constructed his formula, did not present to him the above anomaly, it would be in vain for me to seek an explanation of it. I may just mention, however, that at the second surfaces of bodies the angle of maximum polarization, or $\tan \frac{1}{m}$ is necessarily less for the least refrangible than for the mean rays, which is the reverse of what takes place at the first surface; and since the limit of total reflexion whose sine is $\frac{1}{m}$, or since the sphere of circular polarization commences sooner for the least than for the most refrangible rays, it might be expected that the angle of maximum circular polarization should be less for these rays, as I have found to be the case.

Although we do not understand the nature of the forces by which metals reflect the two oppositely polarized pencils, yet they act exactly like the second surfaces of transparent bodies when producing total reflexion. Setting out from a perpendicular incidence, the least refrangible rays begin to suffer the double reflexion sooner than the mean ray, and they sooner reach their maximum of elliptic polarization, thus exhibiting the inversion as it were of the spectrum, which we have noticed.

The theory of elliptic vibrations as given by Fresnel, will no doubt embrace the phenomena of elliptic polarization; and when the nature of metallic action shall be more thoroughly examined, we may expect to be able to trace the phenomenon under consideration to its true cause.

Allerly, *February* 19*th*, 1830.

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* I am acquainted with M. Fresnel's formula only from the account given of it by Mr. Herschel.

XXII. *Researches in physical astronomy.* *By* JOHN WILLIAM LUBBOCK, *Esq. Fellow of the Royal Society.*

Read April 29, 1830.

IN the first volume of the Mécanique Céleste, LAPLACE has given expressions for the variations of the elliptic constants, which are true when the square and higher powers of the disturbing force are neglected; and he has proved, upon the supposition that the planets move in the same direction, in orbits nearly circular and little inclined one to another, that the eccentricities and inclinations vary within small limits, thereby demonstrating within these conditions the stability of the planetary system. But these conditions are not necessary to the stability of a system of bodies, subject to the law of attraction, which obtains in our system. I have given in the following investigation the expressions for the variations of the elliptic constants, which are rigorously true whatever power of the disturbing force be retained; and it is easy to conclude from the form of their expressions, that however far the approximation be carried, the eccentricity, the major axis, and the tangent of the inclination of the orbit to a fixed plane, contain no term which varies with the time; their variations are all periodic, and they oscillate therefore within certain limits. This theorem is no longer true if the planet moves in a resisting medium.

I have also given some equations which obtain when an angle is taken for the independent variable, which in the elliptic movement is the eccentric anomaly, which are of remarkable simplicity, and which, as far as I know, have never been noticed, and the development of the disturbing function R to the quantities involving the squares and products of the eccentricities inclusive.

Let x, y, z denote the rectangular co-ordinates
r …… distance from the sun
$r^{\backprime}$ …… distance from the sun projected upon the plane x, y
*λ …… longitude reckoned upon the plane of its orbit
$\lambda^{\backprime}$ …… longitude reckoned upon the plane $x\,y$
s …… tangent of the latitude
υ …. a variable, which in the elliptic theory is the eccentric anomaly
m …. the mass
a …… semiaxis major
e …… eccentricity
ϖ …… longitude of the perihelion
ε …… longitude of the epoch
ν …… longitude of the ascending node
ι …… inclination of the orbit to the plane $x\,y$
α …. a constant quantity which accompanies υ
of the planet m.
M …. the mass of the sun.

$$M+m=\mu,\ \sqrt{\frac{\mu}{a^3}}=n.$$

$x=r^{\backprime}\cos\lambda^{\backprime}$, $y=r^{\backprime}\sin\lambda^{\backprime}$, $z=r^{\backprime}s$, and in the elliptic motion $r^{\backprime 2}d\,\lambda^{\backprime}=h\,d\,t$.

$$R=m_{,}\left\{\frac{x\,x_{,}+y\,y_{,}+z\,z_{,}}{\{x_{,}^2+y_{,}^2+z_{,}^2\}^{\frac{3}{2}}}-\frac{1}{\{(x-x_{,})^2+(y-y_{,})^2+(z-z_{,})^2\}^{\frac{1}{2}}}\right\}$$

$$=m_{,}\left\{\frac{r^{\backprime}\{\cos(\lambda^{\backprime}-\lambda^{\backprime}_{,})+s\,s_{,}\}}{r^{\backprime 2}_{,}(1+s_{,}^2)^{\frac{3}{2}}}-\frac{1}{\left\{r^{\backprime 2}(1+s^2)-2r^{\backprime}r^{\backprime}_{,}\{\cos(\lambda^{\backprime}-\lambda^{\backprime}_{,})+s\,s_{,}\}+r^{\backprime 2}_{,}(1+s_{,}^2)\right\}^{\frac{1}{2}}}\right\}$$

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* LAPLACE uses the letter v to denote longitude, u the eccentric anomaly, and φ the inclination of the orbit to a fixed plane; but as v is very frequently used to signify velocity, and φ geographical latitude, and as the letters of the Greek alphabet are generally used for angles, I have taken the letters λ, υ, and ι for these quantities.

$$= - m_{,}\left\{\begin{array}{l} \dfrac{1}{r_{,}^{\backprime}\sqrt{1+s_{,}^{2}}} + \dfrac{r^{\backprime 2}}{4 r_{,}^{\backprime 3}}\{1 + 3\cos(2\lambda^{\backprime} - 2\lambda_{,}^{\backprime}) + 12 s s_{,} \cos(\lambda^{\backprime} - \lambda_{,}^{\backprime}) - 2 s^{2}\} \\ \qquad + \dfrac{r^{\backprime 3}}{8 r_{,}^{\backprime 4}}\{3(1 - 4 s^{2})\cos(\lambda^{\backprime} - \lambda_{,}^{\backprime}) + 5\cos(3\lambda^{\backprime} - 3\lambda_{,}^{\backprime})\} \\ \qquad + \dfrac{r^{\backprime 4}}{64 r_{,}^{\backprime 5}}\{9 + 20\cos(2\lambda^{\backprime} - 2\lambda_{,}^{\backprime}) + 35\cos(4\lambda^{\backprime} - 4\lambda_{,}^{\backprime})\} \end{array}\right\}$$

$$\left(\frac{\mathrm{d}R}{\mathrm{d}r^{\backprime}}\right) = m_{,}\left\{\frac{\cos(\lambda^{\backprime} - \lambda_{,}^{\backprime}) + s s_{,}}{r_{,}^{\backprime 2}(1 + s_{,}^{2})^{\frac{3}{2}}} + \frac{r^{\backprime}(1 + s^{2}) - r_{,}^{\backprime}\{\cos(\lambda^{\backprime} - \lambda_{,}^{\backprime}) + s s_{,}\}}{\{r^{\backprime 2}(1 + s^{2}) - 2 r^{\backprime} r_{,}^{\backprime}\{\cos(\lambda^{\backprime} - \lambda_{,}^{\backprime}) + s s_{,}\} + r_{,}^{\backprime 2}(1 + s_{,}^{2})\}^{\frac{3}{2}}}\right\}$$

$$\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right) = - m_{,}\left\{\frac{r^{\backprime}\sin\lambda^{\backprime} - \lambda_{,}^{\backprime}}{r_{,}^{\backprime 2}(1 + s_{,}^{2})^{\frac{3}{2}}} - \frac{r^{\backprime} r_{,}^{\backprime}\sin(\lambda^{\backprime} - \lambda_{,}^{\backprime})}{\{r^{\backprime 2}(1 + s^{2}) - 2 r^{\backprime} r_{,}^{\backprime}\{\cos(\lambda^{\backprime} - \lambda_{,}^{\backprime}) + s s_{,}\} + r_{,}^{\backprime 2}(1 + s_{,}^{2})\}^{\frac{3}{2}}}\right\}$$

$$\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = m_{,}\left\{\frac{r^{\backprime} s_{,}}{r_{,}^{\backprime 2}(1 + s_{,}^{2})^{\frac{3}{2}}} + \frac{r^{\backprime 2} s - r^{\backprime} r_{,}^{\backprime} s_{,}}{\{r^{\backprime 2}(1 + s^{2}) - 2 r^{\backprime} r_{,}^{\backprime}\{\cos(\lambda^{\backprime} - \lambda_{,}^{\backprime}) + s s_{,}\} + r_{,}^{\backprime 2}(1 + s_{,}^{2})\}^{\frac{3}{2}}}\right\}$$

Let P be the place of the planet m, S the place of the sun, S N the intersection of the orbits of m and $m_{,}$, S L the line from which longitudes are reckoned, $\mathrm{P}_{,}^{\backprime}$, the projection of $\mathrm{P}_{,}$ upon the plane of the orbit of P; then if the plane $x\,y$ coincide with the orbit of m, $\mathrm{S\,P} = r$, $\mathrm{P\,S\,L} = \lambda$, $\mathrm{N\,S\,L} = \nu$, $\mathrm{P}_{,}\mathrm{S\,N} + \mathrm{N\,S\,L} = \lambda_{,}$, $\mathrm{P}_{,}^{\backprime}\mathrm{S\,L} = \lambda_{,}^{\backprime}$, $\mathrm{S\,P}_{,}^{\backprime} = r_{,}^{\backprime}$, $\mathrm{S\,P}_{,} = r_{,} = r_{,}^{\backprime}(1 + s_{,}^{2})^{\frac{1}{2}}$

$$R = m_{,}\left\{\frac{\mathrm{SP} \times \mathrm{SP}_{,}\cos\mathrm{PSP}_{,}}{\mathrm{SP}_{,}^{3}} - \frac{1}{\{\mathrm{SP}^{2} - 2\,\mathrm{SP} \times \mathrm{SP}_{,}\cos\mathrm{PSP}_{,} + \mathrm{SP}_{,}^{2}\}^{\frac{1}{2}}}\right\}$$

$$\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) = m_{,}\left\{\frac{\mathrm{SP}_{,}^{\backprime}\cos\mathrm{PSP}_{,}^{\backprime}}{\mathrm{SP}_{,}^{3}} + \frac{\mathrm{SP} - \mathrm{SP}_{,}^{\backprime}\cos\mathrm{PSP}_{,}^{\backprime}}{\{\mathrm{SP}^{2} - 2\,\mathrm{SP} \times \mathrm{SP}_{,}^{\backprime}\cos\mathrm{PSP}_{,}^{\backprime} + \mathrm{SP}_{,}^{2}\}^{\frac{3}{2}}}\right\}$$

$$r\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) = m_{,}\left\{\frac{\mathrm{SP} \times \mathrm{SP}_{,}\cos\mathrm{PSP}_{,}}{\mathrm{SP}_{,}^{3}} + \frac{\mathrm{SP}^{2} - \mathrm{SP} \times \mathrm{SP}_{,}\cos\mathrm{PSP}_{,}}{\{\mathrm{SP}^{2} - 2\,\mathrm{SP} \times \mathrm{SP}_{,}^{\backprime}\cos\mathrm{PSP}_{,}^{\backprime} + \mathrm{SP}_{,}^{2}\}^{\frac{3}{2}}}\right\}$$

$$\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right) = - m_{,}\left\{\frac{\mathrm{SP} \times \mathrm{SP}_{,}^{\backprime}\sin\mathrm{PSP}_{,}^{\backprime}}{\mathrm{SP}_{,}^{3}} - \frac{\mathrm{SP} \times \mathrm{SP}_{,}^{\backprime}\sin\mathrm{PSP}_{,}^{\backprime}}{\{\mathrm{SP}^{2} - 2\,\mathrm{SP} \times \mathrm{SP}_{,}^{\backprime}\cos\mathrm{PSP}_{,}^{\backprime} + \mathrm{SP}_{,}^{2}\}^{\frac{3}{2}}}\right\}$$

$$\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = m_{,}\left\{\frac{\mathrm{SP} \times \mathrm{P}_{,}\mathrm{P}_{,}^{\backprime}}{\mathrm{SP}_{,}^{3}} - \frac{\mathrm{SP} \times \mathrm{P}_{,}\mathrm{P}_{,}^{\backprime}}{\{\mathrm{SP}^{2} - 2\,\mathrm{SP} \times \mathrm{SP}_{,}^{\backprime}\cos\mathrm{PSP}_{,}^{\backprime} + \mathrm{SP}_{,}^{2}\}^{\frac{3}{2}}}\right\}$$

$$\cos(\lambda - \lambda_{,}) = \cos(\lambda - \nu)\cos(\lambda_{,} - \nu) + \cos\iota\sin(\lambda - \nu)\sin(\lambda_{,} - \nu)$$

$$= \cos^{2}\frac{\iota}{2}\cos(\lambda - \lambda_{,}) + \sin^{2}\frac{\iota}{2}\cos(\lambda + \lambda_{,} - 2\nu)$$

$$\tan(\lambda_{,}^{\backprime} - \nu) = \cos\iota\tan(\lambda_{,} - \nu)$$

$$\sin(\lambda_{,}^{\backprime} - \nu) = \frac{\cos\iota\tan(\lambda_{,} - \nu)}{(1 + \cos^{2}\iota\tan^{2}(\lambda_{,} - \nu))^{\frac{1}{2}}}$$

$$\cos(\lambda_{\prime}^{\backprime} - \nu) = \frac{1}{(1 + \cos^2 \iota \tan^2 (\lambda_{\prime} - \nu))^{\frac{1}{2}}}$$

$$r\, r_{\prime}^{\backprime} \sin(\lambda - \lambda_{\prime}^{\backprime}) = r\, r_{\prime}^{\backprime} . \{\sin\lambda \cos\lambda_{\prime}^{\backprime} - \cos\lambda \sin\lambda_{\prime}\}$$

$$\cos\lambda_{\prime}^{\backprime} = \cos(\lambda_{\prime}^{\backprime} - \nu)\cos\nu - \sin(\lambda_{\prime}^{\backprime} - \nu)\sin\nu$$

$$= \frac{\cos(\lambda_{\prime} - \nu)\cos\nu - \sin(\lambda_{\prime} - \nu)\sin\nu\left(1 - 2\sin^2\frac{\iota}{2}\right)}{(1 - \sin^2\iota \sin^2(\lambda_{\prime} - \nu)^{\frac{1}{2}}}$$

$$= \frac{\cos\lambda_{\prime} + 2\sin^2\frac{\iota}{2}\sin(\lambda_{\prime} - \nu)\sin\nu}{(1 - \sin^2\iota\sin^2(\lambda_{\prime} - \nu))^{\frac{1}{2}}}$$

$$\sin\lambda_{\prime}^{\backprime} = \sin(\lambda_{\prime}^{\backprime} - \nu)\cos\nu + \cos(\lambda_{\prime}^{\backprime} - \nu)\sin\nu$$

$$= \frac{\sin(\lambda_{\prime} - \nu)\cos\nu\left(1 - 2\sin^2\frac{\iota}{2}\right) + \cos(\lambda_{\prime} - \nu)\sin\nu}{(1 - \sin^2\iota\sin^2(\lambda_{\prime} - \nu))^{\frac{1}{2}}}$$

$$\sin \mathrm{P}_{\prime}\mathrm{S}\,\mathrm{P}_{\prime}^{\backprime} = \sin\iota\sin(\lambda_{\prime} - \nu), \quad r_{\prime}^{\backprime} = r_{\prime}\cos\mathrm{P}_{\prime}\mathrm{S}\,\mathrm{P}_{\prime}^{\backprime} = r_{\prime}\left(1 - \sin^2\iota\sin^2(\lambda_{\prime} - \nu)\right)^{\frac{1}{2}}$$

therefore,

$$r\, r_{\prime}^{\backprime}\sin(\lambda - \lambda_{\prime}^{\backprime}) = r\, r_{\prime}\left\{\sin(\lambda - \lambda_{\prime}) + 2\sin^2\frac{\iota}{2}\sin(\lambda_{\prime} - \nu)\cos(\lambda - \nu)\right\}$$

$$= r\, r_{\prime}\left\{\cos^2\frac{\iota}{2}\sin(\lambda - \lambda_{\prime}) + \sin^2\frac{\iota}{2}\sin(\lambda + \lambda_{\prime} - 2\nu)\right\}$$

similarly it may be shown that

$$r\, r_{\prime}^{\backprime}\cos(\lambda - \lambda_{\prime}^{\backprime}) = r\, r_{\prime}\left\{\cos^2\frac{\iota}{2}\cos(\lambda - \lambda_{\prime}) + \sin^2\frac{\iota}{2}\cos(\lambda + \lambda_{\prime} - 2\nu)\right\}$$

$$\mathrm{S}\,\mathrm{P}_{\prime} = \frac{a_{\prime}(1 - e_{\prime}^2)}{1 + e_{\prime}\cos(\mathrm{P}_{\prime}\mathrm{S}\,\mathrm{N} + \mathrm{N}\,\mathrm{S}\,\mathrm{L} - \varpi_{\prime})} = \frac{a_{\prime}(1 - e_{\prime}^2)}{1 + e_{\prime}\cos(\lambda_{\prime} - \varpi_{\prime})}$$

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{\mu x}{r^3} + \left(\frac{\mathrm{d}R}{\mathrm{d}x}\right) = 0, \quad \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \frac{\mu y}{r^3} + \left(\frac{\mathrm{d}R}{\mathrm{d}y}\right) = 0, \quad \frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + \frac{\mu z}{r^3} + \left(\frac{\mathrm{d}R}{\mathrm{d}z}\right) = 0$$

$$\frac{r^{\backprime 2}\mathrm{d}\lambda^{\backprime 2} + \mathrm{d}r^{\backprime 2}(1 + s^2) + 2r^{\backprime}s\,\mathrm{d}r^{\backprime}\mathrm{d}s + r^{\backprime 2}\mathrm{d}s^2}{\mathrm{d}t^2} - \frac{2\mu}{r^{\backprime}(1 + s^2)^{\frac{1}{2}}} + \frac{\mu}{a} + 2\int \mathrm{d}R = 0$$

d R being the differential of R with regard only to the co-ordinates of the planet m.

$$\frac{\mathrm{d}^2 r^{\backprime}(1 + s^2) - r^{\backprime}\mathrm{d}\lambda^{\backprime 2} + 2s\,\mathrm{d}r^{\backprime}\mathrm{d}s + r^{\backprime}s\,\mathrm{d}^2 s}{\mathrm{d}t^2} + \frac{\mu}{r^{\backprime}(1 + s^2)^{\frac{1}{2}}} + \left(\frac{\mathrm{d}R}{\mathrm{d}r^{\backprime}}\right) = 0$$

$$\frac{\mathrm{d}.\dfrac{r^{\backprime 2}\mathrm{d}\lambda^{\backprime}}{\mathrm{d}t}}{\mathrm{d}t} + \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda^{\backprime}}\right) = 0$$

$$\frac{r^{\backprime}s\,\mathrm{d}^2 r^{\backprime} + 2r^{\backprime}\mathrm{d}r^{\backprime}\mathrm{d}s + r^{\backprime 2}\mathrm{d}^2 s}{\mathrm{d}t^2} + \frac{s}{r^{\backprime}(1 + s^2)^{\frac{3}{2}}} + \left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$$\frac{r^{\backprime}\mathrm{d}^2 r^{\backprime} - r^{\backprime 2}\mathrm{d}\lambda^{\backprime 2}}{\mathrm{d}t^2} + \frac{\mu}{r^{\backprime}(1+s^2)^{\frac{3}{2}}} + r^{\backprime}\left(\frac{\mathrm{d}R}{\mathrm{d}r^{\backprime}}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$$\frac{r^{\backprime 2}\mathrm{d}^2 s + 2r^{\backprime}\mathrm{d}r^{\backprime}\mathrm{d}s - r^{\backprime 2}s\,\mathrm{d}\lambda^{\backprime 2}}{\mathrm{d}t^2} + (1+s^2)\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - r^{\backprime}s\left(\frac{\mathrm{d}R}{\mathrm{d}r^{\backprime}}\right) = 0$$

$$\frac{\mathrm{d}^2 . r^{\backprime 2}(1+s^2)}{2\,\mathrm{d}t^2} - \frac{\mu}{r^{\backprime}(1+s^2)^{\frac{3}{2}}} + \frac{\mu}{a} + 2\int \mathrm{d}R + r^{\backprime}\left(\frac{\mathrm{d}R}{\mathrm{d}r^{\backprime}}\right) = 0$$

Making λ the independent variable instead of *t*,

$$\frac{\mathrm{d}^2 r^{\backprime}}{\mathrm{d}t^2} - 2\frac{\mathrm{d}r^{\backprime}\mathrm{d}^2 t}{\mathrm{d}t^3} - r^{\backprime 2}\frac{\mathrm{d}\lambda^{\backprime 2}}{\mathrm{d}t^2} + \frac{\mu}{r^{\backprime}(1+s^2)^{\frac{3}{2}}} + r^{\backprime}\left(\frac{\mathrm{d}R}{\mathrm{d}r^{\backprime}}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$r^{\backprime 4}\frac{\mathrm{d}\lambda^{\backprime 2}}{\mathrm{d}t^2} = h^2 - 2\int r^{\backprime 2}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda^{\backprime}}\right)\mathrm{d}\lambda^{\backprime}$, *h* being a constant,

$$4r^{\backprime 3}\,dr^{\backprime}\frac{\mathrm{d}\lambda^{\backprime 2}}{\mathrm{d}t^2} - 2r^{\backprime 4}\frac{\mathrm{d}\lambda^{\backprime 2}\mathrm{d}^2 t}{\mathrm{d}t^3} = -2r^{\backprime 2}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda^{\backprime}}\right)\mathrm{d}\lambda^{\backprime}$$

$$\left\{\frac{\mathrm{d}^2 . \frac{1}{r^{\backprime}}}{\mathrm{d}\lambda^{\backprime 2}} + \frac{1}{r^{\backprime}}\right\}\left\{1 - \frac{2}{h^2}\int r^{\backprime 2}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda^{\backprime}}\right)\mathrm{d}\lambda^{\backprime}\right\} - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}}$$

$$- \frac{r^{\backprime}}{h^2}\left\{r^{\backprime}\left(\frac{\mathrm{d}R}{\mathrm{d}r^{\backprime}}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - \frac{1}{r^{\backprime}}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda^{\backprime}}\right)\frac{\mathrm{d}r^{\backprime}}{\mathrm{d}\lambda^{\backprime}}\right\} = 0$$

$$\left\{\frac{\mathrm{d}^2 s}{\mathrm{d}\lambda^{\backprime 2}} + s\right\}\left\{1 - \frac{2}{h^2}\int r^{\backprime 2}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda^{\backprime}}\right)\mathrm{d}\lambda^{\backprime}\right\}$$

$$+ \frac{r^{\backprime 2}}{h^2}\left\{(1+s^2)\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - r^{\backprime}\left(\frac{\mathrm{d}R}{\mathrm{d}r^{\backprime}}\right) - \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda^{\backprime}}\right)\frac{\mathrm{d}s}{\mathrm{d}\lambda^{\backprime}}\right\} = 0$$

When the disturbing force is neglected

$$\frac{\mathrm{d}.r^{\backprime 2}\mathrm{d}\lambda^{\backprime}}{\mathrm{d}t^{\backprime}} = 0, \quad \frac{\mathrm{d}^2 . \frac{1}{r^{\backprime}}}{\mathrm{d}\lambda^{\backprime 2}} + \frac{1}{r^{\backprime}} - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}} = 0, \quad \frac{\mathrm{d}^2 s}{\mathrm{d}\lambda^{\backprime 2}} + s = 0$$

of which equations the integrals are,

$$r^{\backprime 2}\,\mathrm{d}\lambda^{\backprime} = h\,\mathrm{d}t, \quad \frac{1}{r^{\backprime}} = \frac{\mu\cos\iota^2}{h^2}\left\{(1+s^2)^{\frac{1}{2}} + e\cos(\lambda^{\backprime} - \varpi)\right\}$$

$$s = \tan\iota\,\sin(\lambda^{\backprime} - \nu).$$

If $\mathrm{d}t = \sqrt{\frac{a}{\mu}}\,r^{\backprime}\,\mathrm{d}v$, and v be taken for the independent variable

$$r^{\backprime}\frac{\mathrm{d}^2 r^{\backprime}}{\mathrm{d}t^2} - r^{\backprime}\,\mathrm{d}r^{\backprime}\frac{\mathrm{d}^2 t}{\mathrm{d}t^3} - r^{\backprime 2}\frac{\mathrm{d}\lambda^{\backprime 2}}{\mathrm{d}t^2} + \frac{\mu}{r^{\backprime}(1+s^2)^{\frac{3}{2}}} + r^{\backprime}\left(\frac{\mathrm{d}R}{\mathrm{d}r^{\backprime}}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$$\mathrm{d}^2 t = \left(\frac{a}{\mu}\right)^{\frac{1}{2}} \mathrm{d}\, r' \, \mathrm{d}\, v$$

$$\frac{\mathrm{d}^2 r'}{\mathrm{d}\, v^2} + \left(\frac{\mathrm{d}.r' s}{\mathrm{d}\, v}\right)^2 - \frac{a}{(1+s^2)^{\frac{3}{2}}} + r' + \frac{a r'}{\mu}\left\{2\int \mathrm{d}\, R + r'\left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r'}\right) - s\left(\frac{\mathrm{d}\, R}{\mathrm{d}\, s}\right)\right\} = 0.$$

If the orbit of the planet m coincide with the plane $x\, y$, s is of the order of the disturbing force, of which therefore neglecting the square, $r' = r$, $\lambda' = \lambda$

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\, v^2} - a + r + \frac{a r}{\mu}\left\{2\int \mathrm{d}\, R + r\left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right)\right\} = 0.$$

When the disturbing force is neglected, the integral of this equation is $r = a\,\{1 - e \cos (v - \alpha)\}$, v being the eccentric anomaly.

$$n\, t + \varepsilon - \varpi = v - \alpha - e \sin (v - \alpha)$$

$$\tan \frac{\lambda - \varpi}{2} = \left\{\frac{1+e}{1-e}\right\}^{\frac{1}{2}} \tan \frac{v - \alpha}{2}.$$

If Q be put for the quantity $\frac{a\, r}{\mu}\left\{2\int \mathrm{d}\, R + r\left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right)\right\}$ and the constant α, which may afterwards be replaced, be omitted for the present,

$$r = a\,\{1 - e \cos v\} - \sin v \int Q \cos v\, \mathrm{d}\, v + \cos v \int Q \sin v\, \mathrm{d}\, v$$

$$\mathrm{d}\, t = \sqrt{\frac{a}{\mu}}\left\{a\{1 - e\cos v\} - \sin v\int Q \cos v\, \mathrm{d}\, v + \cos v \int Q \sin v\, \mathrm{d}\, v\right\} \mathrm{d}\, v$$

$$n\, t + \varepsilon - \varpi = v - e \sin v - \frac{1}{a}\left\{\int Q\, \mathrm{d}\, v - \cos v \int Q \cos v\, \mathrm{d}\, v - \sin v \int Q \sin v\, \mathrm{d}\, v\right\}$$

If $v = \mathrm{f}\,(n\, t + \varepsilon - \varpi)$ in the elliptic theory, then neglecting the square of the disturbing force,

$$v = \mathrm{f}\,(n t + \varepsilon - \varpi) + \frac{\mathrm{d}.\mathrm{f}(n t + \varepsilon - \varpi)}{\mathrm{d}\, t}\,\frac{1}{a}\left\{\int Q\, \mathrm{d}\, v - \cos v \int Q \cos v\, \mathrm{d}\, v - \sin v \int Q \sin v\right\} \mathrm{d}\, v$$

If $\delta\, v$, $\delta\, r$ denote the values of those parts of r and v which are due to the first power of the disturbing force,

$$\delta\, v = \frac{\mathrm{d}\, v}{a\, n\, \mathrm{d}\, t}\left\{\int Q\, \mathrm{d}\, v - \cos v \int Q \cos v\, \mathrm{d}\, v - \sin v \int Q \sin v\, \mathrm{d}\, v\right\}$$

$$\delta\, r = a\, e \sin v\, \delta\, v - \sin v \int Q \cos v\, \mathrm{d}\, v + \cos v \int Q \sin v\, \mathrm{d}\, v$$

$$= \frac{\sin v}{1 - e \cos v}\int Q\, \{e - \cos v\}\, \mathrm{d}\, v - \frac{e - \cos v}{1 - e \cos v}\int Q \sin v\, \mathrm{d}\, v.$$

In the elliptic theory,

$$\mathrm{d}\, v = \frac{n\, \mathrm{d}\, t}{1 - e \cos v}, \qquad \frac{\cos v - e}{1 - e \cos v} = \cos \lambda, \qquad \frac{\sin v\, (1 - e^2)^{\frac{1}{2}}}{1 - e \cos v} = \sin \lambda$$

therefore,

$$\delta r = \frac{a\,n \cos\lambda \int r \sin\lambda \left\{ 2\int \mathrm{d}R + r\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) \right\} \mathrm{d}t - a\,n \sin\lambda \int r \cos\lambda \left\{ 2\int \mathrm{d}R + r\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) \right\} \mathrm{d}t \Big\}}{\mu(1-e^2)^{\frac{1}{2}}}$$

which is the equation X of the Mécanique Céleste, vol. i. p. 258.

Multiplying the equation of p. 114, l. 2, by $\frac{2r'^2 \mathrm{d}\lambda'}{\mathrm{d}t}$, and integrating, $\frac{r'^4 \mathrm{d}\lambda'^2}{\mathrm{d}t^2} + 2\int r'^2 \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\mathrm{d}\lambda' = h_0^2$, if $h^2 = h_0^2 - 2\int r'^2\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\mathrm{d}\lambda'$, h being variable, and h_0 the value of h at a given epoch, $h\,\mathrm{d}h = -\,r'^2\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\mathrm{d}\lambda'$, $r^2\,\mathrm{d}\lambda' = h\,\mathrm{d}t$, and making λ' the independent variable instead of t, $2\,r'\,\mathrm{d}r'\,\mathrm{d}\lambda' = \mathrm{d}h\,\mathrm{d}t + h\,\mathrm{d}^2 t$

$$r'\frac{\mathrm{d}^2 r'}{\mathrm{d}t^2} - \frac{r'\mathrm{d}r'\mathrm{d}^2 t}{\mathrm{d}t^3} - \frac{r^2\mathrm{d}\lambda'^2}{\mathrm{d}t^2} + \frac{\mu}{r'(1+s^2)^{\frac{3}{2}}} + r'\left(\frac{\mathrm{d}R}{\mathrm{d}r'}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$$r'\left(\frac{\mathrm{d}^2 r'}{\mathrm{d}t^2}\right) + \frac{r'\mathrm{d}r'\mathrm{d}t}{h\,\mathrm{d}t^2} - \frac{2r'^3\mathrm{d}r'^2\mathrm{d}\lambda}{h\,\mathrm{d}t^3} - \frac{r'^2\mathrm{d}\lambda'^2}{\mathrm{d}t^2} + \frac{\mu}{r'(1+3^2)^{\frac{3}{2}}} + r'\left(\frac{\mathrm{d}R}{\mathrm{d}r'}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$$\frac{\mathrm{d}^2 . \frac{1}{r'}}{\mathrm{d}\lambda'^2} + \frac{1}{r'} - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}} - \frac{r'}{h^2}\left\{ r'\left(\frac{\mathrm{d}R}{\mathrm{d}r'}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - \frac{1}{r'}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\frac{\mathrm{d}r'}{\mathrm{d}\lambda'} \right\} = 0$$

$$\frac{\mathrm{d}^2 s}{\mathrm{d}\lambda'^2} + s + \frac{r'^2}{h^2}\left\{ (1+s^2)\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - s\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\frac{\mathrm{d}s'}{\mathrm{d}\lambda'} \right\} = 0$$

If all the constants in the elliptic integrals are supposed to vary, subject to the condition that they still satisfy these differential equations, and that the form of the first differential coefficients $\frac{\mathrm{d}r'}{\mathrm{d}\lambda'}$, $\frac{\mathrm{d}s}{\mathrm{d}\lambda'}$, remains unaltered,

$$\frac{\mathrm{d} . \frac{1}{r'}}{\mathrm{d}\lambda'} = \frac{\mu \cos\iota^2}{h^2}\left\{ \frac{s}{(1+s^2)^{\frac{1}{2}}}\frac{\mathrm{d}s}{\mathrm{d}\lambda'} - e\sin(\lambda' - \varpi) \right\}, \qquad \frac{\mathrm{d}s}{\mathrm{d}\lambda'} = \tan\iota\cos(\lambda' - \nu)$$

$$-\,2\left\{(1+s^2)^{\frac{1}{2}} + e\cos(\lambda' - \varpi)\right\}\left\{h^2 \sin\iota\,\mathrm{d}\iota + \cos\iota\,h\,\mathrm{d}h\right\}$$
$$+\,h^2\cos\iota\cos(\lambda' - \varpi)\,\mathrm{d}e + h^2\cos\iota\,e\sin(\lambda' - \varpi)\,\mathrm{d}\varpi = 0$$

$$-\,2\left\{\frac{s\tan\iota\cos(\lambda' - \nu)}{(1+s^2)^{\frac{1}{2}}} - e\sin(\lambda' - \varpi)\right\}\left\{h^2\sin\iota\,\mathrm{d}\iota + \cos\iota\,h\,\mathrm{d}h\right\}$$
$$-\,h^2\cos\iota\sin(\lambda' - \varpi)\,\mathrm{d}e + h^2\cos\iota\,e\cos(\lambda' - \varpi)\,\mathrm{d}\varpi$$
$$-\,\frac{\cos\iota\,s\,r'^2}{(1+s^2)^{\frac{1}{2}}}\left\{(1+s^2)\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - r'\left(\frac{\mathrm{d}R}{\mathrm{d}r'}\right) - \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)\left(\frac{\mathrm{d}s}{\mathrm{d}\lambda'}\right)\right\}\mathrm{d}\lambda'$$

$$- \frac{h^2 r'}{\mu \cos \iota} \left\{ r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - \frac{1}{r'} \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} r'}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$\sin (\lambda' - \nu) \frac{\mathrm{d} \iota}{\cos \iota^2} - \tan \iota \cos (\lambda' - \nu) \mathrm{d} \nu = 0$$

$$\cos (\lambda' - \nu) \frac{\mathrm{d} \iota}{\cos \iota^2} + \tan \iota \sin (\lambda' - \nu) \mathrm{d} \nu + \frac{r'^2}{h^2} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$h \,\mathrm{d} h = - r'^2 \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \mathrm{d} \lambda'$$

Whence by elimination,

$$h^2 \cos \iota \,\mathrm{d} e + 2 \left\{ (1 + s^2)^{\frac{1}{2}} \cos (\lambda' - \varpi) + e - \frac{\tan \iota \, s \cos (\lambda' - \nu) \sin (\lambda' - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \right\}$$

$$\left\{ \sin \iota \cos^2 \iota \, r'^2 \cos (\lambda' - \nu) \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} + \cos \iota \, r'^2 \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda'$$

$$+ \frac{s \cos \iota \, r'^2 \sin (\lambda' - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda'$$

$$+ \frac{h^2 r'}{\mu \cos \iota} \sin (\lambda' - \varpi) \left\{ r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - \frac{1}{r'} \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} r'}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$h^2 \cos \iota \, e \,\mathrm{d} \varpi + 2 \left\{ (1 + s^2)^{\frac{1}{2}} \sin (\lambda' - \varpi) + \frac{s \tan \iota \cos (\lambda' - \nu) \cos (\lambda' - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \right\}$$

$$\left\{ \sin \iota \cos^2 \iota \, r'^2 \cos (\lambda' - \nu) \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} + \cos \iota \, r'^2 \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda'$$

$$- \frac{s \cos \iota \, r'^2 \cos (\lambda - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda'$$

$$- \frac{h^2 r'}{\mu \cos \iota} \cos (\lambda' - \varpi) \left\{ r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - \frac{1}{r'} \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} r'}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$d \iota + \frac{r'^2 \cos \iota^2 \cos (\lambda' - \nu)}{h^2} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} r'}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$d \nu + \frac{r'^2 \sin (\lambda' - \nu)}{h^2 \tan \iota} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

If $x = \frac{r \cos \lambda'}{(1 + s^2)^{\frac{1}{2}}}$, $y = \frac{r \sin \lambda'}{(1 + s^2)^{\frac{1}{2}}}$, $z = \frac{r s}{(1 + s^2)^{\frac{1}{2}}}$,

$$\frac{d r^2 + \frac{r^2}{1 + s^2}\left(\frac{d s^2}{1 + s^2} + d \lambda'^2\right)}{d t^2} - \frac{2 \mu}{r} + \frac{\mu}{a} + 2 \int d R = 0$$

$$\frac{d^2 r}{d t^2} - \frac{r}{1 + s^2} \frac{\left(\frac{d s^2}{1 + s^2} + d \lambda'^2\right)}{d t^2} + \frac{\mu}{r^2} + \left(\frac{d R}{d r}\right) = 0$$

$$\frac{d \, . \frac{r^2}{1 + s^2} \, . \frac{d \lambda'}{d t}}{d t} + \left(\frac{d R}{d \lambda'}\right) = 0$$

$$\frac{r^2 d^2 s + 2 r d r d s - \frac{2 r^2 s d s^2}{(1 + s^2)} + r^2 s d \lambda'^2}{d t^2} + (1 + s^2)^2 \left(\frac{d R}{d s}\right) = 0$$

Of which equations the integrals are

$$\frac{r^2}{1 + s^2} \, . \, d \lambda' = h \, d \, t, \ \frac{1}{r} = \frac{\mu \cos \iota^2}{h^2 (1 + s^2)^{\frac{1}{2}}} \left\{(1 + s^2)^{\frac{1}{2}} + e \cos (\lambda' - \varpi)\right\}$$

$$s = \tan \iota \sin (\lambda' - \nu)$$

If $d \, t = \sqrt{\frac{a}{\mu}} \, r \, d \, v$, and v be taken for the independent variable

$$\frac{d^2 r}{d v^2} - a + r + \frac{a r}{\mu} \left\{2 \int d R + r \left(\frac{d R}{d r}\right)\right\} = 0$$

$r = a \{1 - e' \cos (v - \alpha)\}$ in the elliptic motion.

e' is accented for the present in order to distinguish it from e.

If the constants in the elliptic integrals are supposed to vary, subject to the condition that they still satisfy these differential equations, and that the form of the first differential coefficient $\frac{d r}{d v}$ remains unaltered,

$$d^2 t = \frac{r d v d a}{2 \sqrt{a \mu}} + \sqrt{\frac{a}{\mu}} \, d \, r \, d \, v, \quad \frac{r d^2 r}{d t^2} = \frac{r d r^2}{d t^2} - \frac{r d r d^2 t}{d t^2}$$

$$\frac{d^2 r}{d v^2} - a + r + \frac{a r^2}{\mu} \left(\frac{d R}{d r}\right) - \frac{d r d a}{2 a d v^2} = 0$$

$$\Big(1 - e' \cos (v - \alpha)\Big) \, d \, a - a \cos (v - \alpha) \, d \, e' - a \, e' \sin (v - \alpha) \, d \, \alpha = 0$$

$$e' \sin (v - \alpha) \, d \, a + a \sin (v - \alpha) \, d \, e' - a \, e' \cos (v - \alpha) \, d \, \alpha$$

$$+\frac{a\,r^2}{\mu}\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)\mathrm{d}\,v+\frac{a^2 e}{\mu}\sin\,(v-\alpha)\,\mathrm{d}\,R=0$$

$$\mathrm{d}\,a=-\frac{2\,a^2}{\mu}\,\mathrm{d}\,R$$

$$\mathrm{d}\,e'=\frac{a}{\mu}\{2\,e'-2\cos\,(v-\alpha)-e'\sin\,(v-\alpha)^2\}\,\mathrm{d}\,R-\frac{r^2}{\mu}\sin\,(v-\alpha)\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)\mathrm{d}\,v$$

$$e\,\mathrm{d}\,\alpha=\frac{a}{\mu}\sin\,(v-\alpha)\,\{e'\cos\,(v-\alpha)-2\}\,\mathrm{d}\,R+\frac{r^2}{\mu}\cos\,(v-\alpha)\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)\mathrm{d}\,v$$

$$\int n\,\mathrm{d}\,t+\varepsilon-\varpi=v-\alpha-e'\sin\,(v-\alpha)$$

$$\mathrm{d}\,\varepsilon-\mathrm{d}\,\varpi=-\,\mathrm{d}\,\alpha-\sin\,(v-\alpha)\,\mathrm{d}\,e'+e'\cos\,(v-\alpha)\,\mathrm{d}\,\alpha.$$

The equation of condition which obtains between the constants a, e, ϖ, ν, ι and h may be found from the equation

$$\frac{\mathrm{d}\,r^2+\frac{r^2}{1+s^2}\left(\frac{\mathrm{d}\,s^2}{1+s^2}+\mathrm{d}\,\lambda'^2\right)}{\mathrm{d}\,t^2}-\frac{2\,\mu}{r}+\frac{\mu}{a}=0$$

$$\frac{\mathrm{d}\,r^2}{\mathrm{d}\,t^2}+\frac{h^2}{r^2\cos\iota^2}-\frac{2\,\mu}{r}+\frac{\mu}{a}=0$$

which gives

$$\frac{\mu\cos\iota^2}{h^2}\Big\{e^2\cos^2\iota\sin^2(\nu-\varpi)-\{1+e\cos(\nu-\varpi)\}\{1-e\cos(\nu-\varpi)\}\Big\}+\frac{1}{a}=0$$

Equating the values of r which have been found,

$$\frac{h^2\sqrt{1+s^2}}{\mu\cos\iota^2\{\sqrt{1+s^2}+e\cos(\lambda-\varpi)\}}=a\,\{1-e'\cos\,(v-\alpha)\}$$

since the origin of the angle v is arbitrary, we may suppose $\lambda-\varpi=0$, and $v-\alpha=0$ at the same time,

so that $$\frac{h^2\sqrt{1+\tan\iota^2\sin(\varpi-\nu)^2}}{\mu\cos\iota^2\{\sqrt{1+\tan\iota^2\sin(\varpi-\nu)^2}+e\}}=a\,(1-e')$$

All the equations which have hitherto been proved are rigorously true, whatever powers of the disturbing function be retained. They are susceptible of simplification when the square and higher powers of the disturbing function are neglected: in this case, if the orbit be supposed to coincide with the plane $x\,y$, $\tan\iota=0$, and if the longitude be reckoned from the perihelion of the planet P,

$$h^2\,\mathrm{d}\,e+\{2\cos\lambda+e+e\cos\lambda^2\}\,r^2\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)\mathrm{d}\,\lambda+\frac{h^2r^2\sin\lambda}{\mu}\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)\mathrm{d}\,\lambda=0$$

$$h^2 e\, \mathrm{d}\varpi + \{2 + \cos\lambda\} \sin\lambda\, r^2 \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right) \mathrm{d}\lambda - h^2 r^2 \cos\lambda \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) \mathrm{d}\lambda = 0$$

$$\mathrm{d}\iota + r^2 \frac{\cos(\lambda - \nu)}{\mu} \left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) \mathrm{d}\lambda = 0$$

$$\mathrm{d}\nu + r^2 \frac{\sin(\lambda - \nu)}{\mu} \left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) \mathrm{d}\lambda = 0$$

by equation, p. 336, line 12, $\mu\, a\,(1 - e^2) = h^2$, and by equation, line 17, $e = e'$.

$$\mathrm{d}e = \frac{a}{\mu} \left\{2e - 2\cos(v - \alpha) - e \sin(v - \alpha)^2\right\} \mathrm{d}R - \frac{r^2}{\mu} \sin(v - \alpha) \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) \mathrm{d}v$$

$$e\, \mathrm{d}\alpha = \frac{a}{\mu} \left\{e \cos(v - \alpha) - 2\right\} \sin(v - \alpha)\, \mathrm{d}R + \frac{r^2}{\mu} \cos(v - \alpha) \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) \mathrm{d}v$$

$$\int n\, \mathrm{d}t + \varepsilon - \varpi = v - \alpha - e \sin(v - \alpha)$$

$$\mathrm{d}\varepsilon - \mathrm{d}\varpi = - \{1 - e\cos(v - \alpha)\}\, \mathrm{d}\alpha - \sin(v - \alpha)\, \mathrm{d}e$$

$$\mathrm{d}\varepsilon - \mathrm{d}\varpi = - \frac{r^2}{a^2 \sqrt{1 - e^2}}\, \mathrm{d}\varpi - \left\{\frac{r}{a(1 - e^2)} + 1\right\} \sin(v - \alpha)\, \mathrm{d}e$$

$$\frac{\dfrac{h^2}{\mu}}{1 + e\cos(\lambda - \varpi)} = a\{1 - e\cos(v - \alpha)\}$$

$$\mathrm{d}e = - \frac{a n\, \mathrm{d}t}{\mu \sqrt{1 - e^2}} \left\{2\cos(\lambda - \varpi) + e + e\cos(\lambda - \varpi)^2\right\} \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)$$
$$- \frac{a^2 n\, \mathrm{d}t \sqrt{1 - e^2}}{\mu} \sin(\lambda - \varpi) \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)$$

$$e\, \mathrm{d}\varpi = - \frac{a n\, \mathrm{d}t}{\mu \sqrt{1 - e^2}} \left\{2 + e\cos(\lambda - \varpi)\right\} \sin(\lambda - \varpi) \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)$$
$$+ \frac{a^2 n\, \mathrm{d}t \sqrt{1 - e^2}}{\mu} \cos(\lambda - \varpi) \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)$$

$$\mathrm{d}\varepsilon - \mathrm{d}\varpi = - \sqrt{1 - e^2}\, \mathrm{d}\varpi + \frac{2a^2 n(1 - e^2)\, \mathrm{d}t}{\mu\{1 + e\cos(\lambda - \varpi)\}} \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)$$

If the longitudes be reckoned from the perihelion of the planet P,

$$\frac{\mathrm{d}e}{\mathrm{d}t} = - \frac{a n(\cos\lambda + e)}{\mu \sqrt{1 - e^2}} \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right) - \frac{m_{,} a^2 n \sqrt{1 - e^2}}{\mu} \left\{\frac{r_{,}' \sin\lambda_{,}'}{r_{,}^3} + \frac{r\sin\lambda - r_{,}' \sin\lambda_{,}'}{\{r^2 - 2rr_{,}' \cos(\lambda - \lambda_{,}') + r_{,}'^2\}^{\frac{3}{2}}}\right\}$$

$$\frac{d\varpi}{dt} = -\frac{a n \sin\lambda}{\mu\sqrt{1-e^2}}\left(\frac{dR}{d\lambda}\right) - \frac{m_{,} a^2 n\sqrt{1-e^2}}{\mu}\left\{\frac{r_{,}'\cos\lambda_{,}'}{r_{,}^3} + \frac{r\cos\lambda - r_{,}'\cos\lambda_{,}'}{\{r^2 - 2 r r_{,}'\cos(\lambda-\lambda_{,}') + r_{,}^2\}^{\frac{3}{2}}}\right\}$$

and since $n\,dt = \frac{r}{a}\,dv$,

$$\frac{de}{dv} = -\frac{r(\cos\lambda + e)}{\mu\sqrt{1-e^2}}\left(\frac{dR}{d\lambda}\right) - \frac{m_{,} a r\sqrt{1-e^2}}{\mu}\left\{\frac{r_{,}'\sin\lambda_{,}'}{r_{,}^3} + \frac{r\sin\lambda - r_{,}'\sin\lambda_{,}'}{\{r^2 - 2 r r_{,}'\cos(\lambda-\lambda_{,}') + r_{,}^2\}^{\frac{3}{2}}}\right\}$$

$$\frac{e\,d\varpi}{dv} = -\frac{r\sin\lambda}{\mu\sqrt{1-e^2}}\left(\frac{dR}{d\lambda}\right) + \frac{m_{,} a r\sqrt{1-e^2}}{\mu}\left\{\frac{r_{,}'\cos\lambda_{,}'}{r_{,}^3} + \frac{r\cos\lambda - r_{,}'\cos\lambda_{,}'}{\{r^2 - 2 r r_{,}'\cos(\lambda-\lambda_{,}') + r_{,}^2\}^{\frac{3}{2}}}\right\}$$

$$\frac{d\epsilon - d\varpi}{dv} = -\frac{r^2}{a^2\sqrt{1-e^2}}\frac{d\varpi}{dv} - \left\{\frac{r}{a(1-e^2)} + 1\right\}\sin v\,\frac{de}{dv}$$

$$da = -\frac{2a^2}{\mu}\,dR$$

and since $r^2\,d\lambda = \sqrt{\mu\,a\,(1-e^2)}\,dt = a\sqrt{1-e^2}\,r\,dv$

$$\frac{d\iota}{dv} + \frac{a\sqrt{1-e^2}\,r\cos\lambda}{\mu}\left(\frac{dR}{ds}\right) = 0$$

$$\frac{d\nu}{dv} + \frac{a\sqrt{1-e^2}\,r\sin\lambda}{\mu}\left(\frac{dR}{ds}\right) = 0.$$

The last six equations serve to determine the perturbations of a comet.

Let $(\Delta\,e)_n$ be the variation of any element e during the variation $\Delta\,v$ of v at any given epoch n, neglecting the square and higher powers of $\Delta\,v$,

$$(\Delta\,e)_n = \left(\frac{de}{dv}\right)_n \Delta\,v$$

If the values of $(\Delta\,e)_n$ be calculated for the epochs 0, 1, 2 . . . m corresponding to the values v, $v + i\,\Delta\,v$, $v + 2i\,\Delta\,v$, &c., differing from each other by $i\,\Delta\,v$, then the whole variation $(\delta\,e)$ of v corresponding to the variation $i\,\Delta\,v$ of v

$$= i\left\{(\Delta\,e)_0 + (\Delta\,e)_1 \,.\,.\,.\,.\,.\,.\,. + (\Delta\,e)_{m-1}\right\}$$

$$+ \frac{i-1}{2}\left\{(\Delta\,e)_m - (\Delta\,e)_0\right\} - \frac{(i-1)(i+1)}{12\,i}\left\{(\Delta^2\,e)_m - (\Delta^2\,e)_0\right\} + \&c.$$

$$= i\left\{\left(\frac{de}{dv}\right)_0 + \left(\frac{de}{dv}\right)_1 \,.\,.\,.\,.\,.\,.\,. + \left(\frac{de}{dv}\right)_{m-1}\right\}\Delta\,v$$

$$+ \frac{i-1}{2}\left\{\left(\frac{de}{dv}\right)_m - \left(\frac{de}{dv}\right)_0\right\}\Delta\,v - \frac{(i-1)(i+1)}{12\,i}\left\{\Delta\left(\frac{de}{dv}\right)_m - \Delta\left(\frac{de}{dv}\right)_0\right\}\Delta\,v.$$

When the interval $\Delta\, v$ is indefinitely diminished, $i\,\Delta\, v$ is still equal to the variation of v between the epochs for which the quantities $\left(\frac{d\,e}{d\,v}\right)_0$, $\left(\frac{d\,e}{d\,v}\right)_1$ &c. are calculated, and

$$\delta\, e = i\,\Delta\, v \left\{ \left(\frac{d\,e}{d\,v}\right)_0 + \left(\frac{d\,e}{d\,v}\right)_1 \ldots\ldots + \left(\frac{d\,e}{d\,v}\right)_{m-1} + \tfrac{1}{2}\left\{\left(\frac{d\,e}{d\,v}\right)_m - \left(\frac{d\,e}{d\,v}\right)_0\right\} \right.$$
$$\left. - \tfrac{1}{12}\left\{\Delta\left(\frac{d\,e}{d\,v}\right)_m - \Delta\left(\frac{d\,e}{d\,v}\right)_0\right\} + \&c. \right\}$$

If the radius be taken for unity, and $i\,\Delta\, v$ is the mth part of the circumference; $i\,\Delta\, v = \frac{2 \times 3{\cdot}14159}{m}$, or, in other words, the resulting values of $\delta\, e$ and $\delta\, a$ in the equations given above must be multiplied by 2 × 3.14159, and divided by 360° expressed in the same unit as $i\,\Delta\, v$.

n is equal to the angular circumference divided by the periodic time expressed in the same unit as t; so that if a degree be taken as the unity of angular circumference, $n = 360°$ divided by the periodic time expressed in the same unit as t.

In the elliptic movement or first approximation

$\lambda = n\,t + \varepsilon +$ a series of sines of arcs multiples of $n\,t$ &c.
$\lambda_{,} = n_{,}\,t + \varepsilon_{,} +$ a series of sines of arcs multiples of $n_{,}\,t$ &c.
$\frac{a}{r} =$ constant $+$ a series of cosines.
$s =$ a series of sines.
$s_{,} =$ a series of sines.

These values being substituted in the equations of p. 334 give $\frac{d\,e}{d\,\lambda}$, $\frac{d\,h}{d\,\lambda}$ and $\frac{d\,\iota}{d\,\lambda}$ each equal to a series of sines without any constant quantity, and $\frac{d\,\varpi}{d\,\lambda}$, $\frac{d\,\nu}{d\,\lambda}$ and $\frac{d\,\varepsilon}{d\,\lambda}$ each equal to a series of cosines $+$ a constant quantity.

In the second approximation the values of λ, r and s retain the same form; and it is easy to see from the form of the expressions for $\frac{d\,e}{d\,\lambda}$, $\frac{d\,h}{d\,\lambda}$, &c. p. 334, that the form of the values of these quantities is not altered however far the approximation be carried.

If the sun or primary be a spheroid, ω the angle which the plane of the sun's equator makes with the plane of the orbit of the planet; and if the longitude be reckoned from the line of intersection of the sun's equator with the orbit of the planet; R is increased by the quantity $c\left\{\frac{3\sin^2\omega\sin^2\lambda-1}{r^3}\right\}$, c being a constant dependent upon the figure of the sun; but the partial differential coefficients of this quantity, which are introduced into the values of d e, d ϖ, &c. do not change the form of the expressions for those quantities.

If the planet move in a medium which resists according to any power n of the velocity, if c be a constant and v the velocity, the term $2\,c\int v^{n+1}\,\mathrm{d}\,t$ must be added to $2\int \mathrm{d}\,R$,

$$c\,v^{n-1}\left\{(1+s^2)\frac{\mathrm{d}\,r^{\backprime}}{\mathrm{d}\,t}+r^{\backprime}\,s\,\frac{\mathrm{d}\,s}{\mathrm{d}\,t}\right\} \text{ to } \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r^{\backprime}}\right),$$

$$c\,v^{n-1}\,r^{\backprime 2}\frac{\mathrm{d}\,\lambda^{\backprime}}{\mathrm{d}\,t} \text{ to } \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right), \quad \text{and } c\,v^{n-1}\,r^{\backprime}\,\frac{\mathrm{d}\,.\,r^{\backprime}\,s}{\mathrm{d}\,t} \text{ to } \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right)$$

in the equations of p. 330.

If the orbit of the planet be supposed to coincide with the plane $x\,y$, so that $s=0$, then by the equations of p. 337 after reductions

$$\mathrm{d}\,a=-2\,c\,a^2\left(\frac{\mu}{a}\right)^{\frac{n+1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n+1}{2}}\frac{(1-e\cos v)}{n}\,\mathrm{d}\,v$$

$$\mathrm{d}\,e=-2\,c\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}}\left(\frac{1-e^2}{n}\right)\cos v\,\mathrm{d}\,v$$

$$e\,\mathrm{d}\,\varpi=-2\,c\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}}\sqrt{\frac{1-e^2}{n}}\sin v\,\mathrm{d}\,v$$

$$\mathrm{d}\,\varepsilon-\mathrm{d}\,\varpi=2\,c\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}}\frac{\sin v}{n}\left\{\frac{1-e^2\cos v}{e}\right\}\mathrm{d}\,v$$

The form of these equations differs from that which obtained before, now the variations of e, ϖ and ε are periodical, while that of a has a term which varies with the time. $\frac{\mathrm{d}\,e}{\mathrm{d}\,v}$ contains only odd powers of $\cos v$ and for that reason has no constant term. The periods of the periodic inequalities of all the elliptic constants due to the action of the resisting medium are fractional parts of the periodic time of the planet.

If the origin of t coincides with the instant of the perihelion passage, by LAGRANGE's theorem

$$\cos v = \cos nt - e \sin nt^2 - \frac{e^2}{2} \cdot \frac{d.\sin nt^3}{d.nt} - \frac{e^3}{2.3} \cdot \frac{d^2.\sin nt^4}{(d.nt)^2} - \frac{e^4}{2.3.4} \frac{d^3 \sin nt^5}{(d.nt^3} - \&c.$$

$$\sin v = \sin nt + e \sin nt \cos nt + \frac{e^2}{2} \frac{d.\sin nt^2 \cos nt}{d.nt} + \frac{e^3}{2.3} \frac{d^2.\sin nt^3 \cos nt}{(d.nt)^2} + \frac{e^4}{2.3.4} \frac{d^3.\sin nt^4 \cos nt}{(d.nt)^3}$$

$$\sin n t^2 = \frac{1 - \cos 2nt}{2}, \quad \sin n t^3 = \frac{3 \sin nt - \sin 3nt}{4}$$

$$\sin n t^4 = \frac{3 - 4 \cos 2nt + \cos 4nt}{8}, \quad \sin n t^5 = \frac{10 \sin nt - 5 \sin 3nt + \sin 5nt}{16}$$

$$\frac{d.\sin nt^3}{d.nt} = \frac{3 \cos nt - 3 \cos 3nt}{4}$$

$$\frac{d.\sin nt^4}{d.nt} = \frac{2 \sin 2nt - \sin 4nt}{2}, \quad \frac{d^2.\sin nt^4}{(d.nt)^2} = 2 \cos 2nt - 2 \cos 4nt$$

$$\frac{d.\sin nt^5}{d.nt} = \frac{10 \cos nt - 15 \cos 3nt + 5 \cos 5nt}{16}$$

$$\frac{d^2 \sin nt^5}{(d.nt)^2} = \frac{-10 \sin nt + 45 \sin 3nt - 25 \sin 5nt}{16}$$

$$\frac{d^3.\sin nt^5}{(d.nt)} = \frac{-10 \cos nt + 135 \cos 3nt - 125 \cos 5nt}{16}$$

$$\cos v = \cos nt - \frac{e}{2} + \frac{e}{2} \cos 2nt - \frac{e^2}{2} \left\{ \frac{3 \cos nt - 3 \cos 3nt}{4} \right\} - \frac{e^3}{2.3} \left\{ 2 \cos 2nt - 2 \cos 4nt \right\}$$
$$- \frac{e^4}{2.3.4} \left\{ \frac{-10 \cos nt + 135 \cos 3nt - 125 \cos 5nt}{16} \right\}$$

If the origin of the time does not coincide with the perihelion passage, $nt + \varepsilon - \varpi$ must be substituted for nt, but as ε always accompanies nt, it may be suppressed at present for convenience, and afterwards replaced.

$$\cos v = \left\{ 1 - \frac{3}{8} e^2 + \frac{5e^4}{192} \right\} \cos(nt - \varpi) - \frac{e}{2} + \frac{e}{2} \left\{ 1 - \frac{2e^2}{3} \right\} \cos(2nt - 2\varpi)$$
$$+ \frac{3}{8} e^2 \left\{ 1 - \frac{15}{16} e^2 \right\} \cos(3nt - 3\varpi) + \frac{e^3}{3} \cos(4nt - 4\varpi) + \frac{125}{384} e^4 \cos(5nt - 5\varpi)$$

$$\sin nt^2 \cos nt = \frac{d.\sin nt^3}{3 d.nt} = \frac{\cos nt - \cos 3nt}{4}, \quad \frac{d.\sin nt^2 \cos nt}{d.nt} = \frac{-\sin nt + 3 \sin 3nt}{4}$$

$$\sin n t^3 \cos n t = \frac{\mathrm{d}\,.\sin n t^4}{4\,\mathrm{d}\,.\,nt} = \frac{2 \sin 2 n t - \sin 4 n t}{8}, \qquad \frac{\mathrm{d}\,.\sin n t^3 \cos n t}{\mathrm{d}\,.\,n t} = \frac{\cos 2 n t - \cos 4 n t}{2}$$

$$\frac{\mathrm{d}^2\,.\sin n t^3 \cos n t}{(\mathrm{d}\,.\,n t)^3} = - \sin 2 n t + 2 \sin 4 n t$$

$$\sin n t^4 \cos n t = \frac{\mathrm{d}\,.\sin n t^5}{5\,\mathrm{d}\,.\,n t} = \frac{2 \cos n t - 3 \cos 3 n t + \cos 5 n t}{16}$$

$$\frac{\mathrm{d}\,.\sin n t^4 \cos n t}{\mathrm{d}\,.\,n t} = \frac{- 2 \sin n t + 9 \sin 3 n t - 5 \sin 5 n t}{16}$$

$$\frac{\mathrm{d}^2\,.\sin n t^4 \cos n t}{(\mathrm{d}\,.\,n t)^2} = \frac{- 2 \cos n t + 27 \cos 3 n t - 25 \cos 5 n t}{16}$$

$$\frac{\mathrm{d}^3\,.\sin n t^4 \cos n t}{(\mathrm{d}\,.\,n t)^3} = \frac{2 \sin n t - 81 \sin 3 n t + 125 \sin 5 n t}{16}$$

$$\sin v = \sin n t + \frac{e}{2} \sin 2 n t + \frac{e^2}{2} \left\{\frac{- \sin n t + 3 \sin 3 n t}{4}\right\} + \frac{e^3}{2\,.\,3} \{- \sin 2 n t + 2 \sin 4 n t\}$$
$$+ \frac{e^4}{2\,.\,3\,.\,4} \left\{\frac{2 \sin n t - 81 \sin 3 n t + 125 \sin 5 n t}{16}\right\}$$

and replacing $n t$ by $n t - \varpi$

$$\sin v = \left\{1 - \frac{3 e^2}{8} + \frac{5}{192} e^4\right\} \sin (n t - \varpi) + \frac{e}{2} \left\{1 - 2 \frac{e^2}{3}\right\} \sin (2 n t - 2 \varpi)$$
$$+ \left\{\frac{2 e^3}{8} - \frac{4}{192} e^4\right\} \sin (n t - \varpi) \qquad + \frac{e^3}{6} \sin (2 n t - 2 \varpi)$$
$$+ \frac{3 e^2}{8} \left\{1 - \frac{15}{16} e^2\right\} \sin (3 n t - 3 \varpi) + \frac{e^3}{3} \sin (4 n t - 4 \varpi) + \frac{125}{384} e^4 \sin (5 n t - 5 \varpi)$$
$$+ \frac{3}{8} \cdot \frac{6}{16} e^4 \sin (3 n t - 3 \varpi)$$

$$\cos (v + \varpi) = \left\{1 - \frac{e^2}{4} + \frac{e^4}{64}\right\} \cos n t - \frac{e}{2} \cos \varpi + \frac{e}{2} \left\{1 - \frac{e^2}{2}\right\} \cos (2 n t - \varpi)$$
$$+ \frac{3}{8} e^2 \left\{1 - \frac{3}{4} e^2\right\} \cos (3 n t - 2 \varpi) + \frac{e^3}{3} \cos (4 n t - 3 \varpi) + \frac{125}{384} e^4 \cos (5 n t - 4 \varpi)$$
$$- \frac{e^3}{8} \left\{1 - \frac{e^2}{12}\right\} \cos (n t - 2 \varpi) - \frac{e^3}{12} \cos (2 n t - 3 \varpi) - \frac{9}{128} e^4 \cos (3 n t - 4 \varpi)$$

$$\sin (v + \varpi) = \left\{1 - \frac{e^2}{4} + \frac{e^4}{64}\right\} \sin n t - \frac{e}{2} \sin \varpi + \frac{e}{2} \left\{1 - \frac{e^2}{2}\right\} \sin (2 n t - \varpi)$$
$$+ \frac{3}{8} e^2 \left\{1 - \frac{3}{4} e^2\right\} \sin (3 n t - 2 \varpi) + \frac{e^3}{3} \sin (4 n t - 3 \varpi) + \frac{125}{384} e^4 \sin (5 n t - 4 \varpi)$$

$$+\frac{e^2}{8}\left(1-\frac{e^2}{12}\right)\sin(nt-2\varpi)+\frac{e^3}{12}\sin(2nt-3\varpi)+\frac{9}{128}e^4\sin(3nt-4\varpi)$$

$$\cos(\upsilon-\varpi)=\left(1-\frac{e^2}{4}\right)\cos(nt-2\varpi)-\frac{e}{2}\cos\varpi+\frac{e}{2}\cos(2nt-3\varpi)+\frac{3}{8}e^2\cos(3nt-4\varpi)-\frac{e^2}{8}\cos nt$$

$$\sin(\upsilon-\varpi)=\left(1-\frac{e^2}{4}\right)\sin(nt-2\varpi)+\frac{e}{2}\sin\varpi+\frac{e}{2}\sin(2nt-3\varpi)+\frac{3}{8}e^2\sin(3nt-4\varpi)+\frac{e^2}{8}\sin nt$$

$$r\cos\lambda=r\cos(\lambda-\varpi)\cos\varpi-r\sin(\lambda-\varpi)\sin\varpi=a\{(\cos\upsilon-e)\cos\varpi-(1-e^2)^{\frac{1}{2}}\sin\upsilon\sin\varpi\}$$

$$r\sin\lambda=r\sin(\lambda-\varpi)\cos\varpi+r\cos(\lambda-\varpi)\sin\varpi=a\{(1-e^2)^{\frac{1}{2}}\sin\upsilon\cos\varpi+(\cos\upsilon-e)\sin\varpi\}$$

$$r\cos\lambda=a\left\{\left(1-\frac{e^2}{4}-\frac{e^4}{16}\right)\cos(\upsilon+\varpi)-e\cos\varpi+\frac{e^2}{4}\left(1+\frac{e^2}{4}\right)\cos(\upsilon-\varpi)\right\}$$

$$r\sin\lambda=a\left\{\left(1-\frac{e^2}{4}-\frac{e^4}{16}\right)\sin(\upsilon+\varpi)-e\sin\varpi-\frac{e^2}{4}\left(1+\frac{e^2}{4}\right)\sin(\upsilon-\varpi)\right\}$$

$$\genfrac{}{}{0pt}{}{r\cos\lambda}{r\sin\lambda}=a\left\{\left(1-\frac{e^2}{2}-\frac{e^4}{64}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}nt-\frac{3e}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}\varpi+\frac{e}{2}\left(1-\frac{3}{4}e^2\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-\varpi)\right.$$

$$+\frac{3}{8}e^2\left(1-e^2\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-2\varpi)+\frac{e^3}{3}\genfrac{}{}{0pt}{}{\cos}{\sin}(4nt-3\varpi)+\frac{125}{384}e^4\genfrac{}{}{0pt}{}{\cos}{\sin}(5nt-4\varpi)$$

$$\left.\pm\frac{e^2}{8}\left(1+\frac{e^2}{3}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-2\varpi)\pm\frac{e^3}{24}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-3\varpi)\pm\frac{3}{128}e^4\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-4\varpi)\text{\&c.}\right\}$$

$$rr_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}\{\lambda-\lambda_{,}\}=aa_{,}\left\{\left(1-\frac{e^2}{2}-\frac{e^4}{64}\right)\left(1-\frac{e_{,}^2}{2}-\frac{e_{,}^4}{64}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-n_{,}t)\right.$$

$$\mp\frac{3}{2}e\left(1-\frac{e_{,}^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(n_{,}t-\varpi)+\frac{e}{2}\left(1-\frac{3}{4}e^2\right)\left(1-\frac{e_{,}^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-n_{,}t-\varpi)$$

$$+\frac{3}{8}e^2(1-e^2)\left(1-\frac{e_{,}^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-n_{,}t-2\varpi)+\frac{e^3}{3}\genfrac{}{}{0pt}{}{\cos}{\sin}(4nt-n_{,}t-3\varpi)$$

$$+\frac{125}{384}e^4\genfrac{}{}{0pt}{}{\cos}{\sin}(5nt-n_{,}t-4\varpi)\pm\frac{e^2}{8}\left(1+\frac{e^2}{3}\right)\left(1-\frac{e_{,}^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+n_{,}t-2\varpi)$$

$$\pm\frac{e^3}{24}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+n_{,}t-3\varpi)\pm\frac{3}{128}e^4\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt+n_{,}t-4\varpi)$$

$$-\frac{3}{2}e_{,}\left(1-\frac{e^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-\varpi_{,})+\frac{9}{4}ee_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(\varpi-\varpi_{,})$$

$$-\frac{3}{4}e\,e_{,}\left(1-\frac{3}{4}e^2\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-\varpi-\varpi_{,})-\frac{9}{16}e^2e_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-2\varpi-\varpi_{,})$$

$$-\frac{e^3e_{,}}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(4nt-3\varpi-\varpi_{,})\mp\frac{3}{16}e^2e_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-2\varpi+\varpi_{,})\mp\frac{e^3e_{,}}{16}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-3\varpi+\varpi_{,})$$

$$+\frac{e_{,}}{2}\left(1-\frac{3}{4}e_{,}^{2}\right)\left(1-\frac{e^{2}}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-2n_{,}t+\varpi_{,})\mp\frac{3}{4}ee_{,}\left(1-\frac{3}{4}e_{,}^{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2n_{,}t-\varpi-\varpi_{,})$$

$$+\frac{ee_{,}}{4}\left(1-\frac{3}{4}e^{2}\right)\left(1-\frac{3}{4}e_{,}^{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-2n_{,}t-\varpi+\varpi_{,})+\frac{3}{16}e^{2}e_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-2n_{,}t-2\varpi+\varpi_{,})$$

$$+\frac{e^{3}e_{,}}{6}\genfrac{}{}{0pt}{}{\cos}{\sin}(4nt-2n_{,}t-3\varpi+\varpi_{,})\pm\frac{e^{3}e_{,}}{16}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+2n_{,}t-2\varpi-\varpi_{,})$$

$$\pm\frac{e^{3}e_{,}}{48}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+2n_{,}t-3\varpi-\varpi_{,})+\frac{3}{8}e_{,}^{2}(1-e_{,}^{2})\left(1-\frac{e^{2}}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-3n_{,}t+2\varpi_{,})$$

$$\mp\frac{9}{16}ee_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(3n_{,}t-\varpi-2\varpi_{,})+\frac{3}{16}ee_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-3n_{,}t-\varpi+2\varpi_{,})$$

$$+\frac{9}{64}e^{2}e_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-3n_{,}t-2\varpi+2\varpi_{,})\pm\frac{3}{64}e^{2}e_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+3n_{,}t-2\varpi-2\varpi_{,})$$

$$+\frac{e^{3}}{3}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-4n_{,}t+3\varpi_{,})\mp\frac{e\,e_{,}^{3}}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(4n_{,}t-\varpi-3\varpi_{,})$$

$$+\frac{e\,e_{,}^{3}}{6}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-4n_{,}t-\varpi+3\varpi_{,})+\frac{125}{384}e_{,}^{4}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-5n_{,}t+4\varpi_{,})$$

$$+\frac{e_{,}^{2}}{8}\left(1+\frac{e^{2}}{3}e_{,}^{2}\right)\left(1-\frac{e^{2}}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+n_{,}t-2\varpi_{,})-\frac{3}{16}ee_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(n_{,}t+\varpi-2\varpi_{,})$$

$$+\frac{e\,e_{,}^{2}}{16}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+n_{,}t-\varpi-2\varpi_{,})+\frac{3}{64}e^{2}e_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt+n_{,}t-2\varpi-2\varpi_{,})$$

$$\pm\frac{e^{2}e_{,}^{2}}{64}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-n_{,}t-2\varpi+2\varpi_{,})+\frac{e_{,}^{3}}{24}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+2n_{,}t-3\varpi_{,})$$

$$\mp\frac{e\,e_{,}^{3}}{16}\genfrac{}{}{0pt}{}{\cos}{\sin}(2n_{,}t+\varpi-3\varpi_{,})+\frac{e\,e_{,}^{3}}{48}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+2n_{,}t-\varpi-3\varpi_{,})$$

$$+\frac{3}{128}e_{,}^{4}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+3n_{,}t-4\varpi_{,})+\&c.$$

$$rr_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}\{\lambda+\lambda_{,}\}=aa_{,}\left\{\left(1-\frac{e^{2}+e_{,}^{2}}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+n_{,}t)\right.$$

$$-\frac{3}{2}e\genfrac{}{}{0pt}{}{\cos}{\sin}(n_{,}t+\varpi)+\frac{e}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+n_{,}t-\varpi)$$

$$+\frac{3}{8}e^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt+n_{,}t-2\varpi)\pm\frac{e^{2}}{8}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-n_{,}t-2\varpi)$$

$$-\frac{3}{2}e_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+\varpi_{,})+\frac{9}{4}ee_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(\varpi+\varpi_{,})-\frac{3}{4}ee_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-\varpi+\varpi_{,})$$

$$+ \frac{e_{,}}{2} \genfrac{}{}{0pt}{}{\cos}{\sin} (nt+2n_{,}t-\varpi_{,}) - \frac{3}{4}\, ee_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} (2n_{,}t+\varpi-\varpi_{,}) + \frac{ee_{,}}{4} \genfrac{}{}{0pt}{}{\cos}{\sin} (2nt+2nt-\varpi-\varpi_{,}) + \&c.$$

$$rr_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} \{\lambda+\lambda_{,}-2\nu\} = aa_{,} \left\{ \left(1 - \frac{e^2+e_{,}^2}{2}\right) \genfrac{}{}{0pt}{}{\cos}{\sin} (nt+n_{,}t-2\nu) \right.$$

$$- \frac{3}{2}\, e \genfrac{}{}{0pt}{}{\cos}{\sin} (n_{,}t+\varpi-2\nu) + \frac{e}{2} \genfrac{}{}{0pt}{}{\cos}{\sin} (2nt+n_{,}t-\varpi-2\nu)$$

$$+ \frac{3}{8}\, e^2 \genfrac{}{}{0pt}{}{\cos}{\sin} (3nt+n_{,}t-2\varpi-2\nu) \pm \frac{e^2}{8} \genfrac{}{}{0pt}{}{\cos}{\sin} (nt-n_{,}t-2\varpi+2\nu)$$

$$- \frac{3}{2}\, e_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} (nt+\varpi_{,}-2\nu) + \frac{9}{4}\, e\, e_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} (\varpi+\varpi_{,}-2\nu)$$

$$- \frac{3}{4}\, ee_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} (2nt-\varpi+\varpi_{,}-2\nu) + \frac{e_{,}}{2} \genfrac{}{}{0pt}{}{\cos}{\sin} (nt+2n_{,}t-\varpi_{,}-2\nu)$$

$$- \frac{3}{4}\, ee_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} (2n_{,}t+\varpi-\varpi_{,}-2\nu) + \frac{ee_{,}}{4} \genfrac{}{}{0pt}{}{\cos}{\sin} (2nt+2n_{,}t-\varpi-\varpi_{,}-2\nu) + \&c.$$

$$r = a \left\{ 1 + \frac{e^2}{2} - e\left(1 - \frac{3}{8}\, e^2\right) \cos (nt-\varpi) - \frac{e^2}{2}\left(1 - \frac{2e^2}{3}\right) \cos (2nt-2\varpi) \right.$$

$$- \frac{3}{8}\, e^3 \cos (3nt-3\varpi) - \frac{e^4}{3} \cos (4nt-4\varpi) + \&c.$$

$$r^2 = a^2 \left\{ 1 + \frac{3\,e^2}{2} - 2\,e\left(1 - \frac{e^2}{8}\right) \cos (nt-\varpi) - \frac{e^2}{2}\left(1 - \frac{e^2}{3}\right) \cos (2nt-2\varpi) \right.$$

$$- \frac{e^3}{4} \cos (3nt-3\varpi) - \frac{e^4}{6} \cos (4nt-4\varpi) + \&c.$$

$$r^{-3} = a^{-3} \left\{ 1 + \frac{3}{2}\, e^2 \left(1 + \frac{e^2}{2}\right) + 3\,e\left(1 + \frac{9}{8}\, e^2\right) \cos (nt-\varpi) \right.$$

$$+ \frac{9}{2}\, e^2 \left(1 - \frac{e^2}{18}\right) \cos 2nt-2\varpi + \frac{53}{8}\, e^3 \cos (3nt-3\varpi)$$

$$+ \frac{31}{4}\, e^4 \cos (4nt-4\varpi) + \&c.$$

If the coefficient of cos $n\,\theta$ in the development of $\left\{1 - \frac{2\,a}{a_{,}} \cos \theta + \frac{a^2}{a_{,}^2}\right\}^{-m}$ according to cosines of arcs mulitples of θ is called $b_{2m,n}$

$$b_{1,0} = 1 + \left(\frac{1}{2}\right)^2 \frac{a^2}{a_{,}^2} + \left(\frac{1.3}{2.4}\right)^2 \frac{a^4}{a_{,}^4} + \left(\frac{1.3.5}{2.4.6}\right)^2 \frac{a^6}{a_{,}^6} + \&c.$$

$$b_{1,1} = \frac{a}{a_{\prime}} + \frac{1.3}{2.4}\frac{a^3}{a_{\prime}^3} + \frac{1.3.3.5}{2.4.4.6}\frac{a^5}{a_{\prime}^5} + \&c.$$

$$b_{1,2} = \frac{3}{4}\frac{a^2}{a_{\prime}^2} + \frac{1.3.5}{2.4.6}\frac{a^4}{a_{\prime}^4} + \frac{1.3.3.5.7}{2.4.4.6.8}\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{1,3} = \frac{3.5}{4.6}\frac{a^3}{a_{\prime}^3} + \frac{1.3.5.7}{2.4.6.8}\frac{a^5}{a_{\prime}^5} + \frac{1.3.3.5.7.9}{2.4.4.6.8.10}\frac{a^7}{a_{\prime}^7} + \&c.$$

$$b_{3,0} = 1 + \left(\frac{3}{2}\right)^2\frac{a^2}{a_{\prime}^2} + \left(\frac{3.5}{2.4}\right)^2\frac{a^4}{a_{\prime}^4} + \left(\frac{3.5.7}{2.4.6}\right)^2\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{3,1} = \frac{3\,a}{a_{\prime}} + \frac{3.3.5}{2.4}\frac{a^3}{a_{\prime}^3} + \frac{3.5.3.5.7}{2.4.4.6}\frac{a^5}{a_{\prime}^5} + \&c.$$

$$b_{3,2} = \frac{3.5}{4}\frac{a^2}{a_{\prime}^2} + \frac{3.3.5.7}{2.4.6}\frac{a^4}{a_{\prime}^4} + \frac{3.5.3.5.7}{2.4.4.6.8}\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{3,3} = \frac{3.5.7}{4.6}\frac{a^3}{a_{\prime}^3} + \frac{3.3.5.7.9}{2.4.6.8}\frac{a^5}{a_{\prime}^5} + \frac{3.5.3.5.7.9.11}{2.4.4.6.8.10}\frac{a^7}{a_{\prime}^7} + \&c.$$

$$b_{5,0} = 1 + \left(\frac{5}{2}\right)^2\frac{a^2}{a_{\prime}^2} + \left(\frac{5.7}{2.4}\right)^2\frac{a^4}{a_{\prime}^4} + \left(\frac{5.7.9}{2.4.6}\right)^2\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{5,1} = 5\,\frac{a}{a_{\prime}} + \frac{5.5.7}{2.4}\frac{a^3}{a_{\prime}^3} + \frac{5.7.5.7.9}{2.4.4.6}\frac{a^5}{a_{\prime}^5} + \&c.$$

$$b_{5,2} = \frac{5.7}{4}\frac{a^2}{a_{\prime}^2} + \frac{5.5.7.9}{2.4.6}\frac{a^4}{a_{\prime}^4} + \frac{5.7.5.7.9.11}{2.4.4.6.8}\frac{a^6}{a_{\prime}^6} + \&c.$$

$$b_{5,3} = \frac{5.7.9}{4.6}\frac{a^3}{a_{\prime}^3} + \frac{5.5.7.9.11}{2.4.6.8}\frac{a^5}{a_{\prime}^5} + \frac{5.7.5.7.9.11.13}{2.4.4.6.8.10}\frac{a^7}{a_{\prime}^7}$$

$$b_{3,1} = 3\,\frac{a}{a_{\prime}}\left\{b_{5,0} - \frac{1}{2}\,b_{5,2}\right\} \qquad b_{1,1} = \frac{a}{a_{\prime}}\left\{b_{3,0} - \frac{1}{2}\,b_{3,2}\right\}$$

$$2\,b_{3,2} = \frac{3}{2}\,\frac{a}{a_{\prime}}\left\{b_{5,1} - b_{5,3}\right\} \qquad 2\,b_{1,2} = \frac{1}{2}\,\frac{a}{a_{\prime}}\left\{b_{3,1} - b_{3,3}\right\}$$

$$3\,b_{3,3} = \frac{3}{2}\,\frac{a}{a_{\prime}}\left\{b_{5,2} - b_{5,4}\right\} \qquad 3\,b_{1,3} = \frac{1}{2}\,\frac{a}{a_{\prime}}\left\{b_{3,2} - b_{3,4}\right\}$$

$$R = m_{\prime}\left\{\frac{r\,r_{\prime}'\cos(\lambda - \lambda_{\prime}')}{r_{\prime}^3} - \frac{1}{\{r^2 - 2\,r\,r_{\prime}'\cos(\lambda - \lambda_{\prime}') + r_{\prime}^2\}^{\frac{1}{2}}}\right\}$$

and since

$$r r_{\prime}'\cos(\lambda - \lambda_{\prime}') = r r_{\prime}\left\{\cos^2\frac{\iota_{\prime}}{2}\cos(\lambda - \lambda_{\prime}) + \sin^2\frac{\iota_{\prime}}{2}\cos(\lambda + \lambda_{\prime} - 2\,\nu_{\prime})\right\}, \text{ p. } 330;$$

$$R = m_{,}\Bigg\{\frac{r r_{,}\Big\{\cos^2\frac{\iota_{,}}{2}\cos(\lambda-\lambda_{,})+\sin^2\frac{\iota_{,}}{2}\cos(\lambda+\lambda_{,}-2\nu_{,})\Big\}}{r_{,}^3}$$

$$-\frac{1}{\Big\{r^2-2rr_{,}\Big\{\cos^2\frac{\iota_{,}}{2}\cos(\lambda-\lambda_{,})+\sin^2\frac{\iota_{,}}{2}\cos(\lambda+\lambda_{,}-2\nu_{,})\Big\}+r_{,}^2\Big\}^{\frac{1}{2}}}\Bigg\}$$

Neglecting the cubes and higher powers of e

$$R=m_{,}\frac{a}{a_{,}^2}\Big\{\Big(1-\sin^2\frac{\iota_{,}}{2}-\frac{e^2+e_{,}^2}{2}\Big)\cos(nt-n_{,}t)-\frac{3}{2}e\cos(n_{,}t-\varpi)-\frac{3}{2}e_{,}\cos(nt-\varpi_{,})$$

$$+\frac{e}{2}\cos(2nt-n_{,}t-\varpi)+\frac{e_{,}}{2}\cos(nt-2n_{,}t+\varpi_{,})+\frac{3}{8}e^2\cos(3nt-n_{,}t-2\varpi)$$

$$+\frac{3}{8}e_{,}^2\cos(nt-3n_{,}t+2\varpi_{,})+\frac{e^2}{8}\cos(nt+n_{,}t-2\varpi)+\frac{e_{,}^2}{8}\cos(nt+n_{,}t-2\varpi_{,})$$

$$+\frac{9}{4}ee_{,}\cos(\varpi-\varpi_{,})-\frac{3}{4}ee_{,}\cos(2nt-\varpi-\varpi_{,})-\frac{3}{4}ee_{,}\cos(2n_{,}t-\varpi-\varpi_{,})$$

$$+\frac{ee_{,}}{4}\cos(2nt-2n_{,}t-\varpi+\varpi_{,})+\sin^2\frac{\iota}{2}\cos(nt+n_{,}t-2\nu)\Big\}\Big\{1+\frac{3}{2}e_{,}^2$$

$$+3e_{,}\cos(n_{,}t-\varpi_{,})+\frac{9}{2}e_{,}^2\cos(2n_{,}t-2\varpi_{,})+\&c.\Big\}$$

$$-m_{,}\Bigg\{a^2\Big\{1+\frac{3}{2}e^2-2e\cos(nt-\varpi)-\frac{e^2}{2}\cos(2nt-2\varpi)\Big\}$$

$$-2aa_{,}\Big\{\Big(1-\sin^2\frac{\iota_{,}}{2}-\frac{e^2+e_{,}^2}{2}\Big)\cos(nt-n_{,}t)-\frac{3}{2}e\cos(n_{,}t-\varpi)$$

$$-\frac{3}{2}e_{,}\cos(nt-\varpi_{,})+\frac{e}{2}\cos(2nt-n_{,}t-\varpi)+\frac{e_{,}}{2}\cos(nt-2n_{,}t+\varpi_{,})$$

$$+\frac{3}{8}e^2\cos(3nt-n_{,}t-2\varpi)+\frac{3}{8}e_{,}^2\cos(nt-3n_{,}t+2\varpi_{,})+\frac{e^2}{8}\cos(nt+n_{,}t-2\varpi)$$

$$+\frac{e_{,}^2}{8}\cos(nt+n_{,}t-2\varpi_{,})+\frac{9}{4}ee_{,}\cos(\varpi-\varpi_{,})-\frac{3}{4}ee_{,}\cos(2nt-\varpi-\varpi_{,})$$

$$-\frac{3}{4}ee_{,}\cos(2n_{,}t-\varpi-\varpi_{,})+\frac{ee_{,}}{4}\cos(2nt-2n_{,}t-\varpi+\varpi_{,})$$

$$+\sin^2\frac{\iota}{2}\cos(nt+n_{,}t-2\nu)\Big\}+a_{,}^2\Big\{1+\frac{3}{2}e_{,}^2$$

$$-2e_{,}\cos(n_{,}t-\varpi_{,})-\frac{e_{,}^2}{2}\cos(2n_{,}t-2\varpi_{,})+\&c.\Big\}\Bigg\}^{\frac{1}{2}}$$

$= \text{terms independent of } b$

$$-\frac{m_{,}}{a_{,}}\left\{b_{1,0}+b_{1,1}\cos(nt-n_{,}t)+b_{1,2}\cos(2nt-2n_{,}t)+\&\text{c.}\right\}$$

$$+\frac{m_{,}}{2a_{,}^{3}}\left\{b_{3,0}+b_{3,1}\cos(nt-n_{,}t)+b_{3,2}\cos(2nt-2n_{,}t)\ \&\text{c.}\right\}\left\{a^2\left\{\frac{3}{2}e^2\right.\right.$$

$$\left.-2e\cos(nt-\varpi)-\frac{e^2}{2}\cos(2nt-2\varpi)\right\}-2aa_{,}\left\{\left(-\sin^2\frac{\iota_{,}}{2}-\frac{e^2+e_{,}^2}{2}\right)\cos(nt-n_{,}t)\right.$$

$$-\frac{3}{2}e\cos(n_{,}t-\varpi)-\frac{3}{2}e_{,}\cos(nt-\varpi_{,})+\frac{e}{2}\cos(2nt-n_{,}t-\varpi)$$

$$+\frac{e_{,}}{2}\cos(nt-2n_{,}t+\varpi_{,})+\frac{3}{8}e^2\cos(3nt-n_{,}t-2\varpi)+\frac{3}{8}e_{,}^2\cos(nt-3n_{,}t+2\varpi_{,})$$

$$+\frac{e^2}{8}\cos(nt+n_{,}t-2\varpi)+\frac{e_{,}^2}{8}\cos(nt+n_{,}t-2\varpi_{,})+\frac{9}{4}ee_{,}\cos(\varpi-\varpi_{,})$$

$$-\frac{3}{4}ee_{,}\cos(2nt-\varpi-\varpi_{,})-\frac{3}{4}ee_{,}\cos(2n_{,}t-\varpi-\varpi_{,})+\frac{ee_{,}}{4}\cos(2nt-2n_{,}t-\varpi+\varpi_{,})$$

$$\left.\left.+\sin^2\frac{\iota_{,}}{2}\cos(nt+n_{,}t-2\nu_{,})\right\}+a_{,}^2\left\{\frac{3}{2}e_{,}^2-2e_{,}\cos(n_{,}t-\varpi_{,})-\frac{e_{,}^2}{2}\cos(2n_{,}t-2\varpi_{,})\&\text{c.}\right\}\right\}$$

$$-\frac{1.3m_{,}}{2.4a_{,}^{5}}\left\{b_{5,0}+b_{5,1}\cos(nt-n_{,}t)+b_{5,2}\cos(2nt-2n_{,}t)+\&\text{c.}\right\}\left\{2a^2e\cos(nt-\varpi)\right.$$

$$-3aa_{,}e\cos(n_{,}t-\varpi)-3aa_{,}e_{,}\cos(nt-\varpi_{,})+aa_{,}e\cos(2nt-n_{,}t-\varpi)$$

$$\left.+aa_{,}e_{,}\cos(nt-2n_{,}t+\varpi_{,})+2a_{,}^2e_{,}\cos(n_{,}t-\varpi_{,})\right\}^2+\&\text{c.}$$

$$\left\{2a^2e\cos(nt-\varpi)-3aa_{,}e\cos(n_{,}t-\varpi)-3aa_{,}e_{,}\cos(nt-\varpi_{,})\right.$$

$$\left.+aa_{,}e\cos(2nt-n_{,}t-\varpi)+aa_{,}e_{,}\cos(nt-2n_{,}t+\varpi_{,})+2a_{,}^2e_{,}\cos(n_{,}t-\varpi_{,})\right\}^2$$

$$=2a^4e^2+5a^2a_{,}^2(e^2+e_{,}^2)+2a_{,}^4e_{,}^2+a^2(2a^2-3a_{,}^2)e^2\cos(2nt-2\varpi)$$

$$+\frac{9}{2}a^2a_{,}^2e^2\cos(2n_{,}t-2\varpi)+\frac{9}{2}a^2a_{,}^2e_{,}^2\cos(2nt-2\varpi_{,})+\frac{a^2a_{,}^2}{2}e^2\cos(4nt-2n_{,}t-2\varpi)$$

$$+\frac{a^2a_{,}^2}{2}e_{,}^2\cos(2nt-4n_{,}t+2\varpi_{,})+a_{,}^2(2a_{,}^2-3a^2)e_{,}^2\cos(2n_{,}t-2\varpi_{,})$$

$$-4(a^2e^2+a_{,}^2e_{,}^2)aa_{,}\cos(nt-n_{,}t)-6a^3a_{,}e^2\cos(nt+n_{,}t-2\varpi)$$

$$- 6\, a a_{,}^{3} e_{,}^{2} \cos(nt + n_{,}t - 2\varpi_{,}) - 6\,(a^2 + a_{,}^2)\, a a_{,} e e_{,} \cos(\varpi - \varpi_{,})$$

$$- 2\,(3a^2 - a_{,}^2)\, a a_{,} e e_{,} \cos(2nt - \varpi - \varpi_{,}) - 2\,(3a_{,}^2 - a^2)\, a a_{,} e e_{,} \cos(2n_{,}t - \varpi - \varpi_{,})$$

$$+ 2\, a^3 a_{,} e^2 \cos(3nt - n_{,}t - 2\varpi) + 2\, a a_{,}^3 e_{,}^2 \cos(nt - 3n_{,}t + 2\varpi_{,})$$

$$+ 2\,(a^2 + a_{,}^2)\, a a_{,} e e_{,} \cos(2nt - 2n_{,}t - \varpi + \varpi_{,}) + 9\, a^2 a_{,}^2 e e_{,} \cos(nt - n_{,}t + \varpi - \varpi_{,})$$

$$+ 14\, a^2 a_{,}^2 e e_{,} \cos(nt + n_{,}t - \varpi - \varpi_{,}) - 3\, a^2 a_{,}^2 (e^2 + e_{,}^2) \cos(2nt - 2n_{,}t)$$

$$- 3\, a^2 a_{,}^2 e e_{,} \cos(nt - 3n_{,}t + \varpi + \varpi_{,}) - 3\, a^2 a_{,}^2 e e_{,} \cos(3nt - n_{,}t - \varpi - \varpi_{,})$$

$$- 2\, a^2 a_{,}^2 e e_{,} \cos(nt - n_{,}t - \varpi + \varpi_{,}) + a^2 a_{,}^2 e e_{,} \cos(3nt - 3n_{,}t - \varpi + \varpi_{,})$$

Replacing ε which accompanies nt,

$$R = m_{,} \Big\{ -\frac{b_{1,0}}{a_{,}} + 3\,\frac{(a^2 e^2 + a_{,}^2 e_{,}^2)}{2\,.\,2\,a_{,}^3} b_{3,0} + \frac{a a_{,}}{2 a_{,}^3}\Big(\sin^2\frac{\iota_{,}}{2} + \frac{e^2 + e_{,}^2}{2}\Big) b_{3,1}$$

$$- \frac{1\,.\,3}{2\,.\,4\,.\,a_{,}^5}\Big(2a^4 e^2 + 5a^2 a_{,}^2 (e^2 + e_{,}^2) + 2a_{,}^4 e_{,}^2\Big) b_{5,0} + \frac{1\,.\,3\,.\,2}{2\,.\,4\,a_{,}^5}(a^2 e^2 + a_{,}^2 e_{,}^2)\, a a_{,} b_{5,1}$$

$$+ \frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2}\,\frac{a^2 a_{,}^2}{a_{,}^5}\,(e^2 + e_{,}^2)\, b_{5,2} \qquad [0*]$$

$$+ m_{,} \Big\{ \frac{a}{a_{,}^2}\Big\{\cos^2\frac{\iota_{,}}{2} - \frac{e^2 + e_{,}^2}{2}\Big\} - \frac{b_{1,1}}{a_{,}} + \frac{a a_{,}}{a_{,}^3}\Big\{\sin^2\frac{\iota_{,}}{2} + \frac{e^2 + e_{,}^2}{2}\Big\}\Big\{b_{3,0} + \tfrac{1}{2} b_{3,2}\Big\}$$

$$+ \frac{3\,(a^2 e^2 + a_{,}^2 e_{,}^2)}{2\,.\,2\,a_{,}^3} b_{3,1} + \frac{1\,.\,3\,.\,4}{2\,.\,4\,a_{,}^5}(a^2 e^2 + a_{,}^2 e_{,}^2)\, a a_{,} b_{5,0}$$

$$- \frac{1\,.\,3}{2\,.\,4\,a_{,}^5}\Big\{2a^4 e^2 + \frac{7}{2} a^2 a_{,}^2 (e^2 + e_{,}) + 2a_{,}^4 e_{,}^2\Big\} b_{5,1} + \frac{1\,.\,3\,.\,2}{2\,.\,4\,a_{,}^5}(a^2 e^2 + a_{,}^2 e_{,}^2)\, a a_{,} b_{5,2}$$

$$+ \frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2}\,\frac{a^2 a_{,}^2}{a_{,}^5}\,(e^2 + e_{,}^2)\, b_{5,3}\Big\} \cos(nt - n_{,}t + \varepsilon - \varepsilon_{,}) \qquad [1]$$

$$+ m_{,} \Big\{ -\frac{b_{1,2}}{a_{,}} + \frac{a a_{,}}{2 a_{,}^3}\Big\{\sin^2\frac{\iota_{,}}{2} + \frac{e^2 + e_{,}^2}{2}\Big\}\Big\{b_{3,1} + b_{3,3}\Big\} + \frac{3\,(a^2 e^2 + a_{,}^2 e_{,}^2)}{2\,.\,2 a_{,}^3} b_{3,2}$$

$$+ \frac{1\,.\,3\,.\,3}{2\,.\,4}\,\frac{a^2 a_{,}^2 (e^2 + e_{,}^2)}{a_{,}^5} b_{5,0} + \frac{1\,.\,3\,.\,2}{2\,.\,4}\,\frac{(a^2 e^2 + a_{,}^2 e_{,}^2)}{a_{,}^5}\, a a_{,} b_{5,1}$$

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* The numbers at the side serve to distinguish the arguments.

$$-\frac{1.3}{2.4\,a_,^5}\left\{2a^4e^2+5a^2a_,^2(e^2+e_,^2)+2a_,^4e_,^2\right\}b_{5,2}$$

$$+\frac{1.3.2}{2.4\,a_,^5}(a^2e^2+a_,^2e_,^2)\,b_{5,3}\Big\}\cos(2nt-2n_,t+2\varepsilon-2\varepsilon_,) \qquad [2]$$

$$+m_,\left\{-\frac{b_{1,3}}{a_,}+\frac{a\,a_,}{2\,a_,^3}\left\{\sin^2\frac{\iota_,}{2}+\frac{e^2+e_,^2}{2}\right\}b_{3,2}+\frac{3(a^2e^2+a_,^2e_,^2)}{2.2\,a_,^3}b_{3,3}\right.$$

$$+\frac{1.3.3}{2.4.2}\frac{a^2a_,^2(e^2+e_,^2)}{a_,^5}b_{5,1}+\frac{1.3.2}{2.4}\frac{(a^2e^2+a_,^2e_,^2)}{4a_,^5}aa_,b_{5,2}$$

$$\left.-\frac{1.3}{2.4}\left\{2a^4e^2+5a^2a_,^2(e^2+e_,^2)+2a_,^4e_,^2\right\}b_{5,3}\right\}\cos(3nt-3n_,t+3\varepsilon-3\varepsilon_,) \qquad [3]$$

$$+m_,\left\{-\frac{b_{1,4}}{a_,}+\frac{a\,a_,}{2\,a_,^3}\left\{\sin^2\frac{\iota_,}{2}+\frac{e^2+e_,^2}{2}\right\}b_{3,3}+\frac{1.3.3}{2.4.2}\frac{a^2a_,^2(e^2+e_,^2)}{a_,^5}b_{5,2}\right.$$

$$\left.+\frac{1.3.2}{2.4}\frac{(a^2e^2+a_,^2e_,^2)}{a_,^5}aa_,b_{5,3}\right\}\cos(4nt-4n_,t+4\varepsilon-4\varepsilon_,) \qquad [4]$$

$$+m_,\left\{-\frac{b_{1,5}}{a_,}+\frac{1.3.3}{2.4.2}\frac{a^2a_,^2(e^2+e_,^2)}{a_,^5}b_{5,3}\right\}\cos(5nt-5n_,t+5\varepsilon-5\varepsilon_,) \qquad [5]$$

$$+m_,\left\{-\frac{3\,a}{2\,a_,^2}+\frac{3\,a\,a'}{2\,a_,^3}b_{3,0}-\frac{a^2}{2\,a_,^3}b_{3,1}-\frac{a\,a_,}{2.2\,a_,^3}b_{3,2}\right\}e\cos(n_,t+\varepsilon_,-\varpi) \qquad [6]$$

$$+m_,\left\{-\frac{a^2}{a_,^3}b_{3,0}+\frac{a\,a_,}{2\,a_,^3}b_{3,1}\right\}e\cos(nt+\varepsilon-\varpi) \qquad [7]$$

$$+m_,\left\{\frac{1}{2}\frac{a}{a_,^2}-\frac{a\,a_,}{2\,a_,^3}b_{3,0}-\frac{a^2}{2\,a_,^3}b_{3,1}+\frac{3}{2.2}\frac{a\,a_,}{a_,^3}b_{3,2}\right\}e\cos(2nt-n_,t+2\varepsilon-\varepsilon_,-\varpi) \qquad [8]$$

$$+m_,\left\{-\frac{a\,a_,}{2.2\,a_,^3}b_{3,1}-\frac{a^2}{2\,a_,^3}b_{3,2}+\frac{3}{2.2}\frac{a\,a_,}{a_,^3}b_{3,3}\right\}e\cos(3nt-2n_,t+3\varepsilon-2\varepsilon_,-\varpi) \qquad [9]$$

$$+m_,\left\{-\frac{a\,a_,}{2.2\,a_,^3}b_{3,2}-\frac{a^2}{2\,a_,^3}b_{3,3}\right\}e\cos(4nt-3n_,t+4\varepsilon-3\varepsilon_,-\varpi) \qquad [10]$$

$$-m_,\frac{a\,a_,}{2.2\,a_,^3}b_{3,3}\,e\cos(5nt-4n_,t+5\varepsilon-4\varepsilon_,-\varpi) \qquad [11]$$

$$+m_,\left\{\frac{3}{2.2}\frac{a\,a_,}{a_,^3}b_{3,1}-\frac{a^2}{2\,a_,^3}b_{3,2}-\frac{a\,a_,}{2.2\,a_,^3}b_{3,3}\right\}e\cos(nt-2n_,t+\varepsilon-2\varepsilon_,+\varpi) \qquad [12]$$

$$+m_,\left\{\frac{3}{2.2}\frac{a\,a_,}{a_,^3}b_{3,2}-\frac{a^2}{2\,a_,^3}b_{3,3}\right\}e\cos(2nt-3n_,t+2\varepsilon-3\varepsilon_,+\varpi) \qquad [13]$$

$$+m_,\frac{3\,a\,a_,}{2.2\,a_,^3}b_{3,3}\,e\cos(3nt-4n_,t+3\varepsilon-4\varepsilon_,+\varpi) \qquad [14]$$

$$+m_{,}\left\{-\frac{a_{,}^{2}}{a_{,}^{3}}b_{3,0}-\frac{a\,a_{,}}{2\,a_{,}^{3}}b_{3,1}\right\}e_{,}\cos(n_{,}t+\varepsilon_{,}-\varpi_{,}) \qquad [15]$$

$$+m_{,}\left\{\frac{3\,a\,a_{,}}{2\,.\,2\,a_{,}^{3}}b_{3,1}-\frac{a_{,}^{2}}{2\,a_{,}^{3}}b_{3,2}-\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}b_{3,3}\right\}e_{,}\cos(2nt-n_{,}t+2\varepsilon-\varepsilon_{,}-\varpi_{,}) \qquad [17]$$

$$+m_{,}\left\{\frac{3\,a\,a_{,}}{2\,.\,2\,a_{,}^{3}}b_{3,2}-\frac{a_{,}^{2}}{2\,a_{,}^{3}}b_{3,3}\right\}e_{,}\cos(3nt-2n_{,}t+3\varepsilon-2\varepsilon_{,}-\varpi_{,}) \qquad [18]$$

$$+m_{,}\frac{3\,a\,a_{,}}{2\,.\,2\,a_{,}^{3}}b_{3,3}\,e_{,}\cos(4nt-3n_{,}t+4\varepsilon-3\varepsilon_{,}-\varpi_{,}) \qquad [19]$$

$$+m_{,}\left\{\frac{2\,a}{a_{,}^{2}}-\frac{a\,a_{,}}{2\,a_{,}^{3}}b_{3,0}-\frac{a_{,}^{2}}{2\,a_{,}^{3}}b_{3,1}+\frac{3}{2\,.\,2}\frac{a a_{,}}{a_{,}^{3}}b_{3,2}\right\}e_{,}\cos(nt-2n_{,}t+\varepsilon-2\varepsilon_{,}+\varpi_{,}) \qquad [20]$$

$$+m_{,}\left\{-\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}b_{3,1}-\frac{a_{,}^{2}}{2\,a_{,}^{3}}b_{3,2}+\frac{3}{2\,.\,2}\frac{a a_{,}}{a_{,}^{3}}b_{3,3}\right\}e_{,}\cos(2nt-3n_{,}t+2\varepsilon-3\varepsilon_{,}+\varpi_{,}) \qquad [21]$$

$$+m_{,}\left\{-\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}b_{3,2}-\frac{a_{,}^{2}}{2\,a_{,}^{3}}b_{3,3}\right\}e_{,}\cos(3nt-4n_{,}t+3\varepsilon-4\varepsilon_{,}+\varpi_{,}) \qquad [22]$$

$$-m_{,}\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}b_{3,3}\,e_{,}\cos(4nt-5n_{,}t+4\varepsilon-5\varepsilon_{,}+\varpi_{,}) \qquad [23]$$

$$\begin{aligned}+m_{,}\Big\{&-\frac{a\,a_{,}}{2\,.\,8\,a_{,}^{3}}b_{3,1}-\frac{a^{2}}{2\,.\,4\,a_{,}^{3}}b_{3,2}-\frac{3}{2\,.\,8}\frac{a a_{,}}{a_{,}^{3}}b_{3,3}-\frac{1\,.\,3\,.\,9}{2\,.\,4\,.\,2}\frac{a^{2}a_{,}^{2}}{a_{,}^{5}}b_{5,0}+\frac{1\,.\,3\,.\,3}{2\,.\,4}\frac{a^{3}a_{,}}{a_{,}^{5}}b_{5,1}\\&-\frac{1\,.\,3}{2\,.\,4\,.\,2}\frac{(2\,a^{2}-3\,a_{,}^{2})}{a_{,}^{5}}a^{2}b_{5,2}-\frac{1\,.\,3}{2\,.\,4}\frac{a^{3}a_{,}}{a_{,}^{5}}b_{5,3}\Big\}e^{2}\cos(2n_{,}t+2\varepsilon_{,}-2\varpi) \qquad [24]\end{aligned}$$

$$\begin{aligned}+m_{,}\Big\{&\frac{a}{8\,a_{,}^{2}}-\frac{a a_{,}}{2\,.\,4\,a_{,}^{3}}b_{3,0}-\frac{a^{2}}{2\,.\,4\,a_{,}^{3}}b_{3,1}-\frac{3\,a\,a_{,}}{2\,.\,8\,a_{,}^{3}}b_{3,2}+\frac{1\,.\,3\,.\,6}{2\,.\,4}\frac{a^{3}a_{,}}{a_{,}^{5}}b_{5,0}\\&-\frac{1\,.\,3\,a^{2}}{2\,.\,4\,.\,4}\frac{(4\,a^{2}+3\,a_{,}^{2})}{a_{,}^{5}}b_{5,1}-\frac{1\,.\,3}{2\,.\,4}\frac{a^{3}a_{,}}{a_{,}^{5}}b_{5,2}\\&\cdot\frac{1\,.\,3}{2\,.\,4\,.\,4}\frac{a^{2}a_{,}^{2}}{a_{,}^{5}}b_{5,3}\Big\}e^{2}\cos(nt+n_{,}t+\varepsilon+\varepsilon_{,}-2\varpi) \qquad [25]\end{aligned}$$

$$\begin{aligned}+m_{,}\Big\{&-\frac{a^{2}}{2\,.\,2\,a_{,}^{3}}b_{3,0}-\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}b_{3,1}-\frac{1\,.\,3}{2\,.\,4}\frac{a^{2}(2\,a^{2}-3\,a_{,}^{2})}{a_{,}^{5}}b_{5,0}\\&+\frac{1\,.\,3\,.\,2}{2\,.\,4}\frac{a^{3}a_{,}}{a_{,}^{5}}b_{5,1}-\frac{1\,.\,3\,.\,5}{2\,.\,4\,.\,2}\frac{a^{2}a_{,}^{2}}{a_{,}^{5}}b_{5,2}\Big\}e^{2}\cos(2nt+2\varepsilon-2\varpi) \qquad [26]\end{aligned}$$

$$\begin{aligned}+m_{,}\Big\{&\frac{3}{8}\frac{a}{a_{,}^{2}}-\frac{3}{8}\frac{a\,a_{,}}{a_{,}^{3}}b_{3,0}-\frac{a\,a_{,}}{2\,.\,4\,a_{,}^{3}}b_{3,1}-\frac{a^{2}}{2\,.\,8\,a_{,}^{3}}b_{3,2}-\frac{1\,.\,3\,.\,2}{2\,.\,4}\frac{a^{3}a_{,}}{a_{,}^{5}}b_{5,0}\\&-\frac{1\,.\,3\,a^{2}}{2\,.\,4\,.\,4}\frac{(4\,a^{2}-5\,a_{,}^{2})}{a_{,}^{5}}b_{5,1}+\frac{1\,.\,3\,.\,3}{2\,.\,4}\frac{a^{3}a_{,}}{a_{,}^{5}}b_{5,2}\end{aligned}$$

$$\left.- \frac{1.3.9}{2.4.4} \frac{a^2 a_{,}^2}{a_{,}^5} b_{5,3} \right\} e^2 \cos (3nt - n_{,}t + 3\varepsilon - \varepsilon_{,} - 2\varpi) \qquad [27]$$

$$+ m_{,} \left\{ - \frac{3 a a_{,}}{2.8 a_{,}^3} b_{3,1} - \frac{a^2}{2.4 a_{,}^3} b_{3,2} - \frac{a a_{,}}{2.8 a_{,}^3} b_{3,3} - \frac{1.3 a^2 a_{,}^2}{2.4.2 a_{,}^5} b_{5,0} \right.$$

$$- \frac{1.3}{2.4} \frac{a^3 a_{,}}{a_{,}^5} b_{5,1} - \frac{1.3 a^2}{2.4.2} \frac{(2 a^2 - 3 a_{,}^2)}{a_{,}^5} b_{5,2}$$

$$\left. + \frac{1.3.3}{2.4} \frac{a^3 a_{,}}{a_{,}^5} b_{5,3} \right\} e^2 \cos (4nt - 2n_{,}t + 4\varepsilon - 2\varepsilon_{,} - 2\varpi) \qquad [28]$$

$$+ m_{,} \left\{ - \frac{3 a a_{,}}{2.8 a_{,}^3} b_{3,2} - \frac{a^2}{2.4 a_{,}^3} b_{3,3} - \frac{1.3}{2.4.4} \frac{a^2 a_{,}^2}{a_{,}^5} b_{5,1} - \frac{1.3}{2.4} \frac{a^3 a_{,}}{a_{,}^5} b_{5,2} \right.$$

$$\left. - \frac{1.3 a^2}{2.4.2} \frac{(2 a^2 - 3 a_{,}^2)}{a_{,}^5} b_{5,3} \right\} e^2 \cos (5nt - 3n_{,}t + 5\varepsilon - 3\varepsilon_{,} - 2\varpi) \qquad [29]$$

$$+ m_{,} \left\{ - \frac{3 a a_{,}}{2.8 a_{,}^3} b_{3,3} - \frac{1.3}{2.4.4} \frac{a^2 a_{,}^2}{a_{,}^5} b_{5,2} \right.$$

$$\left. - \frac{1.3}{2.4} \frac{a^3 a_{,}}{a_{,}^5} b_{5,3} \right\} e^2 \cos (6nt - 4n_{,}t + 6\varepsilon - 4\varepsilon_{,} - 2\varpi) \qquad [30]$$

$$- m_{,} \frac{1.3}{2.4.4} \frac{a^2 a_{,}^2}{a_{,}^5} b_{5,3} e^2 \cos (7nt - 5n_{,}t + 7\varepsilon - 5\varepsilon_{,} - 2\varpi) \qquad [31]$$

$$+ m_{,} \left\{ - \frac{a a_{,}}{2.8 a_{,}^3} b_{3,2} - \frac{a^2}{2.4 a_{,}^3} b_{3,3} - \frac{1.3.9}{2.4} \frac{a^2 a_{,}^2}{a_{,}^5} b_{5,1} + \frac{1.3.3}{2.4} \frac{a^3 a_{,}}{a_{,}^5} b_{5,2} \right.$$

$$\left. - \frac{1.3}{2.4} \frac{a^2}{2} \frac{(2 a^2 - 3 a_{,}^2)}{a_{,}^5} b_{5,3} \right\} e^2 \cos (nt - 3n_{,}t + \varepsilon - 3\varepsilon_{,} + 2\varpi) \qquad [32]$$

$$- m_{,} \left\{ - \frac{a a_{,}}{2.8 a_{,}^3} b_{3,3} - \frac{1.3.9}{2.4.4} \frac{a^2 a_{,}^2}{a_{,}^5} b_{5,2} \right.$$

$$\left. + \frac{1.3.3}{2.4} \frac{a^3 a_{,}}{a_{,}^5} b_{5,3} \right\} e^2 \cos (2nt - 4n_{,}t + 2\varepsilon - 4\varepsilon_{,} + 2\varpi) \qquad [33]$$

$$- m_{,} \frac{1.3.9}{2.4.4} \frac{a^2 a_{,}^2}{a_{,}^5} b_{5,3} e^2 \cos (3nt - 5n_{,}t + 3\varepsilon - 5\varepsilon_{,} + 2\varpi) \qquad [34]$$

$$+ m_{,} \left\{ - \frac{5}{2.2} \frac{a a_{,}}{a_{,}^3} b_{3,1} + \frac{1.3.2}{2.4} \frac{a^2 a_{,}^2}{a_{,}^5} b_{5,0} + \frac{1.3.2}{2.4} \frac{(a^2 + a_{,}^2)}{a_{,}^5} a a_{,} b_{5,1} \right.$$

$$\left. - \frac{1.3.5}{2.4} \frac{a^2 a_{,}^2}{a_{,}^5} b_{5,2} \right\} e e_{,} \cos (nt - n_{,}t + \varepsilon - \varepsilon_{,} - \varpi + \varpi_{,}) \qquad [35]$$

$$+ m_{,} \left\{ \frac{a}{a_{,}^2} - \frac{a a_{,}}{2.2 a_{,}^3} b_{3,0} - \frac{9}{2.4} \frac{a a_{,}}{a_{,}^3} b_{3,2} - \frac{1.3.2}{2.4} \frac{(a^2 + a_{,}^2)}{a_{,}^5} a a_{,} b_{5,0} \right.$$

$$+\frac{1.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}+\frac{1.3.3}{2.4}\frac{a^2+a_{,}^2}{a_{,}^5}aa_{,}b_{5,2}$$

$$\left.-\frac{1.3.9}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\right\}ee_{,}\cos(2nt-2n_{,}t+2\varepsilon-2\varepsilon_{,}-\varpi+\varpi_{,}) \qquad [36]$$

$$+m_{,}\left\{-\frac{aa_{,}}{2.4a_{,}^3}b_{3,1}-\frac{9}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,3}-\frac{1.3}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}-\frac{1.3}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}b_{5,1}+\frac{1.3}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}\right.$$

$$\left.+\frac{1.3.3}{2.4}\frac{a^2+a_{,}^2}{a_{,}^5}aa_{,}b_{5,3}\right\}ee_{,}\cos(3nt-3n_{,}t+3\varepsilon-3\varepsilon_{,}-\varpi+\varpi_{,}) \qquad [37]$$

$$+m_{,}\left\{-\frac{aa_{,}}{2.4a_{,}^3}b_{3,2}-\frac{1.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}-\frac{1.3}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}b_{5,2}\right.$$

$$\left.+\frac{1.3}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\right\}ee_{,}\cos(4nt-4n_{,}t+4\varepsilon-4\varepsilon_{,}-\varpi+\varpi_{,}) \qquad [38]$$

$$+m_{,}\left\{-\frac{aa_{,}}{2.4a_{,}^3}b_{3,3}-\frac{1.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}\right.$$

$$\left.-\frac{1.3}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}b_{5,3}\right\}ee_{,}\cos(5nt-5n_{,}t+5\varepsilon-5\varepsilon_{,}-\varpi+\varpi_{,}) \qquad [39]$$

$$-m_{,}\frac{1.3}{2.4}\frac{a^2a_{,}^2}{2a_{,}^5}b_{5,3}\cos(6nt-6n_{,}t-\varpi+\varpi_{,}) \qquad [40]$$

$$+m_{,}\left\{-\frac{9}{2.2}\frac{aa_{,}}{a_{,}^3}b_{3,0}-\frac{aa_{,}}{2.4a_{,}^3}b_{3,2}+\frac{1.3.6}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}b_{5,0}-\frac{1.3.7}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}\right.$$

$$\left.-\frac{1.3}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}a_{,}a\,b_{5,2}-\frac{1.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\right\}ee_{,}\cos(\varpi-\varpi_{,}) \qquad [41]$$

$$+m_{,}\left\{-\frac{9}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,1}-\frac{aa_{,}}{2.4a_{,}^3}b_{3,3}-\frac{1.3.9}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}+\frac{1.3.3}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}b_{5,1}\right.$$

$$\left.+\frac{1.3}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}-\frac{1.3}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}b_{5,3}\right\}ee_{,}\cos(nt-n_{,}t+\varepsilon-\varepsilon_{,}+\varpi-\varpi_{,}) \qquad [42]$$

$$+m_{,}\left\{-\frac{9}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,2}-\frac{1.3.9}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}+\frac{1.3.3}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}b_{5,2}\right.$$

$$\left.+\frac{1.3}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\right\}ee_{,}\cos(2nt-2n_{,}t+2\varepsilon-2\varepsilon_{,}+\varpi-\varpi_{,}) \qquad [43]$$

$$+m_{,}\left\{-\frac{9}{2.4}\frac{aa_{,}}{a_{,}^3}b_{3,3}-\frac{1.3.9}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}\right.$$

$$\left.+\frac{1.3.3}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}b_{5,3}\right\}ee_{,}\cos(3nt-3n_{,}t+3\varepsilon-3\varepsilon_{,}+\varpi-\varpi_{,}) \qquad [44]$$

$$-m_{,}\frac{1.3.9}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\,ee_{,}\cos(4nt-4n_{,}t+4\varepsilon-4\varepsilon_{,}+\varpi-\varpi_{,}) \qquad [45]$$

$$+m_{,}\Big\{-\frac{3a}{a_{,}^2}+\frac{3aa_{,}}{2.2a_{,}^3}b_{3,0}+\frac{3aa_{,}}{2.4a_{,}^3}b_{3,2}+\frac{1.3.2}{1.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}\,aa_{,}\,b_{5,0}-\frac{1.3.11}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,1}$$

$$+\frac{1.3}{2.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}\,aa_{,}\,b_{5,2}+\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\Big\}\,ee_{,}\cos(2n_{,}t+2\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [46]$$

$$+m_{,}\Big\{\frac{3}{2.2}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,1}-\frac{1.3.14}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,0}+\frac{1.3.2}{2.4}\,\frac{(a^2+a_{,}^2)}{a_{,}^5}\,aa_{,}\,b_{5,1}$$

$$+\frac{1.3.3}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,2}\Big\}\,ee_{,}\cos(nt+n_{,}t+\varepsilon+\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [47]$$

$$+m_{,}\Big\{\frac{3aa_{,}}{2.2a_{,}^3}\,b_{3,0}+\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,2}+\frac{1.3.2}{2.4}\,\frac{(3a^2-a_{,}^2)}{a_{,}^5}\,aa_{,}\,b_{5,0}-\frac{1.3.11}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,1}$$

$$+\frac{1.3}{2.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}\,b_{5,2}+\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\Big\}\,ee_{,}\cos(2nt+2\varepsilon-\varpi-\varpi_{,}) \qquad [48]$$

$$+m_{,}\Big\{\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,1}+\frac{3.aa_{,}}{2.4a_{,}^3}\,b_{3,3}+\frac{1.3.3}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,0}$$

$$+\frac{1.3}{2.4}\,\frac{(3a^2-a_{,}^2)}{a_{,}^5}\,aa_{,}\,b_{5,1}-\frac{1.3.7}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,2}$$

$$+\frac{1.3}{2.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}\,aa_{,}\,b_{5,3}\Big\}\,ee_{,}\cos(3nt-n_{,}t+3\varepsilon-\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [49]$$

$$+m_{,}\Big\{\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,2}+\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,1}+\frac{1.3}{2.4}\,\frac{(3a^2-a_{,}^2)}{a_{,}^5}\,aa_{,}\,b_{5,2}$$

$$-\frac{1.3.7}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\Big\}\,ee_{,}\cos(4nt-2n_{,}t+4\varepsilon-2\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [50]$$

$$+m_{,}\Big\{\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,3}+\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,aa_{,}\,b_{5,2}$$

$$+\frac{1.3}{2.4}\,\frac{(3a^2-a_{,}^2)}{a_{,}^5}\,aa_{,}\,b_{5,3}\Big\}\,ee_{,}\cos(5nt-3n_{,}t+5\varepsilon-3\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [51]$$

$$+m_{,}\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\,ee_{,}\cos(6nt-4n_{,}t+6\varepsilon-4\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [52]$$

$$+m_{,}\Big\{\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,1}+\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,3}+\frac{1.3.3}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,0}$$

$$+\frac{1.3}{2.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}\,aa_{,}\,b_{5,1}-\frac{1.3.7}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,2}$$

$$+\frac{1.3}{2.4}\frac{(3a^2-a_{,}^2)}{a_{,}^5}aa_{,}b_{5,3}\Big\}ee_{,}\cos(nt-3n_{,}t+\varepsilon-3\varepsilon_{,}+\varpi+\varpi_{,}) \qquad [53]$$

$$+m_{,}\Big\{\frac{3}{2.4}\frac{a\,a_{,}}{a_{,}^3}b_{3,1}+\frac{3}{2.4}\frac{a\,a_{,}}{a_{,}^3}b_{3,2}+\frac{1.3.3}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}$$

$$+\frac{1.3}{2.4}\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}b_{5,1}+\frac{1.3}{2.4}\frac{(3a^2-a_{,}^2)}{a_{,}^5}aa_{,}b_{5,2}$$

$$-\frac{1.3.7}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\Big\}ee_{,}\cos(2nt-4n_{,}t+2\varepsilon-4\varepsilon_{,}+\varpi+\varpi_{,}) \qquad [54]$$

$$+m_{,}\Big\{\frac{3}{2.4}\frac{a\,a_{,}}{a_{,}^3}b_{3,3}+\frac{1.3.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}$$

$$+\frac{1.3}{2.4}\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}b_{5,3}\Big\}ee_{,}\cos(3nt-5n_{,}t+3\varepsilon-5\varepsilon_{,}+\varpi+\varpi_{,}) \qquad [55]$$

$$+m_{,}\frac{1.3.3}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}ee_{,}\cos(4nt-6n_{,}t+4\varepsilon-6\varepsilon_{,}+\varpi+\varpi_{,}) \qquad [56]$$

$$+m_{,}\Big\{-\frac{a_{,}^2}{2.2a_{,}^3}b_{3,0}-\frac{a\,a_{,}}{2.2a_{,}^3}b_{3,1}-\frac{1.3}{2.4}\frac{(2a_{,}^2-3a^2)}{a_{,}^5}a_{,}^2b_{5,0}$$

$$+\frac{1.3.2}{2.4}\frac{a\,a_{,}^3}{a_{,}^5}b_{5,1}-\frac{1.3.5}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}\Big\}e_{,}^2\cos(2n_{,}t+2\varepsilon_{,}-2\varpi_{,}) \qquad [57]$$

$$+m_{,}\Big\{\frac{a}{8a_{,}^2}-\frac{a\,a_{,}}{2.4a_{,}^3}b_{3,0}-\frac{a_{,}^2}{2.4a_{,}^3}b_{3,1}-\frac{3a\,a_{,}}{2.8a_{,}^3}b_{3,2}+\frac{1.3.6}{2.4}\frac{a_{,}^3a}{a_{,}^5}b_{5,0}$$

$$-\frac{1.3}{2.4.4}\frac{(4a^2+3a_{,}^2)}{a_{,}^5}a_{,}^2b_{5,1}-\frac{1.3}{2.4}\frac{a\,a_{,}^3}{a_{,}^5}b_{5,2}$$

$$-\frac{1.3}{2.4.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\Big\}e_{,}^2\cos(nt+n_{,}t+\varepsilon+\varepsilon_{,}-2\varpi_{,}) \qquad [58]$$

$$+m_{,}\Big\{-\frac{a\,a_{,}}{2.8a_{,}^3}b_{3,1}-\frac{a_{,}^2}{2.4a_{,}^3}b_{3,2}-\frac{3}{2.8}\frac{aa_{,}}{a_{,}^3}b_{3,3}-\frac{1.3.9}{2.4.2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}+\frac{1.3.3}{2.4}\frac{a\,a_{,}^3}{a_{,}^5}b_{5,1}$$

$$-\frac{1.3}{2.4.2}\frac{(2a_{,}^2-3a^2)}{a_{,}^5}a_{,}^2b_{5,2}-\frac{1.3}{2.4}\frac{a\,a_{,}^3}{a_{,}^5}b_{5,3}\Big\}e_{,}^2\cos(2nt+2\varepsilon-2\varpi_{,}) \qquad [59]$$

$$+m_{,}\Big\{-\frac{a\,a_{,}}{2.8a_{,}^3}b_{3,2}-\frac{a_{,}^2}{2.4a_{,}^3}b_{3,3}-\frac{1.3.9}{2.4.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}+\frac{1.3.3}{2.4}\frac{a_{,}^3a}{a_{,}^5}b_{5,2}$$

$$-\frac{1.3}{2.4.2}\frac{(2a_{,}^2-3a^2)}{a_{,}^5}a_{,}^2b_{5,3}\Big\}e_{,}^2\cos(3nt-n_{,}t+3\varepsilon-\varepsilon_{,}-2\varpi_{,}) \qquad [60]$$

$$+m_{,}\Big\{-\frac{a\,a_{,}}{2.8a_{,}^3}b_{3,3}-\frac{1.3.9}{2.4.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}$$

$$+\frac{1.3.3}{2.4}\frac{a_\prime^3 a}{a_\prime^5} b_{5,3}\Big\} e_\prime^2\cos(4nt-2n_\prime t+4\epsilon-2\epsilon_\prime-2\varpi_\prime) \qquad [61]$$

$$-m_\prime\frac{1.3.9}{2.4.4}\frac{a^2 a_\prime^2}{a_\prime^5} b_{5,3} e_\prime^2\cos(5nt-3n_\prime t+5\epsilon-3\epsilon_\prime-2\varpi_\prime) \qquad [62]$$

$$+m_\prime\Big\{\frac{27}{8}\frac{a}{a_\prime^2}-\frac{3\,a a_\prime}{2.4\,a_\prime^3} b_{3,0}-\frac{a_\prime^2}{2.4\,a_\prime^3} b_{3,1}-\frac{a a_\prime}{2.8\,a_\prime^3} b_{3,2}-\frac{1.3.2}{2.4}\frac{a_\prime^3 a}{a_\prime^5} b_{5,0}$$

$$-\frac{1.3}{2.4.4}\frac{(4\,a_\prime^2-5\,a^2)}{a_\prime^5} a_\prime^2 b_{5,1}+\frac{1.3.3}{2.4}\frac{a a_\prime^3}{a_\prime^5} b_{5,2}$$

$$-\frac{1.3.9}{2.4.4}\frac{a^2 a_\prime^2}{a_\prime^5} b_{5,3}\Big\} e_\prime^2\cos(nt-3n_\prime t+\epsilon-3\epsilon_\prime+2\varpi_\prime) \qquad [63]$$

$$+m_\prime\Big\{-\frac{3\,a a_\prime}{2.8\,a_\prime^3} b_{3,1}-\frac{a_\prime^2}{2.4\,a_\prime^3} b_{3,2}-\frac{a a_\prime}{2.8\,a_\prime^3} b_{3,3}-\frac{1.3}{2.4.2}\frac{a^2 a_\prime^2}{a_\prime^5} b_{5,0}$$

$$-\frac{1.3}{2.4}\frac{a a_\prime^3}{a_\prime^5} b_{5,1}-\frac{1.3}{2.4.2}\frac{(2\,a_\prime^2-3\,a^2)}{a_\prime^5} a_\prime^2 b_{5,2}$$

$$+\frac{1.3.3}{2.4}\frac{a a_\prime^3}{a_\prime^5} b_{5,3}\Big\} e_\prime^2\cos(2nt-4n_\prime t+2\epsilon-4\epsilon_\prime+2\varpi_\prime) \qquad [64]$$

$$+m_\prime\Big\{-\frac{3\,a a_\prime}{2.8\,a_\prime^3} b_{3,2}-\frac{a_\prime^2}{2.4\,a_\prime^3} b_{3,3}-\frac{1.3}{2.4.4}\frac{a^2 a_\prime^2}{a_\prime^5} b_{5,1}-\frac{1.3}{2.4}\frac{a a_\prime^3}{a_\prime^5} b_{5,2}$$

$$-\frac{1.3}{2.4.2}\frac{(2\,a_\prime^2-3\,a^2)}{a_\prime^5} a_\prime^2 b_{5,3}\Big\} e_\prime^2\cos(3nt-5n_\prime t+3\epsilon-3\epsilon_\prime+2\varpi_\prime) \qquad [65]$$

$$+m_\prime\Big\{-\frac{3}{2.8}\frac{a a_\prime}{a_\prime^3} b_{3,3}-\frac{1.3}{2.4.4}\frac{a^2 a_\prime^2}{a_\prime^5} b_{5,2}$$

$$-\frac{1.3}{2.4}\frac{a a_\prime^3}{a_\prime^5} b_{5,3}\Big\} e_\prime^2\cos(4nt-6n_\prime t+4\epsilon-6\epsilon_\prime+2\varpi_\prime) \qquad [66]$$

$$-m_\prime\frac{1.3}{2.4.4}\frac{a^2 a_\prime^2}{a_\prime^5} b_{5,3} e_\prime^2\cos(5nt-7n_\prime t+5\epsilon-7\epsilon_\prime+2\varpi_\prime) \qquad [67]$$

$$+m_\prime\Big\{\frac{a}{a_\prime^2}\Big(1+\frac{3}{2}e_\prime^2\Big)-\frac{a a_\prime}{a_\prime^3} b_{3,0}\Big\}\sin^2\frac{\iota_\prime}{2}\cos(nt+n_\prime t+\epsilon+\epsilon_\prime-2\nu_\prime) \qquad [68]$$

$$-m_\prime\frac{a\,a_\prime}{2\,a_\prime^3} b_{3,1}\sin^2\frac{\iota_\prime}{2}\cos(2n_\prime t+2\epsilon_\prime-2\nu_\prime) \qquad [69]$$

$$m\,\frac{a\,a_\prime}{2\,a_\prime^3} b_{3,1}\sin^2\frac{\iota_\prime}{2}\cos(2nt+2\epsilon-2\nu_\prime) \qquad [70]$$

$$-m_\prime\frac{a\,a_\prime}{2\,a_\prime^3} b_{3,2}\sin^2\frac{\iota_\prime}{2}\cos(nt+3n_\prime t+\epsilon-3\epsilon_\prime+2\nu_\prime) \qquad [71]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}\,b_{3,2}\sin^2\frac{\iota_{,}}{2}\cos(3nt-n_{,}t+3\epsilon-\epsilon_{,}-2\nu_{,}) \qquad [72]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}\,b_{3,3}\sin^2\frac{\iota_{,}}{2}\cos(2nt-4n_{,}t+2\epsilon-4\epsilon_{,}+2\nu_{,}) \qquad [73]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}\,b_{3,3}\sin^2\frac{\iota_{,}}{2}\cos(4nt-2n_{,}t+4\epsilon-2\epsilon_{,}-2\nu_{,}) \qquad [74]$$

In the development of R, I have supposed $\iota = 0$, so that $\iota_{,}$ is the angle contained between the orbits of the planets P and P$_{,}$, or P$_1$ and P$_2$; in the general case, when ι_1, and ι_2, are retained, $\cos\iota_{,} = \cos\iota_1\cos\iota_2 + \sin\iota_1\sin\iota_2\cos\cdot(\iota_1-\iota_2)$, ι_1 and ι_2, being the inclinations of the orbits of the planets P$_1$, P$_2$ to any plane $x\,y$, of which the direction is arbitrary,

$$r' r_{,}' \sin(\lambda' - \lambda_{,}') = r_1 r_2 \Big\{ \cos^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\sin(\lambda_1-\lambda_2)$$

$$-\sin^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\sin(\lambda_1+\lambda_2-2\nu_1) + \sin^2\frac{\iota_2}{2}\cos^2\frac{\iota_1}{2}\sin(\lambda_1+\lambda_2-2\nu_2)$$

$$-\sin^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\sin(\lambda_1-\lambda_2-2\nu_1+2\nu_2)\Big\}$$

$$r' r_{,}' \Big\{\cos(\lambda'-\lambda_{,}') + s\,s_{,}\Big\} = r_1 r_2 \Big\{\cos^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\cos(\lambda_1-\lambda_2)$$

$$+\sin^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\cos(\lambda_1+\lambda_2-2\nu_1) + \sin^2\frac{\iota_2}{2}\cos^2\frac{\iota_1}{2}\cos(\lambda_1+\lambda_2-2\nu_2)$$

$$+\sin^2\frac{\iota_1}{2}\sin^2\frac{\iota_2}{2}\cos(\lambda_1-\lambda_2-2\nu_1+2\nu_2)$$

$$+\sin\iota_1\sin\iota_2\cos(\lambda_1-\lambda_2-\nu_1+\nu_2) - \sin\iota_1\sin\iota_2\cos(\lambda_1+\lambda_2-\nu_1-\nu_2)\Big\}$$

ERRATA.

In page 330, *for* ι and ν, *read* $\iota_{,}$ and $\nu_{,}$.

XXIII. *On the Error in Standards of Linear Measure, arising from the thickness of the bar on which they are traced. By Captain* Henry Kater, *V.P. and Treasurer of the Royal Society.*

Read June 17th, 1830.

In the course of the adjustment and verification of the copies of the Imperial standard yard, destined for the Exchequer, Guildhall, Dublin, and Edinburgh, I discovered a source of error till then, I believe, wholly unsuspected, arising from the thickness of the bar upon the surface of which measures of linear dimension are traced. The difficulties which I experienced, and the remedy which suggested itself upon that occasion, and which was found efficient, are shortly detailed in the account of the construction and adjustment of the new standards of weights and measures of the United Kingdom of Great Britain and Ireland, published in the Philosophical Transactions for 1826.

But as the notice there given occupies little more than a single page, and might therefore pass unremarked, I cannot but think that a fact of such importance in inquiries where linear measures are concerned, and which may be sufficient to account for the discrepancies to be found in the experiments of different observers, ought to be placed before the Royal Society in a more prominent point of view than that which it at present occupies. I shall, therefore, first extract from the paper alluded to the part to which I refer, and then add an account of such experiments as I have since made on the subject; and describe a scale which I have caused to be constructed so as almost entirely to obviate the source of error of which I am treating.

It may be seen in the paper before mentioned, "On the Construction of the Standard Measures," that Sir George Shuckburgh's scale was employed as the medium of comparison; the distance from the zero point of which to that marked thirty-six inches, had been found by comparisons detailed in the Philosophical Transactions for 1821, not to differ sensibly from the Imperial standard yard.

The bars intended for the Exchequer, &c. were half an inch thick and one inch and a quarter wide; the thickness being nearly the same as that of Sir GEORGE SHUCKBURGH's scale.

The dots upon the surface of the bars having been adjusted, so that their distance appeared to be equal to the distance from zero to thirty-six inches on Sir GEORGE SHUCKBURGH's scale, further comparisons were made after the plugs carrying the dots had been securely fixed; "and it was with surprise and disappointment that I found the whole of them apparently too short. They had been adjusted upon a board of mahogany carefully planed; and the table upon which they were now placed was so flat as to occasion little alteration in a spirit-level passed along it. The error of the standards was, however, far too considerable to be attributed to any curvature which on this occasion could take place; and it was not until after several days that I discovered the cause of this perplexing circumstance. I found that by placing a card, the thickness of which was accurately one-fiftieth of an inch, under the middle of the standard, the distance of the dots was much increased; and by placing a card of the same thickness under each of the extremities, and withdrawing that which was under the centre, the distance of the dots was considerably diminished. The total difference amounted to no less than .0016 of an inch; whilst the double of the error, which would have arisen from mere curvature under similar circumstances, would not have been one ten-thousandth of an inch.

"The cause was now evident: by elevating the middle of the standard, the under surface was shortened and the upper surface extended; and on the contrary, when the extremities were elevated, the upper surface was compressed and the lower surface lengthened; the quantity of the effect evidently depending upon the thickness of the bar.

"Having thus assured myself of the source of the error, a method of obviating it soon presented itself. As the upper and under surfaces of the bar are in different states, the one being compressed and the other extended, there must be an intermediate plane which suffers neither extension nor compression; and this plane must be nearly midway between the two surfaces. I therefore caused Mr. Dollond to reduce the thickness of the bar for the distance of an inch and three-quarters from its extremities to one-half; the gold disks and plugs were then inserted as before, and the adjustment completed in the manner which has been described. The plugs being secured and the projecting parts removed, the standards were repeatedly compared with Sir George Shuckburgh's scale (the standard being placed upon the scale), when no perceptible difference could be detected. Pieces of card were now placed under the standard as before, without occasioning any appreciable alteration; and I had thus experimental proof of the perfect efficiency of the remedy I had employed*."

It will now be necessary for me to state some circumstances which led to the experiments I am about to detail.

In the year 1820, I very carefully compared a scale, now in the possession of Mr. Dollond, with Sir George Shuckburgh's standard, when the distance from zero to thirty-six inches upon both scales appeared to be precisely the same. In the year 1824, I again compared Mr. Dollond's scale with Sir George Shuckburgh's, upon the occasion of examining a scale for M. Schumacher and another for M. Svanberg. Mr. Dollond's scale by these comparisons appeared to differ from Sir George Shuckburgh's only .000181 of an inch in defect. I had thus good reason to suppose the value of Mr. Dollond's scale to be well ascertained; and as it was much thinner than that of Sir George Shuckburgh's, I considered it preferable for determining the value of the scales intended for Denmark and Sweden.

Having recently received from Mr. Dollond a scale which I had instructed him to make for the Government of Russia, and also the Imperial standard yard from the House of Commons, I commenced the necessary preparations for comparing them together.

Finding by stretching a thread along the table, which I have always used for such comparisons, that its surface was concave, I had it carefully planed until the thread no longer indicated any irregularity. My first comparisons at once showed a considerable difference between the scale intended for Russia (copied from Mr. Dollond's scale) and the Imperial standard yard; at the same time that a scale which I had formerly laid off for my own use from Mr. Dollond's, and which might be taken as equal to it, differed very little from the Russian scale. I now procured Sir GEORGE SHUCKBURGH's scale and Mr. DOLLOND's, and set myself carefully to investigate the cause of this difference.

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* Phil. Trans. for 1826, Part II. p. 44–46.

It was immediately evident that Sir GEORGE SHUCKBURGH's scale still nearly agreed with the Imperial standard yard, and that there was no great difference between Mr. DOLLOND's scale, my scale, and that intended for Russia. But the difference between Mr. DOLLOND's scale and Sir GEORGE SHUCKBURGH's was so great, as to leave no doubt on my mind that in determining the value of the former, I had committed some considerable error; and as other scales are dependent upon this, it became an object of importance to ascertain its origin and amount. The origin of this error I could not for a moment hesitate in pronouncing to be the thickness of Sir GEORGE SHUCKBURGH's scale; the very great care I have been accustomed to bestow in comparisons of this kind, leaving me no other cause to which it could with probability be referred.

I have said that a thread stretched along the table indicated no irregularity of surface. I now examined the surfaces of the Imperial standard yard by stretching a thread along that surface which was perpendicular to the table. Every face of this bar was perceptibly concave; yet when laid upon the table and pressed horizontally at one end, it moved about its centre, proving that the surface of the table was convex, though the thread was not capable of indicating it.

I next procured a marble slab nearly sixty-four inches long. This I preferred in its unpolished state; as the operation of polishing being performed upon small portions of the marble in succession might destroy the plane surface procured by grinding. Upon this slab I placed the Imperial standard yard, Sir GEORGE SHUCKBURGH's scale, my own, and Mr. DOLLOND's scale; the Russian scale being laid, for want of room, upon the table. The Imperial standard yard seemed now to rest with nearly its whole surface in contact with the marble, and this, in addition to the test of the thread, I considered to be a sufficient indication that the marble was plane.

The following comparisons were then made between the different scales, in which the same microscopes and apparatus were employed as are described in an account of the comparison of various British standards of linear measure, published in the Philosophical Transactions for 1821. It is to be observed that as the microscopes invert, an increase in the readings of the micrometer indicates that the scale is shorter than that with which it is compared, and vice versâ. The value of one division of the micrometer is .0000428742 of an inch.

Date. 1830.	Therm.	Imperial Standard Yard.	Sir George Shuckburgh's Scale.	Russian Scale.	Mr. Dollond's Scale.	Captain Kater's Scale.
		div.	div.	div.	div.	div.
May 1	66°	7	0.5	23	27.5	32.7
		8.2	2.0	23.7	25	30
2	63	104	98	120	124.5	126
		76.5	72.5	95	101	101
		8.7	5	22.5	28.5	37
	66	88	80.5	95	104	110.5
3	62	3.5	2	23	27	28.2
		3	0	24.5	26	....
		102.5	96	117	125	131
		17	14	35.5	42	46
	64	10.7	4	22.5	28.3	37
4	61	20.2	16	39	40	51
		20	16.5	37	43.6	47.5
	62	18	16	34.6	42	46
5	63	15	10.7	28	37	37.7
		14	10	26.7	33.5	42
Means...		32.02	27.73	47.94	53.43	60.24
Readings of Imperial Stand. Yd.			32.02	32.02	32.02	32.02
Differ. from Imperial Stand. Yd.			4.29	15.92	21.41	28.22

Converting these differences of the readings of the micrometer into inches, we obtain the distances from zero to thirty-six inches on each scale in parts of the Imperial standard yard.

Sir George Shuckburgh's scale . . .	36.00018 inches
The Russian scale	35.99932 ——
Mr. Dollond's scale	35.99908 ——
Captain Kater's scale	35.99879 ——

Here it is seen that Mr. Dollond's scale, which by careful comparisons in the years 1820 and 1824 appeared to differ little, if at all, from Sir George Shuckburgh's scale, is now no less than .0011 of an inch in defect.

I now resolved to determine the amount of the error which might arise from the flexure of a bar of known thickness, the curvature of the surface upon which it is placed being given.

For this purpose I prepared sets of wires, which by careful measurement I found to be, .012, .02, .03, .04, .05, and .10 of an inch in diameter.

In order to subject the surface of the marble slab to a more rigid examination, I prepared some fine copper wire by annealing it, and then stretched it along the slab, applying as much weight as it would bear at each end without breaking. This detected a curvature of one end of the slab, which the thread did not indicate, and which was removed by placing a thick card between it and the table.

The Imperial Standard Yard.

I commenced with the Imperial standard yard. The bar upon which this yard is laid off, is 1.07 inch square.

On placing a wire of .012 of an inch diameter under the middle of the bar, the two extremities were raised above the marble; a thin wedge was placed under one extremity of the bar, the other being then in contact with the marble, in order to prevent the effect which would arise from the weight of the apparatus carrying the microscopes when placed upon the bar. This precaution, I shall content myself with remarking, was used in every instance where necessary.

The following comparisons were then made between the length of the yard when the bar lay flat upon the marble, when it was curved upwards by a wire placed under its middle, and when curved downwards by wires placed under the dots marking the yard. In the latter position the middle of the bar touched the marble.

With wires of .012 of an inch diameter.

(A.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
95.5	89.5	112.0			
96.5	91.7	111.5			
95.7	88.0	111.7			
79.0	72.5	94.5			
Mean 91.7	85.4	107.4	6.5	15.5	22.0
0.2 Reduction to the Chord.					
91.9					

As in the preceding experiments the bar was clear of the marble when a wire was placed under its middle, and touched the slab when wires were placed under its extremities; I next tried whether any greater errors would be produced by the curvature which the bar would assume by its own weight when clear of the marble in both cases. For this purpose I used wires of .05 of an inch diameter, by which the bar was wholly supported.

With wires of .05 of an inch diameter.

(B.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
67.0	60.0	79.0			
61.0	54.0	78.5			
57.5	54.0	74.5			
60.0	52.0	73.0			
57.0	55.5	74.5			
Mean 60.5	55.1	75.9	5.6	15.2	20.8
0.2 Reduction to the Chord.					
60.7					

Here it may be seen that the results are very nearly the same as in the preceding Table, and consequently that the errors of the Imperial standard yard are the greatest possible with a curvature, the versed sine of which is less than one-hundredth of an inch.

In order to obtain accurate conclusions from the above Tables, it must be considered that when the bar is lying flat upon the marble it is the length of the scale that is taken, but when the bar is curved it is the chord of the arc which is measured. It therefore becomes necessary to add to the mean of the column headed "without wires" the number of divisions of the micrometer, which is equal to the excess of the arc above its chord.

This correction being made, if the difference between the mean of the readings so corrected of the micrometer, when the bar lies flat upon the marble, and the mean of the readings when a wire is placed under the middle of the bar be taken, it will give the amount of the error arising from flexure independent of the chord when the bar is curved upwards.

In like manner the difference between such corrected mean and the mean of the readings of the column headed "wires under the ends" will give the error proceeding from the same source when the bar is curved downwards: and the sum of the errors thus obtained will show the greatest error which can arise from a curvature, the versed sine of which is equal to the diameter of the wires employed. The same amount of errors may at once be obtained by taking the difference of the mean readings of the second and third columns.

In the Imperial standard yard it appears that with a curvature, the versed sine of which is less than .012 of an inch, the amount of the errors is .000943, or nearly one-thousandth of an inch; whilst the error which would result from the difference between the arc and its chord is absolutely insensible, not amounting to one hundred-thousandth of an inch. Now it must be obvious, that if a scale were compared at two different periods with the Imperial standard yard, the yard at one of such periods being placed upon a part of the table deviating from a plane surface .012 of an inch in a yard, and at another period the same quantity, but in a contrary direction,—the difference in the resulting values of the scale so compared would be no less than .000943 of an inch. This supposes the scale which is the subject of comparison either to be very thin, or to be placed upon a part of the table which is perfectly flat; a circumstance which it is not difficult to imagine very possible, or even that different parts of the surface of the table may be curved in contrary directions, when the small amount of the curvature in question is considered.

I may here observe, that the thickness of a single shaving which the plane takes off from the table is sufficient to occasion an error equal to that resulting from the preceding comparisons; for I found by careful measurements with a micrometer, that the mean thickness of such a shaving of deal was about .009 of an inch.

I have hitherto supposed the surface of the table not to be plane; but if the table were plane and the surface of the bar were curved, the bar would by its weight apply itself to the plane surface of the table; and a like error in either case would be the consequence.

It has been stated that the surface of the bar of the Imperial standard yard is concave; and by my present comparisons the distance from zero to thirty-six inches on Sir GEORGE SHUCKBURGH's scale, which by former measurements appeared not to differ sensibly from the Imperial standard yard, is in excess .0001945, or nearly two ten-thousandths of an inch. Now if the surface of the marble slab be supposed to be plane, the curvature of the Imperial standard yard would have the same effect in producing error as in the case of wires being placed under its ends; and it will be seen that a curvature of the bar, the versed sine of which is only .002 of an inch in length, would be sufficient to produce the difference in question.

On examining the preceding Table, it will be perceived that the difference between the length of the yard when lying flat on the marble, and its length when curved upwards, is much less than the difference when the bar is supported at its ends, and consequently curved downwards; and hence that the neutral plane is not as I supposed in the middle of the bar, but nearer to the convex surface: but on this important part of the subject I shall have to state more hereafter.

Sir George Shuckburgh's *Scale.*

The bar upon which this scale is traced is 67.7 inches long, 1.4 inch wide, and 0.42 inch thick. The scale comprises 60 inches, the distance from the end of the bar to zero is 3.7 inches, and from 60 to the other extremity of the bar 4 inches.

The scale being placed upon the marble slab, its ends projected two inches beyond it. I could perceive light between the bar and the marble, and could pass a sheet of letter-paper under the zero end to the distance of eight inches from its extremity, that is to the fourth inch of the scale. The bar was in contact with the marble for two inches and a quarter at the other end; and from this part letter-paper could be passed between the bar and the marble for the distance of eighteen inches. The remainder of the bar rested upon the marble.

In the following experiments, the wires in elevating the extremities were placed under 0 and 60 of the scale, and under 30 when the scale was to be curved upwards.

With wires .012 of an inch diameter.

From 0 to 36 inches.

(C.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
99.0	95.5	105.0			
99.0	95.5	106.0			
Mean 99.0	95.5	105.5	3.5	6.5	10.0
0.0 Reduction to the Chord.					
99.0					

With wires .02 of an inch diameter.

(D.)

Without Wires.	A wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
6.0	3.0	14.0			
3.0	0.5	17.0			
97.0	92.0	109.3			
96.0	94.0	109.0			
Mean 50.5	47.4	62.3	3.1	11.8	14.9
0.2 Reduction to the Chord.					
50.7					

With wires .03 of an inch diameter.

(E.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
97.0	92.7	110.2			
96.5	92.0	111.0			
Mean 96.7	92.3	110.6	4.8	13.5	18.3
0.4 Reduction to the Chord.					
97.1					

With wires .04 of an inch diameter.

(F.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
16.0	7.5	32.0			
12.0	8.0	30.7			
94.5	87.0	114.0			
Mean 40.6	34.2	58.9	7.2	17.5	24.7
0.8 Reduction to the Chord.					
41.4					

Wishing to ascertain whether the curvatures of both halves of the scale were similar, I took the distance from 24 to 60 inches.

With wires .04 of an inch diameter.

From 24 to 60 inches.

(G.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
41.0	24.0	44.0			
38.0	21.0	44.0			
126.0	104.0	129.0			
Mean 68.3	49.7	72.3	19.4	3.2	22.6
0.8 Reduction to the Chord.					
69.1					

Here we perceive that the sum of the errors is very nearly the same as before; but there is an irregularity in the curvature, or rather a difference in the place of the neutral plane of the two halves of the bar, which deserves particular attention; for instead of the error being less, as in all the preceding experiments, when a wire is placed under the middle, than it is when the extremities of the bar are supported, it is here greater. We may also remark that the difference between the means of the readings "without wires" is 27.7 divisions, which shows that the distance from 24 to 60 inches of the scale is shorter than that from 0 to 36 inches .001187 of an inch.

With wires .05 of an inch diameter.

From 0 to 36 inches.

(H.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
14.5	7.5	39.5			
13.0	3.5	33.0			
11.5	4.0	31.2			
8.5	4.0	36.7			
7.0	0.0	32.0			
Mean 10.9	3.8	34.5	8.3	22.4	30.7
1.2 Reduction to the Chord.					
12.1					

With wires 0.5 of an inch diameter.

From 24 to 60 inches.

(I.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
41.5	23.0	48.0			
39.5	18.5	50.0			
39.5	19.0	49.0			
38.7	15.7	48.5			
35.0	16.0	45.0			
Mean 38.8	18.4	48.1	21.6	8.1	29.7
1.2 Reduction to the Chord.					
40.0					

From this Table the same conclusions may be drawn as from Table (G), as to the place of the neutral plane in the two halves of the bar. I am disposed to attribute this irregularity to the manner in which the scale is constructed, as it consists of a bar of cast brass, to which is soldered a piece of plate brass, upon which the divisions are traced. The difference between the distance from 24 to 60 inches and from 0 to 36 inches does not differ sensibly from that before found, namely .001187 of an inch. I should have observed that the comparisons of the distance from 0 to 36 and from 24 to 60 inches on the scale immediately followed each other, that is, the microscopes were transferred immediately from one part of the scale to the other.

Wishing to try the effect of an increased curvature, the following comparisons were made.

With wires 0.1 of an inch diameter.

From 12 to 48 inches.

(K.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
81.0	51.5	121.0			
83.5	51.0	117.0			
86.0	53.0	120.0			
83.0	51.0	120.0			
Mean 83.4	51.6	119.5	36.8	31.1	67.9
5.0 Reduction to the Chord.					
88.4					

The above Table shows that employing the middle portion of the scale, the error in either position is nearly the mean of the errors in the two preceding Tables.

Mr. Dollond's *Scale.*

This scale is upon plate brass, 42 inches long, 1.6 inch wide, and 0.17 inch thick; it contains 41 inches. The scale when placed upon the marble slab appeared to be in contact with it throughout its whole extent.

The wires for elevating the ends were placed under 0 and 41 inches, and the wire when the middle was raised was under 20½ inches.

With wires .05 of an inch diameter.

From 0 to 36 inches.

(L.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
32.5	30.0	52.0			
32.0	30.0	52.5			
32.0	30.0	56.0			
31.8	31.2	57.0			
30.0	28.0	52.6			
30.3	28.0	52.6			
Mean 31.4	29.5	53.8	4.4	19.9	24.3
2.5 Reduction to the Chord.					
33.9					

Captain Kater's *Scale.*

This scale is of cast brass, 45.2 inches long, $\frac{5}{8}$ wide, and 0.29 inch thick. The wires, when the ends were elevated, were placed under the extremities.

With wires 0.5 of an inch diameter.

From 0 to 36 inches.

(M.)

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
105.0	97.0	137.0			
89.0	82.5	128.0			
92.5	80.5	123.0			
91.0	79.0	128.0			
95.0	80.0	128.5			
94.0	80.0	131.0			
Mean 94.4	83.2	129.3	13.2	32.9	46.1
2.0 Reduction to the Chord.					
96.4					

I shall now state the results which have been obtained from the comparisons I have detailed.

It has been already shown that a correction must be applied to the observed length of the scale, to reduce it to the chord of the given curvature. This, if the scale is only 36 inches long, may be applied at once; but with Sir George Shuckburgh's scale, the length of which is 60 inches, it becomes necessary first to find the difference between an arc of 60 inches and its chord, and then to take the proportional part of this difference for 36 inches. The same must be done with Mr. Dollond's scale, which is 41 inches long, and with my own, the length of which is 45.2 inches.

The following Table gives the difference between the arc and its chord, and the proportional part where necessary for 36 inches.

Versed Sine.	Difference of the Arc and its Chord on 36 inches.	Proportional part for 36 inches.	Equivalent Divisions of Micrometer.
inches.	inches.	inches.	
.012	.00001		
.02	.00002		
.03	.00006		
.04	.00009.....	....	2.1
.05	.00014		
Versed Sine.	Difference of the Arc and its Chord on 60 inches.		
.012	.000005	.000003	0.0
.02	.000013	.000008	0.2
.03	.000030	.000018	0.4
.04	.000054	.000032	0.8
.05	.000084	.000050	1.2
.10	.000400	.000240	5.0
Versed Sine.	Difference of the Arc and its Chord on 41 inches.		
.05	.000122	.000107	2.5
Versed Sine.	Difference of the Arc and its Chord on 45.2 inches.		
.05	.000112	.000089	2.0

Having applied the reduction to the chord, we obtain the error occasioned by the extension of the surface of the bar when it is curved upwards, and the error when it is curved downwards; but in Sir GEORGE SHUCKBURGH's scale, when a wire is placed under the middle, the surface of the bar rests upon the edges of the marble slab; the length to be considered of the bar is therefore in this position equal to the length of the marble 63.7 inches, and the error of the curvature upwards, as given in the Tables, must be increased in the proportion of 63.7 to 60.

The same method must be pursued with Mr. DOLLOND's scale, the bar being forty-two inches long.

Lastly, these results must be reduced, but in inverse proportion, to what they would have been had the length of the scale been thirty-six inches.

As the distance from 0 to 36 inches is the part of Sir GEORGE SHUCKBURGH's scale, which has been considered upon every occasion as equal to the Imperial standard yard, it is this portion to which all the subsequent deductions refer.

The following Table contains the results of the foregoing experiments, with each scale reduced to a bar of thirty-six inches in the manner which has been described, the bar being taken as equal in thickness to each scale respectively.

	Thickness of Bar.	Table.	Versed Sine.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
	inches.		inches.	inches.	inches.	inches.
Imperial Standard Yard	1.07	(A)	.012	.00028	.00066	.00094
Sir GEORGE SHUCKBURGH's Scale.	0.42	(C)*	.01	.00022	.00039	.00061
		(D)	.02	.00023	.00084	.00107
		(E)	.03	.00036	.00096	.00132
		(F)	.04	.00054	.00125	.00179
		(H)	.05	.00063	.00160	.00223
Mr. DOLLOND's Scale	0.17	(L)	.05	.00022	.00097	.00119
Captain KATER's Scale	0.29	(M)	.05	.00071	.00176	.00247

By referring to the results in this Table, derived from Sir GEORGE SHUCKBURGH's scale, it will be perceived that the sum of the errors increases as the versed sine or diameter of the wire employed. This will be more readily shown by dividing the sum of the errors by the corresponding diameter of the wire, or reducing each to a versed sine of .01 of an inch.

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* Reduced by proportion from a versed sine of .012 to that of .01 of an inch.

Versed Sine.	Sum of Errors.	Sum of Errors Reduced to a Versed Sine of .01 of an inch.
inches.	inches.	inches.
.01	.00061	.00062
.02	.00107	.00053
.03	.00132	.00044
.04	.00179	.00045
.05	.00223	.00045
	Mean..	.00050

The errors arising from the thickness of different scales, though nearly in proportion to their thickness, do not seem to follow any very regular law, but to depend in some measure upon the manufacture of the material employed, as well as upon the thickness of the bar: thus if we reduce the error of each scale to what it would have been had the scale been half an inch thick, thirty-six inches long, and the versed sine equal to .01 of an inch, we have the following results for comparison.

Errors resulting from a varied sine of .01 of an inch, the bar being supposed thirty-six inches long, and half an inch thick.

	Error when curved upwards.	Error when curved downwards.	Sum of Errors with a Versed Sine of .01 of an inch.
	inches.	inches.	inches.
Imperial Standard Yard	.00013	.00031	.00044
Sir George Shuckburgh's Scale..	.00017	.00042	.00059
Mr. Dollond's Scale	.00012	.00056	.00068
Captain Kater's Scale	.00025	.00061	.00086

Here it may be seen that the sum of the errors of my scale is the greatest, and this scale is upon a bar of cast brass not hammered.

Mr. Dollond's scale is of plate brass, and Sir George Shuckburgh's appears to be made of cast brass faced with plate brass.

The Imperial standard yard is upon a bar of brass, which I should judge from its apparent hardness to have been well hammered; but the preceding result given from this bar must be too little, as it has been shown that a wire .012 of an inch diameter was more than sufficient to produce the greatest errors to which the bar is liable.

From the last Table it appears that the error of the scale when curved upwards is scarcely more than one half of its error when curved downwards; and from this it should seem that the neutral plane is not in the middle of the bar as I had supposed.

If this inference be correct, the distance of the neutral plane from that surface of the bar, the curvature of which is convex, may be found by considering the sum of the errors in the preceding Table to represent the thickness of the bar, when the corresponding error resulting from the curvature upwards will be the distance of the neutral plane from the convex surface. This distance appears to be scarcely equal to one-third of the thickness of the bar.

From the experiments which have been detailed, we are led to the following conclusions.

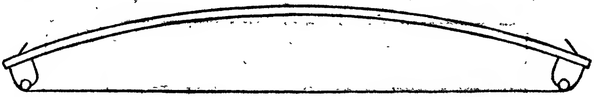
1. That in a standard of linear measure traced upon the surface of a bar, an error arises from the thickness of the bar when it is placed upon a table the surface of which is not plane.

2. That this error in bars of the same material and of unequal thickness is within certain limits as the thickness of the bar, and depends upon the extension of that surface of the bar which becomes convex, and the compression of the surface which is concave.

3. That the error to which the same scale is liable from this cause, is directly as the versed sine of the curvature of the surface upon which the scale is placed.

4. That this error very far exceeds that which would arise from the difference of length between the arc and its chord under similar circumstances; so much so, that the sum of the errors from this cause in a bar one inch thick, with a versed sine of not one-hundredth of an inch, is nearly one-thousandth of an inch; whilst double the difference between the chord and the arc is not one fifty-thousandth.

It was not until I had written thus far, that a method, with which I am perfectly satisfied, occurred to me of trying a surface supposed to be plane. The difficulty, if not the impossibility of procuring what is called a straight edge is well known to workmen; but this desideratum I supplied by the very easy process of stringing a bow six feet long with piano-forte wire one-hundredth of an inch diameter, which bears a considerable degree of tension without breaking. The wire was passed over two thick wires half bedded in pieces glued near the extremities of the bow; these served as bridges, and one was of a sufficient length to support the whole when laid upon a horizontal surface. The nature of the surface upon which it is placed may thus be examined at leisure by observing its proximity to the wire. Should the surface be convex, a wire must be placed under each extremity of the bow-string of sufficient diameter just to clear it from all but the most convex part of the surface. As this simple contrivance may be applied to a great variety of useful purposes where a straight edge is required, I trust I may be pardoned for giving the accompanying sketch of it.



By means of this apparatus I examined a variety of surfaces, any one of which I formerly should have considered as well calculated to serve as a support for scales, which were to be compared together. The following were the results.

	Length in inches.	Curvature.	Versed Sine inches.
	inches.		inches.
A mahogany dining-table....................	42	Concave.	0.04
A marble chimney-piece	61	Ditto	0.12
Another..................................	62	Ditto	0.04
Another..................................	62	Ditto	0.10
Top of a piano-forte of rosewood finely finished..	48	Ditto	0.03

The front board of a piano-forte of rosewood forty-seven inches long. This was very highly finished, and the general surface was found to be nearly plane, but irregular. Here it is worthy of observation, that I could detect the nature and extent in some degree of the irregularities of the surface, by tapping with my fingers upon the wire whilst it was pressed by the weight of the bow upon the board. Where it yielded no sound, the wire was of course in contact with the surface, which in that case was either convex or plane. When the wire yielded a sound by tapping upon it with the finger, the surface was concave; and some idea might be formed of the extent, by the acuteness or gravity of the sound produced; the edges of the concavity serving as bridges, which limited the length of the string. So delicate is this test by sound, that a concavity can be detected by this method where the interval between the wire and the surface under examination is not to be perceived by the eye.

Having now a means superior to any I before possessed of examining a plane surface, I applied it to the marble slab, and detected a slight concavity not exceeding one-hundredth of an inch. This was corrected by placing cards under different parts of the marble. When I had adjusted it so that no irregularity was perceptible, I thought it necessary to repeat my former comparisons.

The marble most carefully made plane by the bow.

Date. 1830.	Imperial Standard Yard.	Sir George Shuckburgh's Scale.	Russian Scale.	Mr. Dollond's Scale.
	div.	div.	div.	div.
May 31	7	5.5	26	31.0
	4.5	6.2	25	33.0
	4.7	7.0	24	33.2
	4.7	4.8	23.2	32
	3.3	3.3	20	30
June 1	4.0	6.0	22	31.2
	7.0	4.0	22	30.0
	5.0	4.0	21	32
2	10	6.0	23.5	31.3
	8	3.0	25	31
	6.3	3.0	22	32.5
	6	4.5	21.2	30
	3	3.5	23.5	31
	7	4	21.5	30.5
3	98.0	97	114.5	122.3
	97.5	96.5	115	123.5
4	28	22	43	53
	27.5	20	40.5	51
5	38.0	33.5	53	59.5
	37.3	31	50	59.3
Mean	20.34	18.24	36.79	45.36
Read^{s} of Imp^{l}. Stand. Y^{d}		20.34	20.34	20.34
Diff. from Imp^{l} Stand. Y^{d}		2.10	16.45	25.02

From these differences converted into inches, we have now the following distances from 0 to 36 inches on each scale in parts of the Imperial standard yard.

Sir George Shuckburgh's scale	36.00009 inches.
The Russian scale	35.99929 ——
Mr. Dollond's scale	35.99893 ——

These results differ but little from those of the former comparisons, to which, however, I think they are to be preferred.

It has been shown that from the present results, the value which has been hitherto given to Mr. DOLLOND's scale requires a considerable correction. Mr. DOLLOND's scale, by former measurements, appeared to be equal to 35.99991 inches of Sir GEORGE SHUCKBURGH's scale, and consequently to be shorter than that scale only .00009 of an inch; but it now appears to be shorter than Sir GEORGE SHUCKBURGH's scale .00116 of an inch. Sir GEORGE SHUCKBURGH's scale, therefore, when the former comparisons were made, must have been curved downwards in consequence of that part of the surface of the table upon which it was placed being concave; and we may remark that by consulting the preceding Tables, it will be seen that a curvature, the versed sine of which is .03 of an inch in a yard, would have been sufficient to occasion the error in question.

With reference to this error in the former estimation of the value of Mr. DOLLOND's scale, it is important to add that in the year 1828 I employed it (not having then access to Sir GEORGE SHUCKBURGH's standard) in determining the value of a scale for the Government of Hanover. This scale was made by Mr. DOLLOND, according to the mode which will hereafter be described; and when referred to Sir GEORGE SHUCKBURGH's scale, then taken as equivalent to the Imperial standard yard, it appeared to be equal to 35.99973 inches: upon that occasion the scale for Hanover was found by numerous comparisons to be shorter than Mr. DOLLOND's scale .00018 of an inch, which being subtracted from 35.99893 inches, the last determination of the value of Mr. DOLLOND's scale, leaves for the true length of the Hanoverian scale 35.99875 inches of the Imperial standard yard.

Having now shown the nature and magnitude of the error, which is the subject of this paper, I shall proceed to point out the means of obviating it.

It has been seen that the error in question results from the extension and compression of the surfaces of the bar upon which the scale is laid off dependent upon its curvature, and that there is a neutral surface which suffers neither extension nor compression, and which appears from the preceding experiments to be at about one-third of the thickness of the bar from that surface which becomes convex. When it is the object to have two points only on the bar, marking for example the yard, it has been already shown that by cutting away one-half of the thickness of the bar at its ends and placing the points upon the new surface, the error which arises from flexure is reduced to the least possible quantity; as this (the difference between one-half and one-third of the thickness of the bar) is the nearest approach that can be made to the neutral surface when the bar is curved upwards and when it is curved downwards.

When a scale of inches is required, this method is not available; as the whole surface of the bar must in that case be employed. But by diminishing the thickness of the bar, the magnitude of the error is diminished proportionably; and it is evident that the thickness of the bar might be so reduced as to render the error scarcely appreciable.

Having prepared a bar, or rather a plate, of a thickness which is no more than sufficient to receive the divisions of the scale, and to preserve an even surface, the next object is to provide this plate with a proper support. For this purpose a bar of brass well hammered is to be employed, of a sufficient width, and half an inch or even an inch thick. Upon the surface of this bar the thin plate intended to receive the divisions is to be placed, and made to slide freely in a dovetailed groove, formed by two side-plates of similar thickness, screwed to the surface of the bar. Lastly, the thin plate upon which the scale is to be laid off is to be fixed at its middle point to the bar by a single screw passing through it.

Now if the thick bar which forms the support be curved, its surfaces will suffer extension and compression in proportion to the thickness of the bar. The thin plate accommodating itself to the curvature of the bar will follow the same law, and the resulting error will be less in the proportion of the thickness of the plate to that of the bar. Now it has been shown that the greatest errors to which the Imperial standard yard (one inch thick) is liable, amount to nearly one-thousandth of an inch; as a wire of one-hundredth of an inch diameter placed under the middle of the bar is more than sufficient to produce the greatest curvature of which it is susceptible; and it follows that if such a bar were to form the support of a plate of one-tenth of an inch in thickness in the manner just described, the sum of the greatest errors, to which a scale so constructed would be liable from curvature, could not exceed one ten-thousandth of an inch.

The support of the scale which I have caused to be made for the Government of Russia, and the comparisons of which, with the Imperial standard yard, are given in the present paper, is a brass bar one inch and three-quarters wide, and half an inch thick. The plate carrying the divisions is one-tenth of an inch thick; but were I to construct another, it should not exceed half this thickness. This plate is made to slide freely upon the surface of the bar in a dovetailed groove, and is fixed to the bar at its middle point by a single screw in the manner before described. The divisions are thirty-six inches, marked by dots upon the surfaces of gold pins let into the brass plate. An additional inch beyond zero is divided in like manner into tenths of an inch. This description will serve also for that of the scale before alluded to, made for the Government of Hanover.

The following experiments were made with the Russian scale, the arrangements being the same as before detailed.

Russian scale with wires .04 of an inch diameter.

From 0 to 36 inches.

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
69	66	75			
71	66	73.5			
71	65	74			
70	67.5	75			
70	63.5	75			
70	64	73.5			
Mean 70.5	65.3	74.3	7.3	1.7	9.0
2.1 Reduction to the Chord.					
72.6					

I now employed wires of .05 of an inch in diameter, and these supported the bar wholly above the surface of the marble.

With wires .05 of an inch diameter.
From 0 to 36 inches.

Without Wires.	A Wire under the middle.	Wires under the ends.	Error when curved upwards.	Error when curved downwards.	Sum of Errors.
div.	div.	div.	div.	div.	div.
38.5	35.5	43.0			
39.5	32.5	39.5			
36.2	31.5	39.0			
Mean 38.1	33.2	40.5	7.0	0.3	7.3
2.1 Reduction to the Chord,					
40.2					

From these last Tables it appears that the sum of the greatest errors to which the Russian scale is liable from flexure is the greatest possible when the versed sine is .04 of an inch, and is then equal to nine divisions of the micrometer, or .00038 of an inch, which, reduced as in the foregoing experiments with the other scales to a versed sine of .01 of an inch, gives for the comparative sum of the errors due to such versed sine .00009, or about one ten-thousandth of an inch.

It is evident that the advantage of this construction consists merely in the employment of a thin plate to receive the divisions, instead of a bar of greater thickness; but a plate sufficiently thin could not well be used without a firm support, which is necessary to protect it from injury, as well as to prevent the plate from being affected by any sudden variation of temperature, either from partial currents of air, or from the vicinity of the person of the observer.

XXIV.—*On the Illumination of Light-houses. By Lieut.* THOMAS DRUMMOND, *of the Royal Engineers. Communicated by Lieut. Colonel* COLBY *of the Royal Engineers, F.R.S.*

Read June 17th, 1830.

IN a former paper which the Royal Society has done me the honour to place among its Transactions*, a method of producing intense light is described, the object of which is to render visible distant stations in geodetical operations. At the conclusion of that paper I ventured to suggest that this method might be found useful for other purposes besides that for which it had been contrived; and more especially for the illumination of light-houses.

Soon after its publication, I received, through my friend Colonel COLBY, a communication from the Trinity House, under whose jurisdiction and management the greater number of light-houses on the coast of England and Wales are placed, intimating the intention of that Corporation to give the method proposed a fair trial, whenever the apparatus should be brought to such a state of perfection as would ensure the steady continuance of the light, while at the same time it admitted of being entrusted with safety to the hands of the ordinary attendants. A good deal remained to be accomplished before this degree of simplicity could be considered as attained: nevertheless, the offer of the Corporation was prompt and liberal; and, with the assurance that the time and labour devoted to this object would not be vainly and unprofitably expended, I was anxious to undertake the necessary experiments without delay.

The survey of Ireland, however, had just been commenced; and being employed on that service, I found it impossible to continue these experiments, in the first instance from constant occupation and absence from London, and latterly from a long and severe illness, the consequence of a very laborious and anxious duty in Ireland.

During the last winter, however, I was again able to return to this subject, and take part in a course of experiments instituted by order of the Trinity House, and carried on under the direction of the Committee for the Management of Light-houses, with a view to ascertain the relative merits of different methods either adopted or proposed to be adopted in the illumination of light-houses. The result of these experiments I have received permission to communicate to the Royal Society; but in order that the particular methods to which they refer may be better understood and more fully appreciated, a few preliminary observations may be necessary.

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* Phil. Trans. 1826, Part III. page 324.

The more rude and ancient methods of illuminating light-houses with open coal fires, with common lamps or candles, sometimes aided by reflectors composed of small facettes or plane mirrors*, have in this country been completely superseded even in light-houses of secondary importance; and it may be said that there is only one method now in use for this purpose along the coast of Great Britain and Ireland. This consists in the use of a parabolic reflector of about three or four inches focal length, and from twenty-one to thirty inches in diameter, illuminated by an Argand lamp, seven-eighths of an inch in diameter, placed in the focus. The reflector is hammered out of a plane surface consisting of two plates of silver and copper rolled out together, and though executed with great skill, considering the means, cannot be regarded as a very perfect instrument. This description must be understood as applying only to light-houses under the management of public bodies: with respect to those which have been let to private individuals, I have no very accurate information; but, if they should, on examination, prove to be of an inferior order, it would only be the natural consequence of so pernicious a system.

In fixed lights the number of these reflectors varies according to the portion of the circumference required to be illuminated; but it should not be less than this arc divided by the angle of divergence of the reflected light. At the Eddystone, where the whole circle requires to be illuminated, the number should not be less than $\frac{360}{17} = 21$: if it be less than this, there must be dark spaces diverging from the light-house as a centre, in which nothing but the unassisted light of a single Argand will be visible.

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* The Eddystone till the year 1811 was lighted with 24 wax candles. Up to that time it was in the hands of private individuals; but on the expiration of the lease the Trinity House took it under their own management, and immediately substituted lamps and reflectors. The Bidstone, a leading light to Liverpool, consisted of a large built reflector, about 10 feet in diameter, lighted by an immense spout-lamp with a wick about 12 inches wide, from which a volume of smoke arose that completely intercepted the light from the upper part of the reflector.

In revolving lights there are five, seven, and even ten reflectors on a side, the number of sides being usually three or four. In the light-houses lately erected on Beachy Head, and on the Perch Rock at the mouth of the river Mersey, there are thirty reflectors in each, disposed on three sides, each bearing ten reflectors. These are the latest, and may be considered as the best specimens of this method of illumination; being about ten times more powerful than the ordinary fixed lights. In some few instances oil gas has been introduced, but the intensity of the flame being very little, if at all, superior to that of an Argand lamp supplied with the best spermaceti oil, little or no advantage can be expected from this introduction, as far as regards the brilliancy of the light, when reflectors are used.

About thirty-eight years ago the experiment was tried, in three or four light-houses, of substituting glass lenses instead of metallic parabolic reflectors. These lenses were 20 inches in diameter, of 19 inches focal length, and about 5 inches thick; but from the imperfection of form and the badness of material, the light transmitted by them appears by our late experiments to be about one-third of that of the reflectors now in use, while their divergence is so small, that at $1\frac{1}{3}$° on each side of the axis they cease to be visible. With a view probably to remedy these defects, a somewhat extraordinary arrangement was adopted, viz.—the addition of parabolic reflectors behind the lenses. It is true that by this means some addition is made to the direct light of the lens, and, what is of more consequence, the divergent light is increased so that at an angle of about 3° with the axis, it is equal to about thirteen times the light of an Argand. So far therefore the reflector, though but a small portion of it comes into use, contributes to the effect of the lens; but the converse experiment does not appear to have been tried, viz.—how far the reflector was improved by the lens placed before it; otherwise it would quickly have been perceived that the effect of the reflector alone was about double the united effects of the reflector and lens; while at the same time its effective divergence was also greater, being about eight times that of the combined lens and reflector, at an angle of 3° on either side of the axis.

This plan was fortunately never very extensively adopted; and in those light-houses belonging to the Trinity House, where it was tried, it has subsequently been discontinued, and the lenses replaced by reflectors. The North Foreland, however, under the management of the governors of Greenwich Hospital, still remains a solitary example of a method which cannot be too soon abandoned, more especially since the remedy is so easy,—merely to remove the lenses, and leave a free and unobstructed passage to the light of the reflectors.

Another mode, differing from any of those now described, has lately been introduced into France by MM. ARAGO and FRESNEL, which rivals the most powerful of our lights in brilliancy, and surpasses them in economy and facility of management. A large Argand lamp with four concentric wicks, the exterior of which is 3⅓ inches in diameter, occupies the centre of the light-house. Around this powerful light eight magnificent lenses 30 inches square are disposed, touching each other at the edges, and forming a hollow octagonal prism about the lamp. Above these, smaller lenses of a similar construction, but in the form of trapezoids, are placed, inclining towards the centre till their axes form angles of about 50° with the horizon, at which inclination their sides come into contact, and thus completely inclose the central light. By the intervention of plane mirrors, the beams of light issuing from the secondary lenses are rendered parallel to those of the principal; but by the same means a horizontal deviation of about 7° is given to them, so that this addition to the light is made to contribute to the divergence and consequent duration of light when revolving, rather than to add to its brilliancy. The lens, which is plano-convex, is of a peculiar construction, being formed of separate rings or zones, whose convex surfaces preserve nearly the same curvature as if they constituted portions of one complete lens, the interior and useless part of the glass being removed; so that a section of these zones resembles a wedge placed with the edge uppermost; one side, that next the lamp, being a straight line, the other an arc of a circle*.

The idea of such a lens appears first to have occurred to the celebrated BUFFON, when engaged in some experiments on burning-glasses; but he supposed, what is not possible, that it might be ground out of one large piece of glass. Dr. BREWSTER, in an article on the same subject in the Edinburgh

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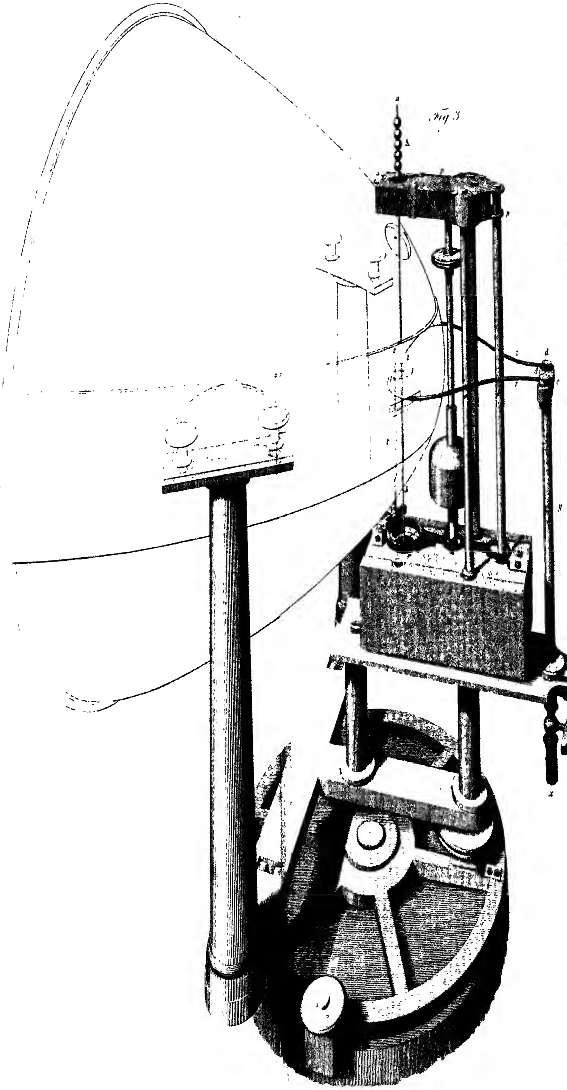
* FRESNEL, Mémoire sur un nouveau systême d'eclairage, lu à l'Academie des Sciences, 1822.

Fig 3



a
A
p
d
c
z
r
y
t
x
x

Fig 4

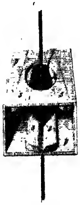


Fig 1

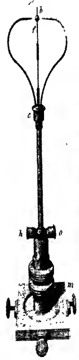


Fig 2

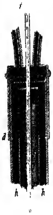
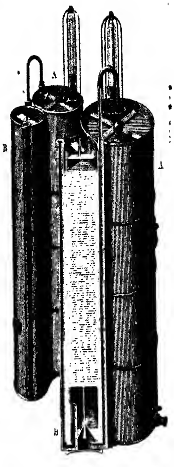


Fig 5

Fig 6



Encyclopedia, in 1811, showed that it might be built of separate pieces; and this was an important step, inasmuch as it rendered of easy execution what was before impracticable. To Dr. BREWSTER therefore the priority of suggesting this improvement is due. To MM. ARAGO and FRESNEL, obviously unacquainted with what had been previously done or recommended, belongs the praise of having first got such a lens constructed, of combining it with a very powerful lamp, and above all of giving it a most useful and beneficial practical application.

The Cordouan light-house at the mouth of the Garonne, the difficult entrance to Bourdeaux, has been fitted up in this manner; and as the lens and lamp used in our experiments were purchased at Paris of the same makers, a pretty accurate estimate may be formed of its merits when compared with the light-houses of this country.

Such are the methods at present in use in the best light-houses of Great Britain or France. The third and last method is that which I have ventured to propose, and in which the light is derived from a source altogether different from the preceding two; a ball or cylinder of lime, intensely ignited, being substituted for the Argand lamps.

For the purposes of a survey, when portability rather than economy is the more important object, this intense heat was obtained by directing a stream of oxygen gas through a flame of spirit of wine: but for a light-house, where, from the long continuance of the light, economy is a primary object, and portability is no longer required, it was desirable if possible to substitute hydrogen gas for the alcohol. As the effects produced in this manner are very remarkable and considerably exceed those formerly obtained, I shall now give a short description of the apparatus, and then proceed to state the results of our experiments.

Plate XII. fig. 1. represents the lamp. The two gases, oxygen and hydrogen, proceding from separate gasometers, enter at *o* and *h*, but do not mix till they arrive at the small chamber *c*, of which fig. 2. is a section: into this chamber the oxygen gas from the inner tube is projected horizontally through a series of very small apertures, and the hydrogen gas rises vertically through a series of similar apertures at *d*. The united gases then pass through two or three pieces of wire-gauze placed at *e*, and being thus thoroughly mixed, issue through the two jets against the ball *b*. To prevent the wasting of the ball opposite the two jets, and at the same time to diffuse the heat more equably, it is made to revolve once in a minute, by means of a movement placed underneath the plate *m*, and with which the wire *f*, carrying the ball and passing through the stem, is connected. Notwithstanding, however, this arrangement, the effect of the heat is such as gradually to cut a deep groove in the ball, so that at the end of about 45 minutes it becomes necessary to change it*. In a light-house where it is of essential consequence to maintain a constant light, it would be unsafe to entrust this to an attendant, and hence the necessity of devising some means for remedying this inconvenience. The apparatus represented by fig. 3. is designed for this purpose, and is drawn in the manner in which it is applied to a reflector, the dotted outline of which is shown.

The wire *a b* passes through the focus of the reflector, and upon it are placed the number of balls at A, required for any given time; these, by means of the shears *s*, as shown in fig. 4, are admitted between the plates *p p*, and thence permitted to fall in succession to the focus. No. 1. represents the focal ball; about two minutes before the change, the ball 3. falls into the position 2, where it becomes gradually heated. At the end of that time, the curved support *t*, moving on a pivot, is thrown into the position represented by the dotted line, by the momentary descent of the ring *r*, which, receiving an impulse from the weight *w*, acts upon the extremity *u* of the support. No. 1. falls, but is prevented from descending more than its own diameter by the loop *l*, and No. 2. following it, occupies the focus. The support *t*, being immediately released, returns by the action of a spring to its former position, retains No. 2, and suffers No. 1. to escape through the loop into the cistern.

The wire *a b* and the support *t* revolve together, and carry round the focal ball, which is ignited as in fig. 1. by the two jets *z z*. These jets, which are moveable round the joints *d d*, enter through small apertures cut in the sides of the reflector, and are easily adjusted to the proper distance from the ball.

Wherever the light is required to be diffused equally around, the renewal of the lime may be effected still more easily, by using a cylinder as represented in fig. 5, instead of a ball, which being gradually raised while revolving, brings fresh portions in succession opposite the jets. In a reflector, a cylinder occasions partial shadows at the top and bottom; still, however, the simplicity and certainty with which it may be renewed will probably entitle it to a preference even in this case.

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* When a cylinder is used instead of a ball, a ring of minute crystals is found adhering to the surface above and below this groove.

These different instruments were prepared by Mr. Simms of Fleet-street, to whom, for various ingenious suggestions, for the trouble which he has bestowed upon them, and the assistance which he contributed during the progress of the experiments, I am greatly indebted.

The apparatus for supplying the lamps with gas is represented in fig. 6. It consists of two strong cylinders, A, 3 feet high, the one for oxygen, the other for hydrogen: the gas is compressed two or three times in each, the latter by being generated under pressure, the former by being pumped in. To each of these gas-holders, a governor, B, is attached, of one of which a section is shown; by which means, whatever be the variation of pressure in the gas-holder, provided it exceed that of the governor, the gas will issue at x with a uniform and constant stream; in the present instance under a pressure of 30 inches of water. Although this apparatus was of great use in the experiments at the Trinity House, and subsequently at Purfleet, by enabling me to keep within a small compass a supply of gas sufficient for two hours consumption, and even to renew it without impeding the progress of the experiments, yet I may remark, that on a large scale the gasometers required would be much more simple, since compression would no longer be required. This apparatus was made for me by Mr. Samuel Crosley, the ingenious inventor of the gas-governor.

Our first experiments were on the illuminating powers of the different lights, independently of the lenses or reflectors with which they are generally used. The method of shadows and that of equally illuminated surfaces, both depending on the same principle, but requiring different instruments, were employed*; the former after the manner of, and with all the precautions recommended by, Count Rumford†; the latter according to the arrangements proposed by Mr. Ritchie‡, who was kind enough on this occasion to make several experiments with his own instruments, and without being made acquainted with the results previously obtained by us. The standard we used was an excellent

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* Bouguer, Traité d'Optique.

† Phil. Trans. 1794, Part I. page 67.

‡ Phil. Trans. 1825, Part I.

Argand lamp ⅞ inch in diameter, supplied with the finest spermaceti oil, and capable of supporting a flame 1¾ inch in height. The following results were obtained:

		Standard.		Wax candles, 1⅛ inch diam., the wick consisting of 40 threads.
French lamp, Mean of	5 observations by shadows . .	1	. .	= 9.1
———	9 ——— . .	8.7	10.4*	= 94.6
———	6 ———by illuminated surfaces	12.1		
Oil gas burner, same diam. as standard	4 ———by shadows	0.95		= 8.7
Lime-ball or cylinder ⅜ inch diam.	2 ———	9.2	13.5	=122.9
	2 ———	10.4		
	4 ———by illuminated surfaces	18.6		
	10 ———	16.0		

The light of the ball, depending upon the intensity of the heat, is very different at different parts, being greatest opposite the jets, and diminishing towards the sides. The mean of the greatest and least intensity is taken in the above Table; and moreover, though the greater number of observations might appear to warrant giving greater weight to their results, yet being made on the same day, and under the same circumstances, it was found that the results seldom differed, whatever might be the number of observations: hence the arithmetical mean is taken, and we obtain this remarkable result,—that the light emitted by a lime-ball only ⅜ of an inch in diameter, heated by two jets, is equal to 13 Argand lamps.

With respect to the intensity or intrinsic brightness of the different lights, the property on which their utility in light-houses more immediately depends, we have the following results:

The intensity of			Mean.	
French lamp, Mean of	11 observations by shadows . .	= 4.1	4.0	times the intensity of the Standard.
———	8 ——— . .	= 3.8		
Oil gas	6 ——— . .	= 0.85 †		
Lime	6 ——— . .	= 263.9	264.1	
——	3 —— by illuminated surfaces	= 264.4		

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* The result given by Fresnel, in the memoir quoted above, considerably exceeds this, being stated at 17 lamps of Carcel. I know not to what cause this difference is to be attributed.

† This low degree of intensity indicates impurity in the gas.

These results were obtained by screening the different lights, and then placing equal apertures opposite each, changing the apertures and taking the mean to destroy the effect of any inaccuracy in size. The intensity of the lime-ball being therefore 264 times that of the Argand lamp, a single reflector illuminated by the former will be equal to 264 reflectors illuminated by the latter; but the divergence of the reflected light, depending upon the size of the luminous body in the focus, will be smaller with the ball than with the lamp in the proportion of about 3 to 8: hence in such a light-house as that of Beachy Head, 8 reflectors may be substituted for 30, and yet an effect would be produced 26 times greater than that of the present light, the most perfect of its kind in this country.

By similar experiments it was found that the French lens was equal to 9.1 reflectors; and if the effect of the additional lenses and reflectors which ought to accompany it, and which has been estimated at one-seventh, be added, then the lens is equal to 10.4 reflectors. In like manner, therefore, the effect of a single reflector with a lime-ball would be equal to 25 times that of such a combination of lenses.

Such appear to be the singular and important results of our late experiments at the Trinity House, made as they have been with every precaution by different individuals, with different instruments, and unbiassed by the knowledge of each other's results. I see no reason to doubt their accuracy; and the comparative appearances of these different lights, when exhibited at a distance of ten miles, to which I shall presently allude, though not admitting of being reduced to numbers, confirm the striking superiority of this method of illumination.

It may now perhaps be asked, At what expense can such a light be maintained? Can the gases by which the requisite heat is produced be procured at such a price as to compete with oil or coal gas? The data I possess for forming an estimate of the expense of the gases are very scanty, but the quantity consumed was accurately determined; at the same time the consumption of the other lights was also tried, and the results are as follow:

	Consumption in 3¼ hours.	Expense per hour.
An Argand lamp seven-eighths of an inch in diameter	1 gill	0.69 penny
The same placed in a reflector . .	1⅕ gill . . .	0.83 penny
The French lamp	2 qts. ½ pt. . . .	1*s.* 2½*d.*

The lime requires 4 cubic feet of hydrogen and 2 of oxygen per hour, and the probable expense is 5*d.* per hour.

In a revolving light of the first class, containing 30 reflectors, the expense per hour would therefore be about 2*s.* 1*d.* If the French method were employed, the increase of light would be $\frac{1}{20}$th, and the expense only 1*s.* 2½*d.* per hour. If six reflectors illuminated with lime-balls were used, which would probably be sufficient, the probable expense would be 2*s.* 6*d.* per hour, and the increase of light 26 times.

If this estimate be erroneous, I think it will prove to be so in excess: admitting, however, that the expense should, in the first instance, somewhat exceed what has been stated, it may in this as in every similar instance be expected that after a little experience a considerable reduction would be effected. This is a new source of artificial light, differing from every other at present in use, and the materials by which it is produced are among the most abundant products of nature; but never having yet been applied on a great scale to any practical purpose, it has not hitherto been an object to obtain them in a separate state at a small expense. When this is effected, it will no doubt receive many useful and important practical applications.

Meanwhile, however, the case in question may perhaps be regarded as one where expense ought not to be a primary object of consideration. On all ordinary occasions, the preference of one mode of illumination to another is a question of convenience, luxury or economy; but in this it assumes a more important character, for it involves to a great extent the preservation of life and property.

To complete the preceding account, it only remains to add a description of the appearances presented by the different lights when exhibited at a distance; and to those who have entered with any degree of interest into the above details, such a description, it is hoped, cannot fail of proving acceptable.

The experiments at the Trinity House being concluded, the whole of the apparatus was removed to Purfleet, where on a knoll of chalk about 100 feet above the river a temporary light-house had been erected, and being fitted with the requisite machinery, the different lights were made to revolve in succession, and the appearance which they presented, as well as the duration of the light, were observed from the Trinity Wharf at Blackwall, a distance in a straight line of 10¼ miles.

The four faces of the revolving machine were thus occupied:

No. 1. A single reflector 21 inches diameter, 3 inches focal distance, with an Argand lamp.

No. 2. Seven reflectors with ditto.

No. 3. French lens, with its lamp.

No. 4. Single reflector with lime-ball.

The respective lights were accurately placed in focus.

On the evening of the 10th of May, the machine performing one revolution in eight minutes, Captain Pelly of the Trinity House made the following observations on the different lights from the Trinity House Wharf, Blackwall, 10¼ miles distant.

No.	Duration.		Divergence.	Computed maximum Divergence.
	min.	sec.		
1	0	25	17°.40	17°.81
2	0	25	17.40	17.81
3	0	7	5.17	5.18
4	0	9	6.12	6.7

When No. 4, the reflector lighted with the lime-ball, was turned towards the Wharf, the light was so great that the shadow of the hand and fingers was distinctly visible even on a dark brick wall, while no such effect was discernible when the other lights were turned in the same direction.

In order more justly to estimate their comparative effects, No. 4. was removed to a temporary tent about twenty-five yards to the right of the light-house, as far as the edge of the cliff would permit, and on the evenings of the 25th and 31st May regular series of experiments were made. Being engaged at Purfleet, directing these exhibitions, I never had an opportunity of witnessing their effects at Blackwall; but Captain Basil Hall, R.N., who from the interest which he took in these experiments was an attentive observer of all that occurred, has at my request kindly favoured me with the following interesting account:

4, St. James's Place, 1st June 1830.

"My dear Sir,

"You wished me to take particular notice of last night's experiments with the different kinds of lights exhibited at Purfleet, and observed at the Trinity

Wharf, Blackwall; but I have little to add to what I told you respecting those on the evening of the 25th instant: indeed it is not within the compass of language to describe accurately the details of such experiments, for it is by ocular evidence alone that their merits can be understood.

"Essentially the experiments of last evening were the same as those of the 25th, and their effects likewise. The degrees of darkness in the evenings however were so different, that some particular results were not the same. The moon last night, being nine or ten days old, lighted up the clouds so much, that even when the moon herself was hid, there was light enough to overpower any shed upon the spot where we stood by your distant illumination: whereas on the 25th, when the night was much darker, the light cast from the temporary light-house at Purfleet, in which your apparatus was fixed, was so great that a distinct shadow was thrown upon the wall by any object interposed. Not the slightest trace of any such shadow, however, could be perceived when your light was extinguished, and any of the other lights were exposed in its place.

"In like manner on the evening of the 25th it was remarked by all the party at the Trinity Wharf, that, in whatever direction your light was turned, an immense coma, or tail of rays, similar to that produced by a beam of sun-light in a dusty room, but extending several miles in length, was seen to stream off from the spot where we knew the light to be placed, although, owing to the reflector being turned too much on one side, the light itself was not visible.

"Now, last night there was none of this singular appearance visible; but whether this was caused by the presence of the moonlight, or by the absence of the haze and drizzling rain which fell during the evening of the 25th, I cannot say. I had hoped that the appearance alluded to was to prove a constant accompaniment to your light, in which case it might, perhaps, have been turned to account for the purposes of light-houses. If in hazy or foggy weather this curious effect of reflected light from the atmosphere be constant, it may help to point out the position of light-houses, even when the distance of the observer is so great that the curvature of the earth shall render it impossible for him to see the light itself.

"The following experiments tried last night were the same as those of the 25th, and certainly no comparative trials could be more fairly arranged.

"Exp. I. The first light exposed was the single Argand burner with a reflector. This was quite distinctly seen, and all the party admitted it to be a good light. After several minutes this was put out.

"Exp. II. The seven Argand burners were next shown, each in its reflector; and this was manifestly superior to the first; but how much so I cannot say, perhaps four times as conspicuous. Both these lights had an obvious tinge of brown or orange.

"Exp. III. The third light which was exposed, (on the seven Argands being put out,) was that behind the French lens; and I think it was generally admitted by the party present, that this light was whiter and more intense than that from the seven Argands, though the size appeared very much the same.

"Exp. IV. The fourth light was that which you have devised, and which, instead of the clumsy word 'Lime', ought to bear the name of its discoverer. The Drummond light, then, the instant it was uncovered, elicited a sort of shout of admiration from the whole party, as being something much more brilliant than we had looked for. The light was not only more vivid and conspicuous, but was peculiarly remarkable from its exquisite whiteness. Indeed there seems no great presumption in comparing its splendour to that of the sun; for I am not sure that the eye would be able to look at a disk of such light, if its diameter were made to subtend half a degree.

"The next series of experiments was the most interesting and decisive of all. Each of the lights above enumerated, viz. the single Argand burner, the seven Argands, and the French lens, were exposed, one at a time, in company with your light, in order to try their relative brilliancy.

"First comparative Experiment.—The single Argand burner was first exposed to this comparative ordeal, and nothing could be more pitiable than the figure it cut. Many of the party could not see the Argand light at all; while others could just detect it 'away in a corner,' as some one described it. It was also of a dusky orange tinge, while your light was of the most intense whiteness*.

"Second comparative Experiment.—The seven Argand burners were now substituted in place of the single light. All the party could now see both lights, but the superiority was not much less obvious. I really cannot affix a proportion either as to size or brilliancy; but I should not hesitate to say that your light was at least six or eight times as conspicuous; while in brilliancy, or purity, or intensity of light, (for I know not precisely what word to use to describe the extreme whiteness,) the superiority was even more remarkable. All this which I have been describing was expressed, and appeared to be quite as strongly felt by the rest of the company, to the number, I should suppose, of five-and-twenty or thirty persons, who were all closely on the watch.

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* To many, the rays from the brighter light appeared, when seen with the naked eye, to extend across and envelope the fainter light, though the perpendicular distance between them was twenty-five yards.

"Third comparative Experiment.—The next comparative trial was between the French lens and your light. The superiority here was equally undeniable; though the difference in the degree of whiteness was not so remarkable. The French light, however, is so nearly similar to that from the seven Argands, that the comparison of each of them with your light gave nearly the same results, and all equally satisfactory on the score of your discovery.

"Final Experiment.—The flashes with which the experiments concluded were very striking, and might I think be turned to great account in rendering light-houses distinct from one another. The revolutions were not effective, and, as I said before, there was no appearance last night of those enormous comets' tails which swept the horizon on the night of the 25th, to the wonder of all who beheld them: neither could there be detected the slightest trace of any shadow from the light thrown towards us, and I suspect none will ever be seen, when the moon, whether the night be clouded or not, is of so great a magnitude.

"Such is the best account I can give of what we witnessed; and I need only add that there seemed to be amongst the company but one opinion of the immense superiority of your light over all the others brought into comparison with it.

"I am, &c.

"Basil Hall."

The advantage of such a light being fully recognised, attention may now be exclusively directed to remove some of those minor obstacles that might render its use in light-houses objectionable; and I have great pleasure in adding that the Trinity Corporation are desirous that every facility in their power should be afforded with a view to effect this object, and that a series of preliminary experiments is accordingly to be carried on at their expense.

Simple as the apparatus and the experiments now described may appear, they have occasioned more trouble and anxiety than would be supposed by those who have not been engaged in similar pursuits; but, on the other hand, I fully acknowledge the encouragement derived from the interest which they appeared to excite.

His Royal Highness the Duke of CLARENCE, Master of the Trinity Corporation, was pleased to be present on one occasion, and remained more than an hour, entering with great interest into the details, and expressing himself much gratified with the effects which were produced.

Sir GEORGE COCKBURN and Mr. BARROW from the Admiralty, and several other naval men whose opinions on such subjects are entitled to the utmost deference, not only attended at the Trinity House, but went afterwards to Blackwall to observe the relative appearance of the lights when exhibited at a distance. The night of the 31st May had been appointed by the Deputy Master for this purpose; and, being desirous that the subject should be fully scrutinized, I was glad to learn on returning from Purfleet that, besides the gentlemen immediately connected with the Trinity House, the experiments described in Captain HALL's letter had been witnessed by Admiral Sir THOMAS HARDY; by the LORD ADVOCATE of Scotland, one of the Commissioners for the northern light-houses; by Sir THOMAS BRISBANE, Colonel COLBY, Captain BEAUFORT, Hydrographer to the Admiralty, and several other individuals eminent for their professional and scientific attainments; and as far as I have been able to learn, the opinions of these distinguished persons coincide with those expressed by Captain HALL, who in fact, as he has given me authority to state, endeavoured to frame his account of what passed in strict conformity with the general sentiments of the party, and neither to exaggerate nor under-rate any of the results.

It now only remains for me to perform the agreeable task of bearing testimony to the liberal spirit evinced by the Trinity Corporation on this occasion, and to the desire which they have manifested of facilitating, by every means in their power, the introduction of this method of illumination into light-houses. Indeed I hesitate not to express my belief that, if this do not take place, it will arise from some insurmountable difficulties in the way of its practical application, and not from the want of a full and impartial trial on the part of that body, to whom these establishments are entrusted.

I owe this acknowledgement in an especial manner to **John Woolmore**, Esq. the Deputy Master of the Corporation, to Captains **Clarke**, **Pelly** and **Browne**, the gentlemen constituting the Light Committee, for the fairness and impartiality of their decisions, as well as the indulgence which they extended to those defects inseparable from a new apparatus; and to Mr. **Herbert** the Secretary, for his uniform desire to promote every arrangement that appeared likely to bring this inquiry to a satisfactory termination.

To these acknowledgements I may be permitted to add my obligations to my commanding officer and friend Colonel **Colby**, for the facilities he has afforded me in carrying on these experiments, and for the advice and assistance which he has on this and many other occasions so kindly rendered me.

London, June 17th, 1830.

XXV. *On the electro-magnetic properties of metalliferous veins in the mines of Cornwall. By* Robert Were Fox *of Falmouth. Communicated by the President.*

Read June 10th, 1830.

In one of my communications to the Cornwall Geological Society on the high temperature of the interior of the earth, I ventured to express a belief that mineral veins, and the internal heat, are connected with electrical action. This opinion, founded as it was on the curious arrangement of the veins, &c. in primitive rocks, I have had the satisfaction to find confirmed by experiments made in some of the mines of Cornwall; and, I doubt not that the existence of electricity in metalliferous veins similarly circumstanced, and capable of conducting it, will prove to be as universal a fact, as the progressive increase of temperature under the earth's surface is now admitted to be, much as my conclusions on this point were at one time controverted.

In my first experiment I did not succeed in detecting any electricity; but in my second I had the gratification to observe considerable electrical action.

My apparatus consisted of small plates of sheet copper, which were fixed in contact with ore in the veins by copper nails, or pressed closely against it by wooden props, stretched across the "levels" or galleries. Between two of these plates at different stations, and a galvanometer*, a communication was made by means of copper wire one-twentieth of an inch in diameter, which was at first coated with sealing-wax; but afterwards this precaution was dispensed with.

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* It may be proper to describe the galvanometer employed in making many of my electro-magnetic experiments. The magnetic needle was three inches and a quarter long, one-eighth of an inch wide, and one-twenty-eighth thick. It was inclosed in a box four inches square and one inch in depth, having a plated copper wire one-fiftieth of an inch in diameter coiled round it twenty-five times. No magnet was used to neutralize the terrestrial polarity. In my earlier experiments a less delicate galvanometer was employed.

The accompanying sketches are intended to represent the plans adopted, and the results obtained in various mines, and under different circumstances. In some instances nearly 300 fathoms of copper wire were employed*.

The intensity of the electro-magnetic action differed greatly in different places:—in some cases the deviation of the needle was inconsiderable, in others it went completely round the circle. In general it was greater, *cæteris paribus*, in proportion to the greater abundance of copper ore in the veins, and in some degree perhaps to the depth of the stations;—and where there was little or no ore, there was little or no action. Hence it seems likely that electro-magnetism may become useful to the practical miner in determining with some degree of probability at least, the relative quantity of ore in veins, and the directions in which it most abounds.

When the distance of the plates from each other in a horizontal direction was only a few fathoms, and the copper ore between them was plentiful, and uninterrupted by non-conducting substances, or the workings in the mine, no action occurred, owing no doubt to the good conducting power of the vein; but where a cross vein of quartz or clay happened to be between the plates under similar circumstances, the action was usually great.

When the communication was established between two plates at different depths on the same vein, or between different veins, whether at the same level or otherwise, the electrical action was in general the most decisive. In fact, veins which in some instances were almost destitute of ore, and did not affect the needle *per se*, did so, though perhaps only in a slight degree, when electrical communications were made between them.

It will be seen that the direction of the positive electricity was in some cases from east to west, and in others from west to east; and when parallel veins were compared, its general tendency was, I think, from north to south, though in several instances it was the reverse. In veins having an underlie towards the north, the east was commonly positive with respect to the west; but in veins dipping towards the south, the contrary was observed, with one exception only, and that under rather unusual circumstances. (See Fig. 27.) In comparing the relative states of veins at different depths, the lower stations appeared to be negative to the upper; but exceptions sometimes occurred when a cross vein of quartz or clay intervened between the plates, and the higher one was on the negative side with respect to the horizontal currents. (See Fig. 8. & 12.)

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* I am indebted to the kindness of my brother, Lewis Fox, for the experiments referred to in Fig. 3, 13, & 22; to W. J. Henwood for those in Fig. 8, 25, & 26, and to Richard Tregaskis for those represented in Fig. 16, 17, & 27. They moreover all assisted me materially on several occasions in making my experiments, especially my brother, who was my companion in most of my descents into the mines. To the various mine agents I am also under particular obligations, for their great attentions and personal assistance; some of them having remained with us underground ten or eleven hours together.

In such cases it may be supposed that there is an accumulation of electricity in different states, on the opposite sides of the non-conducting vein. Such intersections of ore veins, and their being often very rich to a great depth in one direction and not in another, added to their varying underlie at different depths, which is not unfrequently reversed, may tend to produce apparent anomalies in experiments of this nature.

At Huel Jewel mine, I obtained results between a heap of copper ore at the surface, and a plate fixed at different depths against the ore in the vein; the latter becoming more negative, in proportion to the depth at which it was placed. Piles of copper ore at the surface did not act on the needle when tried together, independently of veins, nor was it to be anticipated that they would.

It is not improbable that the progressive increase of negative electricity observed in descending into our mines, if hereafter confirmed, may be found to be connected with the progressive increase of temperature. I have not, however, discovered any distinct connection between them at the same level, but then the differences of temperature are comparatively small. Nor does the electricity appear to be influenced by the presence of the workmen and candles, or by the explosion of gunpowder, although some veins of copper ore were blasted on different occasions in the immediate vicinity of the copper plates. And at a very productive copper vein in Great St. George Mine, the ground is so soft that gunpowder is not used; yet the needle was powerfully acted upon by the electricity it contained. On this occasion, as well as on some others, I remained with the galvanometer at the surface, letting the wires down through the shafts; and in this manner I have sometimes found the electricity act with considerable energy, so as even to cause the needle to revolve with some velocity.

In connection with the electricity of veins, I deemed it desirable to ascertain the relative power of conducting galvanic electricity possessed by many of the metalliferous minerals; and it appeared to be in about the following order, viz.

Conductors.

Copper nickel,
$\left\{\begin{array}{l}\text{Purple copper,}\\ \text{Yellow sulphuret of ditto,}\\ \text{Vitreous ditto,}\end{array}\right.$
Sulphuret of iron,
Arsenical pyrites,
Sulphuret of lead,
Arsenical cobalt,
Crystallized black oxide of manganese,
Tennantite,
Fahlerz.

Very imperfect conductors.

Sulphuret of molybdenum,
Sulphuret of tin, or rather bell-metal ore.

Non-conductors.

Sulphuret of silver,
Ditto of mercury,
Ditto of antimony,
Ditto of bismuth,
Cupriferous ditto,
Realgar,
Sulphuret of manganese,
Ditto of zinc,
Mineral combinations of metals with oxygen, and with acids.

All the conductors of galvanic electricity were so likewise of common electricity; to which may be added the oxide of tin, and, in a less degree, the sulphurets of bismuth and silver, the phosphate of manganese, and a few of the oxides. Sulphuret of zinc appeared to be a more perfect non-conductor of common electricity as well as the sulphuret of antimony, than the red oxides of those metals.

Amongst the rocks prevalent in Cornwall, clay-slate or "killas" seemed to possess the property of conducting common electricity in a slight degree, but only in the direction of its cleavage, perhaps owing to the moisture it retained.

I mention these facts in some detail, because it is curious to observe that the conducting power of metallic ores appears to have no reference to any of the electrical or other properties of the metals in a pure state, or to the proportion of them in combination. Silver and mercury, for example, are combined with, comparatively, very small quantities of sulphur;—and zinc, which seems to hold an opposite place to silver in the electrical scale, is also found in combination with a much less proportion of sulphur than is contained in copper pyrites, though the latter is one of the best mineral conductors of electricity.

There are many other analogous examples, which prove that no conclusion can be drawn à priori, from the nature or chemical arrangements of minerals, as to their relative electrical properties.

Much time and attention have been bestowed by geologists on the consideration of the origin and comparative ages of veins, and but little, I apprehend, on the purposes for which they are designed.

It appears to me that it will prove a vain attempt to reconcile a multitude of facts observable in our mines with any known natural causes.

I may refer to a few of them:—

1st. The very oblique descent of a large proportion of the veins into the earth, in some cases in very hard rock, and in others in ground so soft that it would immediately fall in, however small the excavation, without being completely supported by timber. Were it possible to conceive fissures to exist under such circumstances, it is not reasonable to suppose that they would not take the direction in which the resistance would be least, that is, either the vertical, or the line of the cleavage of the rocks.

2nd. Veins are often divided into branches, which unite again at a considerable depth, including between them vast portions of rock perfectly insulated by the ore or vein-stones from the general mass: these, it is evident, could not have existed as fissures for a moment.

3rd. Veins are continually subject to changes in their horizontal direction and underlie; their size also often varies exceedingly, one part being many times wider than another, without any reference to their relative position or depth under the surface.

4th. Although a portion of their vein-stones are usually quite distinct in their characters from the rocks they traverse, they are generally, in part, of the same nature, and vary with the containing rocks, whether granite, elvan, killas, &c.; and they are commonly too regularly arranged in the veins, and are found inclosing insulated portions of the ore, &c. in their very substance, to admit of the idea of their having been originally mere broken fragments of the inclosing rocks.

At Dolcoath Mine there is an instance of one ore vein intersecting another at different depths, and being itself intersected and even shifted by the same vein at a greater depth. I have given a sketch of these intersections in Fig. 27, (for which I am indebted to Captain PETHERICK of that mine,) as I am not aware that it has been before noticed.

Many other facts might, if it were necessary, be accumulated, relative to the position and intersections of veins, as well as the nature and arrangement of their contents, which, with those I have stated, are calculated to throw entire discredit on the various hypotheses which have been invented to account for their origin. But my object is, rather to suggest whether the arrangement of veins, &c. does not argue design, and a probable connection with other phænomena of our globe.

Metalliferous veins, and those of quartz, &c. appear to be channels for the circulation of the subterraneous water and vapour; and the innumerable clay veins or "flucan courses" (as they are termed in Cornwall,) which intersect them, and are often found contained in them, being generally impervious to water, prevent their draining the surface of the higher grounds as they otherwise would, and also facilitate the working of mines to a much greater depth than would be practicable without them.

With respect to their electrical properties, it may be observed, that ores which conduct electricity have generally, in this county at least, non-conducting substances interposed in the veins between the ore and the surface. Thus a brown iron ochre with quartz, &c. named "gossan" by the miners, is almost invariably found resting on copper. Sulphuret of zinc occurs sometimes in the same situation, both with regard to copper and lead; but tin ore, which is a non-conductor, is without either, and is mostly found nearer the surface than copper.

Tin veins are usually intersected by those of copper when they do not coincide in their horizontal direction or underlie; thus, in this case, the conducting veins traverse the non-conducting ones. And when two veins of copper meet at opposite angles in descending, they are, I apprehend, generally found to be unproductive at and near the place of junction; but when they unite, proceeding downward in the same direction but at different angles, they are commonly observed to be enriched. These facts appear curious when regarded in connection with the opposite currents of electricity in veins having opposite dips.

There are some districts in this county in which the ore veins have generally a north underlie, and in others the south prevails; and it often happens that when lodes occur which deviate from the prevalent underlie of the others, in any district, the former are intersected, and sometimes shifted by the latter. This is strikingly the case in numerous mines in the parishes of St. Agnes and Perran.

The usual horizontal bearing of the copper and tin veins in our principal mining districts, appears to be nearly E. and W., or rather from E.N.E. to W.S.W.; but in others they deviate materially from these directions, sometimes to E.S.E. and W.N.W.: indeed, in some places this is the prevailing course of the veins of ore.

When veins containing the sulphuret of silver occur, (which as I have before stated is a non-conductor of electricity,) they are generally found nearly at right angles to the copper and tin veins, and seem thus to assume in great measure the character of cross veins of quartz, clay, &c.

With respect to the two latter, it has been observed that when they shift the ore veins, there is frequently to be found in them scattered stones of ore, or a small vein of it, or "leader" (to use a mining term), between the dislocated parts of the lode. This is also the case often with slides; so that, although the horizontal transfer of the electricity may be much impeded, it does not seem to be wholly intercepted. The quartz contained in cross veins is usually of a fibrous or radiated texture, and differs materially from that found in the east and west veins.

All our mining districts abound more or less with veins or dykes of a rock generally possessing a porphyritic character, termed by the miners "Elvan courses." Their width is extremely various, sometimes as much as fifty fathoms and upwards. Their direction in general is nearly N.E. or E.N.E. to S.W. or W.S.W., and their underlie is with few exceptions towards the N.W., and at various angles from the perpendicular, often exceeding 45°. They are penetrated by ore-veins in almost every direction, from their greater underlie, and usually more considerable deviation from an east and west bearing than the latter. It has been observed that copper and tin lodes generally become changed in quality whilst in the elvan*; and indeed this remark applies to any change of rock: thus a vein productive in granite commonly becomes barren in killas, and vice versâ.

Many of the phænomena above referred to, bear striking analogies to common galvanic combinations, and the discovery of electricity in veins seems to complete the resemblance.

I have been informed by intelligent persons who have visited some of the mining districts of Mexico, Guatimala, and Chili, that there is a general resemblance between the veins, elvan courses, &c. in some parts of those countries and our own; and I think it has been noticed by Baron HUMBOLDT, that the stratification of primitive rocks in different, and far distant parts of the world, has a general tendency from the N.E. towards the S.W.

Such analogies become highly interesting when regarded in connection with terrestrial electricity, magnetism, and heat; for if it be granted that the two latter increase in intensity at great depths in the earth, they are evidently so connected with electrical action that the augmentation of it also, in the interior of the globe, may be reasonably inferred.

However this may be, assuming that metalliferous veins exist more or less in primitive rocks generally, (and experience favours this assumption, whether we refer to the new mines which have been discovered in various parts of North and South America, Siberia, Ireland, &c. or to the mining county of Cornwall, in which whole districts have comparatively of late been found abounding with mineral treasure, where none had been formerly suspected to exist,) it may I think be presumed, that the electrical currents, which so affect the needle in the galvanometer, may likewise influence the direction of the magnetic needle on the surface of the earth: at least no explanation of this phenomenon appears to be so plausible, or so well connected with ascertained facts. Even the cause of the variations of the needle, mysterious as it has hitherto appeared to be, may probably be referred to the relative energies of the opposing electrical currents, which are perhaps subject to occasional modifications; and the appearance of earthquakes and volcanic action, from time to time, seems to countenance the probability of such changes.

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* It has been remarked of copper lodes, that they are often improved in quality whilst in elvan, particularly if it be not very hard.

Nor should it be overlooked in reference to this view of the subject, that the oblique bearing which is generally observable in the strata and veins, with respect to the equator, causes them, as it were, to cross at opposite sides of the globe in the same parallels of latitude, so that their tendency, if any, must necessarily be to produce more than one magnetic pole in each hemisphere. Thus, in this respect also, the hypothesis accords with the interesting fact lately announced;—of Professor Hansteen having ascertained the existence of a second magnetic pole within the arctic circle. The revolution of the earth on its axis from west to east seems moreover to harmonize with the idea of oblique electrical currents; since rotation in the same direction may be produced by corresponding electro-magnetic arrangements.

Before I conclude, I will briefly mention a few facts relative to the temperature of some of the mines in Cornwall.

At Tingtang copper mine, in the parish of Gwennap, at the bottom of the engine shaft, which is in killas, and 178 fathoms deep, the water about two months ago was at the temperature of 82°. In 1820, when the same shaft was 105 fathoms deep, the temperature of the water was 68°: thus an increase of 14° has been observed in sinking 73 fathoms, which is equal to 1° in 5 fathoms.

At Huel Vor tin mine, near Helston, the water was 69° at the bottom of a shaft 139 fathoms deep, in the year 1819. It is now 209 fathoms deep, and the temperature is 79°, which gives a mean increase of 1° in sinking 7 fathoms. This part of the mine is in killas.

The highest temperature of the water at the bottom of Poldice copper and tin mine in the parish of Gwennap, in 1820, which was then 144 fathoms under the surface, was 80°. It is now 176 fathoms deep, and the temperature is 99°; and in a cross level 20 fathoms further north, the water is 100°*.

The two last-mentioned temperatures are the highest hitherto observed in any of the mines of this county; and the increase is equal to 19° in one case, and 20° in another, in sinking 32 fathoms, or 1° for 1½ fathom. Three persons only were employed at a time near each of these stations, and the water pumped up from this part of the mine was estimated at 1,800,000 gallons in twenty-four hours; and I found on examination that it contained a considerable quantity of common salt in solution.

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* The thermometer used on these occasions was compared with others, and corrected one degree.

EXPLANATIONS.

Sections of Cross Veins marked thus

Section dotted, Granite.

Short line section, Clay Slate.

The arrows mark the direction of the positive electricity through the wire, which is represented by the black lines, and the number of cross marks on the arrows are increased according to the increase of electrical intensity.

⊕ Marks the stations of the Galvanometer.

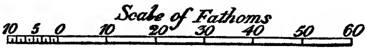
◠ The copper plate at the bottom of a level.

◡ Ditto at the back of a level. They are shaded in proportion to the richness of the veins, which are all of copper ore, except Fig. 11 & 18, and those otherwise marked in Fig. 19 & 21.

B.S. Bottom of the Shaft.

f'D. Depth from Surface in fathoms.

00° Temperature at the Stations of the plates.



Wires corresponding with the directions of the ore Veins or Lodes, from Fig. 1. to 18.

HUEL JEWEL.

South Lode.

North Underlie 23°.

Fig. 1.

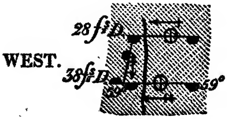
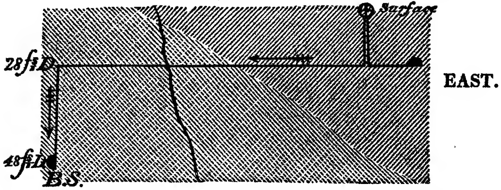


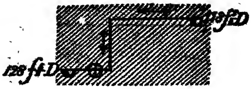
Fig. 2.



TINGTANG MINE.

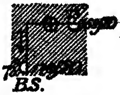
On Roche's Lode.
North Underlie 12°.

Fig. 3.


WEST.
128 fmD

On junction of Old and Middle Lodes.
North Underlie 24°.

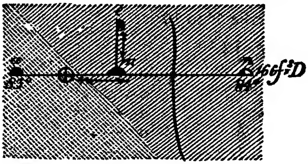
Fig. 4.



POLDICE.

Old Ore Lode.
North Underlie 23°.

Fig. 5.

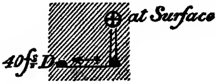

83°
84°
166 fmD

positive.		negative.	
a	with	n	
a	—	m	strong action.
e	—	m	ditto

HUEL UNITY.

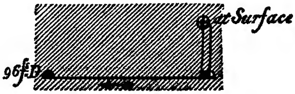
On William's Lode.
North Underlie 27°.

Fig. 6.


at Surface
40 fmD

On Garby's Lode.
North Underlie 27°.

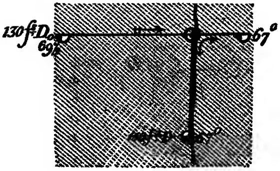
Fig. 7.


at Surface
96 fmD

HUEL DAMSEL.

On South Lode.
North Underlie 20°.

Fig. 8.


130 fmD
69½
67°

DOLCOATH MINE.

On Richard's Lode, perpendicular.

Fig. 9.

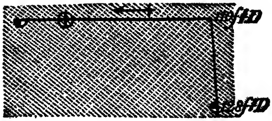
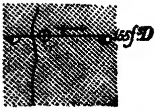


Fig. 10.

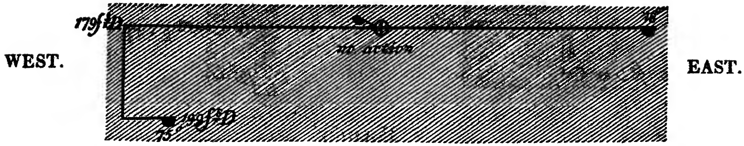


HUEL VOR.

On Tin Lode.

North Underlie 33°.

Fig. 11.

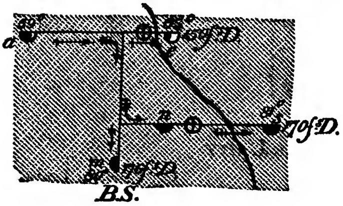

WEST.
EAST.
179f.D.
no action
199f.D.
75°

TRESAVEAN MINE.

On Main Lode.

South Underlie 5° to 10°.

Fig. 12.

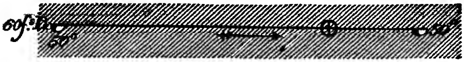

a
60f.D.
n
170f.D.
170f.D.
B.S.

positive.		negative.	
a	with	*e*	
a	—	*m*	
a	—	*n*	
a	—	*s*	strong action.
n	—	*s*	ditto
e	—	*s*	
m	—	*s*	
m	—	*e*	very slight.
n	—	*e*	ditto

North Lode.

South Underlie 3°.

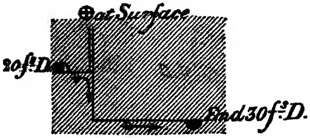
Fig. 13.



GREAT ST. GEORGE.

South Underlie 5°.

Fig. 14.

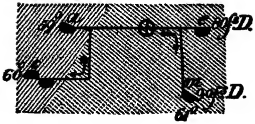

at Surface
20f.D.
End 30f.D.

DOLCOATH.

On Caunter Lode.

South Underlie 12°.

Fig. 15.



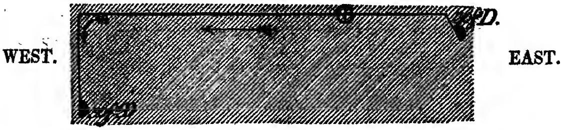
positive.		negative.
a	with	*m*
e	—	*c*
a	—	*c*
e	—	*m*

DOLCOATH.

On Caunter Lode.

South Underlie 12°.

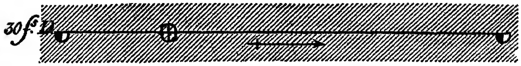
Fig. 16.



On Harriet's Lode.

South Underlie 9°.

Fig. 17.



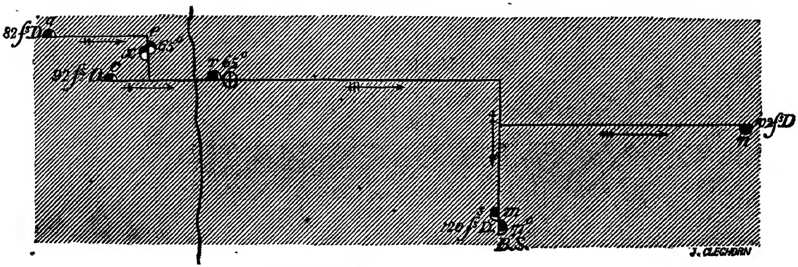
(See intersections of these Lodes, Fig. 27.)

HUEL ROSE LEAD MINE.

Lode 3 feet wide.

South Underlie 25° to 30°.

Fig. 18.


82 fs D
a
e
x
65°
92 fs D
c
r 65°
102 fs D
n
s
m
120 fs D
n
B.S.
J. CLEGHORN

positive.		negative.	
a	with	*r*	strong action
a	—	*m* or *s*	ditto
a	—	*n*	stronger
r	—	*m* or *s*	still stronger
r	—	*n*	ditto
c	—	*r*	
e	—	*r*	
x	—	*r*	
a	—	(*r n*)	

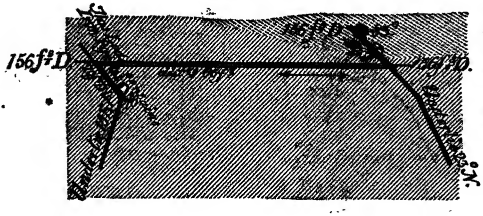
Transverse sections of parallel Lodes connected by the Wires, from Fig. 19. to Fig. 26.

POLDICE MINE.

Tin Lode. Old Ore Lode (copper).

Fig. 19*.

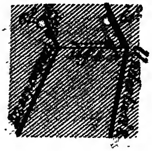
SOUTH.


$156 f^s$ D.
$156 f^s$ D.

HUEL LEISURE.

Copper Lodes.

Fig. 20.

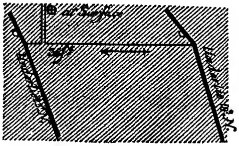


NORTH.

HUEL PROSPER.

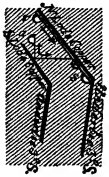
Copper Lode. Lead Lode.

Fig. 21.


$13 f^s$ D.
at Surface

TRESAVEAN MINE.

Fig. 22.

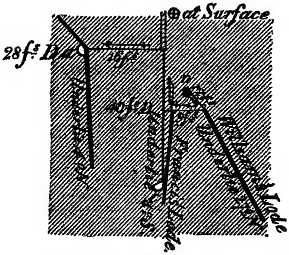


positive. negative.
a with r
a — s more strongly.
r — s

HUEL UNITY.

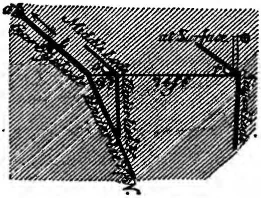
Copper Lodes.

Fig. 23.


at Surface
$28 f^s$ D.
$40 f^s$ D.

positive. negative.
c with a slight action
n — c stronger
n — a ditto

Fig. 24.


at Surface
$96 f^s$ D.

positive. negative.
o with r very slight action
e — r more
r — a great action

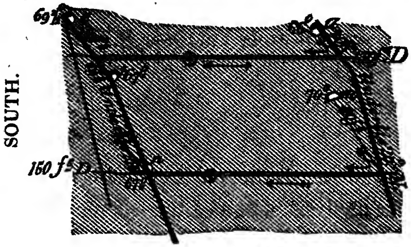
* The parts above the bends or angles in the veins or lodes are intended to represent the horizontal directions in perspective. The parts under the angles show their underlie.

HUEL DAMSEL.

Fig. 25.

S. Lode 2½ feet wide.

N. Lode 4 to 8 feet wide.


SOUTH.
69 f
D
150 f
D

positive. negative.

130 fathoms deep { a with e slight
a — f greater
e — f still more action

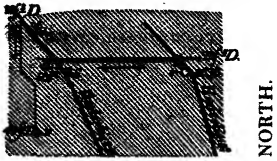
150 fathoms deep m — n

HUEL JEWEL.

Fig. 26.

S. Lode 2 to 4 feet wide.

N. Lode 1½ foot wide.



positive. negative.

r with s

r — x great action

Huel Jewel and Huel Damsel Lodes when connected; distance, 130 fathoms; former north of latter.

positive. negative.

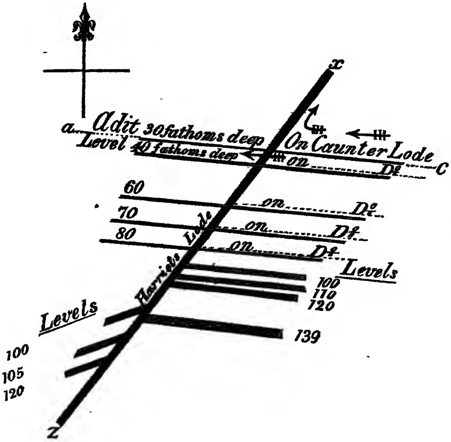
a with s

r — a very slight action

DOLCOATH MINE.

Supposed bird's-eye view of the alternate intersections of the Caunter and Harriet's Lodes, both copper.

Fig. 27.


x
a
Adit 30 fathoms deep
Level 40 fathoms deep
On Caunter Lode
on
Do
c
60
70
80
on
Do
Harriet's Lode
Levels
100
110
120
139
Levels
100
105
120
z

Direction of electricity (positive) at the adit from the Caunter Lode towards the E. or rather N.E. end of Harriet's, shown by the arrows.

On Harriet's Lode, pos. elect. from z to x = an arc of 40°

On Caunter Lode . c to a = —— 190°

c to x = —— 110°

c to(a&x)= —— 210°

(c x) to a = —— 120°

XXVI. *Sequel to a paper on the tendency to Calculous Diseases, and on the Concretions to which such diseases give rise.* *By* John Yelloly, *M.D. F.R.S. &c.*

Read June 17, 1830.

In a paper which the Royal Society did me the honour to publish in the last volume of its Transactions, I gave the analysis of 328 of the calculi contained in the cabinet of the Norfolk and Norwich Hospital; but was prevented from extending my observations over the whole series of specimens, from the circumstance of the remainder not being then divided. Since that period, however, the division of the whole has been effected; and I have therefore been able to complete the analysis, of which I have now the honour to lay the result before the Society.

In my former paper I stated it as probable, that the proportions of the different descriptions of calculi, which formed the undivided part of the cabinet, would not differ materially from that of those which I had analysed; and this proves, in a considerable degree, to be the case. But it may be remarked, that some of the specimens which had been broken in the extraction, and whose interior had been considered as sufficiently exposed for the correct examination of all their laminæ, were found to exhibit some slight differences in their centres, on a more complete division.

I shall adopt the same plan which I pursued in my former paper, and present the results of the analysis in a tabular form, giving, in the order of their occurrence from the centre, the consecutive deposits of the different materials of which the calculi are composed, according to the most prominent character of such material. I shall include in the Table, both those of the former, and the present analysis, amounting together to 663 specimens, in order to present, at one view, a summary of the chemical composition of the whole cabinet *.

---

* I have placed in the cabinet, a book containing an outline of each calculus, with references; and would also observe, that some of the facts stated in this communication, have been added since it was presented to the Society.

Calculi consisting principally of one deposit.

Lithic acid	164
Lithate of ammonia	55
Oxalate of lime	21
Phosphate of lime	5
Mixed phosphates	35

Calculi consisting af two deposits.

Lithic acid and lithate of ammonia	49
——— and oxalate of lime	10
——— and mixed phosphates	15
——— and phosphate of lime	8
Lithate of ammonia and lithic acid	21
——— and oxalate of lime	63
——— and mixed phosphates	22
——— and phosphate of lime	9
Oxalate of lime and lithic acid	15
——— and lithate of ammonia	3
——— and mixed phosphates	20
——— and phosphate of lime	7
——— and silex	1
Mixed phosphates and oxalate of lime	1
——— and phosphate of lime	2
Phosphates of lime and mixed phosphates	3
——— and oxalate of lime	1

Calculi consisting of three deposits.

Lithic acid, oxalate of lime, and phosphate of lime	2
——— oxalate of lime, and lithate of ammonia	4
——— oxalate of lime, and lithic acid	5
——— lithate of ammonia, and oxalate of lime	2
——— lithate of ammonia, and lithic acid	2
——— lithate of ammonia, and mixed phosphates	2

Lithic acid, oxalate of lime, and mixed phosphates	3
Lithate of ammonia, oxalate of lime, and mixed phosphates	13
——— oxalate of lime, and phosphate of lime	13
——— oxalate of lime, and lithic acid	16
——— oxalate of lime, and lithate of ammonia	7
——— phosphate of lime, and lithate of ammonia	1
——— phosphate of lime, and lithic acid	1
——— phosphate of lime, and oxalate of lime	1
——— phosphate of lime, and mixed phosphates	4
——— lithic acid, and mixed phosphates	6
——— lithic acid, and lithate of ammonia	1
——— lithic acid, and phosphate of lime	4
——— lithic acid, and oxalate of lime	3
Oxalate of lime, lithic acid, and lithate of ammonia	3
——— lithic acid, and oxalate of lime	3
——— lithic acid, and mixed phosphates	5
——— lithic acid, and phosphate of lime	1
——— lithate of ammonia, and phosphate of lime	3
——— lithate of ammonia, and oxalate of lime	2
Mixed phosphates, phosphate of lime, and mixed phosphates	1

Calculi consisting of four deposits.

Lithic acid, lithate of ammonia, lithic acid, and lithate of ammonia	1
——— oxalate of lime, lithate of ammonia, and phosphate of lime	1
——— oxalate of lime, lithic acid, and oxalate of lime	1
——— oxalate of lime, lithic acid, and lithate of ammonia	2
Lithate of ammonia, oxalate of lime, lithate of ammonia, and mixed phosphates	5
——— oxalate of lime, lithate of ammonia, and oxalate of lime	3
——— oxalate of lime, mixed phosphates, and oxalate of lime	2
——— oxalate of lime, lithic acid, and lithate of ammonia	1
——— oxalate of lime, phosphate of lime, and mixed phosphates	1
——— oxalate of lime, lithic acid, and mixed phosphates	1
——— oxalate of lime, lithic acid, and oxalate of lime	1

Lithate of ammonia, oxalate of lime, lithate of ammonia, and lithic acid	1
——— phosphate of lime, oxalate of lime, and lithate of ammonia	1
Oxalate of lime, lithic acid, lithate of ammonia, and lithic acid	1
——— lithic acid, oxalate of lime, and phosphate of lime	1
——— lithic acid, oxalate of lime, and mixed phosphates	1
——— lithic acid, lithate of ammonia, and mixed phosphates	1
	663

The general results of the annexed Table, differ in no very material respect from those mentioned in my former paper. The great preponderance of calculi of lithic acid, and lithate of ammonia, or of their nuclei; and the usual presence of carbonate of lime, with phosphate of lime, and the mixed phosphates, are prominent characteristics of the whole cabinet. But it will be seen, with interest, that *silex* enters into the composition of one specimen; and as this is a point of great rarity, I shall state to the Society the circumstances under which it exhibited itself, and the mode by which its existence has been ascertained.—In examining a dark brown calculus of oxalate of lime, of about five grs. in weight, which was removed many years since from a boy of nine years of age, I found some minute, colourless, transparent crystals, diffused irregularly in the substance of the dark oxalate, which, from their great hardness, and their insensibility to all the usual reagents, I suspected to be siliceous. The specimen was however so small, as to make it important to establish its nature by one set of experiments only; and I therefore, in my first subsequent visit to London, in February last, requested a valuable member of this Society, Mr. FARADAY of the Royal Institution, to examine it with me; which he obligingly did; and the following is an account of the experiments which he employed.

A portion of the calculus being separated, which contained about nine of these granules, the oxalate of lime, and whatever other substance might be in combination with it, was destroyed by heat, and afterwards by muriatic acid. The granules were then left transparent and colourless; capable of scratching glass and agate, and unaffected by nitric or muriatic acids. These granules were then dried and exposed to heat, with a fused mixture of carbonate of soda and potash. They gradually dissolved, evolving carbonic acid; and a solution of the mass, when cold, being made in water, and neutralized by muriatic acid, gelatinized silica was thrown down from it. A slight excess of muriatic acid was then added, and the whole evaporated to dryness. After withdrawing the muriate of potash and soda by distilled water, the silica was left in its usual white, insoluble state. By comparing the magnitude of these granules with some which were taken from a sand bath, it was calculated that they did not average more than the 400th part of a grain in weight. The granules were thus unequivocally proved to be of silex; and as they were imbedded in, and diffused through, oxalate of lime, a substance of known urinary origin, it is impossible to avoid the conclusion, that the production, or deposition of these two substances, went on simultaneously.

I am the more particular in mentioning the circumstances under which the siliceous deposit exhibited itself, because much discrimination is occasionally necessary, on the part of medical men, to prevent their being deceived, by the mistakes of patients, or their friends, in matters of an unusual nature. And as if the love of exciting surprise and admiration by the marvellous, were not confined to the traveller, there is sometimes found in patients, however singular the fact may appear, an inclination to impose on their professional attendants, by the description or exhibition of something strange and anomalous*.

There are only three instances on record, as far as I know, of the existence of silex in urinary calculi. Two are mentioned by MM. FOURCROY and VAUQUELIN as occurring among 600 calculi which they analysed; and here the silex was found blended with oxalate of lime, as in the specimen which I have mentioned. The third was observed by Prof. WURZER, and its principal ingredients were phosphate of lime and lithic acid, the weight of the calculus being 870 grains, and the quantity of silex being one per cent.†. In none of these calculi, however, is the magnitude of the siliceous particles stated.

The deposition of siliceous gravel is mentioned by Dr. VENABLES of Chelmsford, in a communication published in the Quarterly Journal of Science, Literature, and Art, for Oct.—Dec. 1829*; and the correctness of that gentleman's observations, receives a strong and important confirmation by the instance which I have brought forward; though they might, at first view, be considered as liable to some doubt, both from the circumstance of the granules being uncombined, and from its being necessary to depend, to a certain extent, on the fidelity of the patient as to their source.

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* Portions of coal, brick, common gravel, and sea shingle, have occasionally been produced, as of urinary origin.

† Annales de Chimie, tome lx. p. 313.

Dr. Venables has kindly allowed me the inspection of his specimens, which bear some resemblance (though they are much more minute, and are of an amber tinge,) to those which I have mentioned as coming under my own view; and he has stated in a letter to me, that in one instance, after carefully filtering and putting aside for a fortnight, a portion of the urine from which some of the granules mentioned by him had been derived, he found the inside of the glass studded, in two or three places, with minute crystals of silex, strongly resembling those which were thrown down by the urine. The precise modes in which silex is capable of being held in solution, have not all of them been distinctly ascertained; but this fact bears a considerable analogy to the deposition of regular crystals of rock crystal, from solutions of silex in fluoric acid, or in alkalis, after such solutions have been put aside for a considerable period.

I have not much to add to the statistical observations which I made in my former communication. I may remark, however, that it appears, from information lately obtained by Mr. Copland Hutchison, that the calculations relative to the tendency to calculous diseases in Scotland, in which I followed Mr. Smith of Bristol, have been a good deal under-rated; and that the average disposition of that part of the kingdom to such complaints, differs but little from that of England in general†.

There seems to be much of the same variation with regard to the prevalence of calculous diseases in Scotland, that there is in England; some districts being exceedingly liable to these complaints, while others are very free from them. My own inquiries, in addition to the valuable information communicated by Mr. HUTCHISON, in the paper already referred to, enable me to state the proportional frequency of the disease in a few districts.

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* New Series, No. XII. p. 234.

† Further Inquiry into the comparative Infrequency of Calculous Diseases among Sea-faring People; with some Observations on their Infrequency in Scotland. Medico-Chirurgical Transactions, vol. xvi. p. 94.

The operations which have been performed at the Dundee Hospital, on cases belonging to Dundee, and to the county of Forfar generally, are about 54 in 36 years, or 1.38 per annum*; this, as the population is 113,000, is at the rate of one case for 107,000 inhabitants.

In the Aberdeen Hospital, the proportion is much more considerable; for in the course of 77 years, as by a list which Mr. CROMAR of that establishment was so good as to transmit to me, 285 operations have occurred, on cases belong to Aberdeenshire, and to the town of Aberdeen, containing together, a population of 155,000; which is at the rate of one case for every 42,000 inhabitants. On the other hand, in the Hospital of Inverness, not more than 5 operations, as Mr. HUTCHISON states, have occurred in the last 20 years, which, for the population of 90,000 contained in Inverness-shire, including the town of Inverness, is not at the rate of more than one for every 300,000 inhabitants. But when it is considered, that this hospital is resorted to by the poor of the contiguous counties of Ross and Nairn, as well as by those of Cromarty and Sutherland, the extreme paucity of stone cases, in this mountainous district, must be manifest.—In the Infirmary of Glasgow, 49 cases have occurred, in the course of 15 years, according to a list which Dr. MACFARLANE did me the favour to forward to me, or about 3.26 per annum; and of these, 31 belonged to the city and suburbs, containing about 147,000 inhabitants, which is at the rate of 2 per annum, or one for every 71,000 inhabitants. The remaining 18 were from the country; but I am unable to state the districts from which they were derived. There is reason, however, to suppose, that the tendency of the neighbouring counties to this disease, is very much less than that of Glasgow, for they have only afforded as small a number of calculous diseases as 18 to the Infirmary of that city, which may be regarded as the principal establishment for cases requiring capital operations in the West of Scotland.

In the neighbouring town of Paisley, consisting of 38,000 inhabitants, about 18 cases have occurred in 10 years, all of them of poor inhabitants of the town, which is at the rate of 1.8 per annum, or one for every 21,000 inhabitants.

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* Observations on the Operation of Lithotomy, by JOHN CRICHTON, Esq. Edin. Med. & Surg. Journ. vol. xxix. p. 225.

In the Infirmary of Edinburgh, 41 cases have occurred in the last 10 years; but I have not been able to procure, separately, the numbers which were derived from the city and the country respectively. Taking, however, the proportions as similar to what is found to be the case in the Glasgow Hospital (which is probably not far from the truth), there would be 24 of that number belonging to the city of Edinburgh, including Leith, and containing a population of 138,000, which would be at the rate of 2.4 per annum, or one case for every 57,000 inhabitants.

In the southern and south-west parts of Scotland, as well as in the northern, the disease is exceedingly rare; for Dr. Craigie of Edinburgh informs me, that it is hardly known in the Dumfries Hospital, which, as being the only establishment of this kind south of Edinburgh and Glasgow, takes in a very large district in that part of the kingdom; and Sir George Ballingall, the Regius Professor of Military Surgery in the University of Edinburgh, states to me, on the most respectable authority, that in Kelso and its neighbouring district, calculous complaints scarcely ever occur.

The county of Northumberland, and that part of the county of Durham which is contiguous to the Tweed, a good deal resemble, in the unfrequency of calculous diseases, the adjoining districts of Scotland; for by a list which was obligingly transmitted to me by Dr. Headlam of Newcastle, it appears, that 95 cases of stone operation occurred in the Newcastle Infirmary during the last 30 years, which is at the rate of 3.6 per annum. Of this number, 64 belonged to the above district, including Newcastle, with the addition of Gateshead, which lies on the opposite bank of the Tyne, in the county of Durham; and these afforded 2.13 cases per annum, which, as the population was 213,000, gave one case for every 100,000 inhabitants. But if the country district be taken without Newcastle or Gateshead, there will then be 29 cases in 30 years for a population of 166,000, and one case for every 172,000 inhabitants.

As very little has been hitherto known concerning the proneness to calculous diseases in Ireland, though it is generally believed that such complaints are unfrequent, I have lately instituted inquiries on the subject, at the various hospitals in that kingdom, to which Sir Charles Flint, the resident Under-Secretary of State for Ireland in this country, has obligingly given me every facility. From the result of those inquiries it appears, that calculous diseases in Ireland are very rare, particularly in its country population. In various extensive districts from which I have been favoured with returns, stone is entirely unknown; and in others, it occurs with extreme unfrequency. Thus in the counties of Antrim, Armagh, Londonderry, Donegal, Fermanagh, Tyrone, Carlow, Kildare, Kilkenny, and Longford; in King's County; and in the counties of Louth, Wicklow, Clare, Kerry, Galway, Roscommon, Tipperary, and Mayo, containing, together, a population of above three millions and half of persons, not a single operation of lithotomy has occurred in any of their respective hospitals since their establishment; nor has one example, among the poor of those extensive districts, come within the cognizance of the eminent and well-informed practitioners, who have done me the favour of replying to the queries which I transmitted to them on the subject.

In the counties of Down, Monaghan, Leitrim, Sligo, Limerick, and Waterford, and in Queen's County, the population of which amounts together to about 1,200,000 persons, 9 cases of stone operation only have occurred, during the whole time to which the records of the hospitals, or the information or inquiries of their medical officers extend, and which embrace a period hardly short of 40 years. This is at the rate of not more than 0.25 per annum, or one case in 4 years.

In the city and county of Cork, containing, together, above 800,000 inhabitants, about 13 operations of lithotomy have been performed in the last 18 years, or about 0.66 per annum, of which 10 occurred to Dr. Woodroffe of the South Hospital.

In the hospitals of Dublin, including the Meath or county of Dublin Hospital, it appears from information with which I have been favoured by Mr. Roney, late President of the Royal College of Surgeons of Dublin, and Mr. Crampton, the Surgeon-general of Ireland, that about 6 cases occur in the course of the year; and this estimate is confirmed by Mr. Carmichael's opinion, as stated by Mr. Hutchison.

Making a suitable allowance, therefore, for those counties from which I have been disappointed in not yet receiving returns*, it does not appear that more than 8 operations of lithotomy occur annually among the poor of Ireland, the whole population of which was considered, in 1821, to be about seven millions†. But when we refer 5 of those cases to the city and county of Dublin, containing, together, a population of about 350,000, (which, from the documents with which I have been favoured, seems to be very near the truth,) it will be seen how minute the proportion of calculous cases must be in the remaining population of Ireland, when not more than 3 operations of lithotomy occur, annually, among the poor of a population of between six and seven millions.

I have mentioned in my former paper, a suspicion, from some facts there stated by me, that the principal occurrence of calculous diseases is in towns. This idea is strengthened by a consideration of the lists of stone cases, for which I have been indebted to Dr. Headlam of Newcastle, and Mr. Eddison of Nottingham. In the town of Newcastle and Gateshead, calculous cases have occurred, within the last 30 years, in the proportion of about one for every 40,000 inhabitants; while in the county of Northumberland, independently, together with that part of the county of Durham which borders on the Tweed, the proportion, (as I have already remarked,) has only been one for 172,000 inhabitants. —In the town of Nottingham, the proportion, in 48 years, has been at the rate of one for every 67,000 inhabitants; while in the county of Nottingham only, it has been one for every 146,000 inhabitants.

In the town of Dundee, the calculous operations have occurred in the proportion of one to 41,000 inhabitants; yet in the county of Forfar, in which Dundee is situated, the proportion has been one to 82,000.

In Glasgow, the proportion has been, in the last 15 years, about one in 71,000; in Paisley, in 10 years, one in 21,000; and in Edinburgh, in a similar time, one in 57,000; while there is every reason for supposing (though I cannot speak from direct evidence upon the subject) that the stone cases afforded to any of those establishments, by country population, is exceedingly small. It is to be remarked, however, that the proportional numbers furnished by the town and country population of Aberdeenshire, during a period of 70 years, are nearly alike; or rather, that those of the country population have somewhat preponderated; for while Aberdeen has afforded to the amount of one case for 44,000 inhabitants, the proportion of Aberdeenshire alone, without any obvious cause for such difference, has been one for 40,000 inhabitants.

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* These are Cavan, Meath, Westmeath, and Wexford.

† Statistical Illustrations, 3rd edition, 1827.

In Ireland, the great preponderance of calculous cases originating in towns, is likewise strongly marked; for in Dublin, to judge from the returns of the county hospital for the last 12 years, two thirds of the cases, or 4 per annum, seem to be furnished by that city, which is about one half of the number afforded by the whole remaining part of Ireland, and is at the rate of one for every 45,000 inhabitants. In Cork likewise, the proportions are much less than those of Dublin, being about one for 160,000 inhabitants; yet this infinitely exceeds the usual product, either of the county of Cork, or of Ireland generally.

Where the circumstances which have a tendency to produce calculous diseases, are so very obscure, so difficultly traceable, and so full of anomalies, I have thought it useful to notice the local situations which are remarkable either for the frequency or unfrequency of such maladies, because the attention of observers may thus be directed to analogies, or discrepancies, which may not before have been sufficiently the subject of remark. That there are certain affections of the digestive organs which favour the occurrence of calculous complaints, is very generally admitted; and that these affections are likewise exceedingly prevalent in towns, and more especially among persons who practise sedentary occupations, is likewise well known. But while dyspeptic complaints are so common, that they form a very large share of the diseases which present themselves in the medical charities, both of London and the country, it still remains a problem, to what particular circumstances of constitution or habit, the origin of stone is to be attributed, which, in comparison of other chronic maladies, is so rare; and what are the peculiarities which render some dyspeptics liable to the complaint, and thousands of others exempt from it.

That there is a certain connection between calculous tendency, and some prevailing diathesis of towns, is supported by the greater frequency of stone in town, than country populations; and I was inclined to consider it not improbable, that this diathesis might be the scrophulous, both from mesenteric diseases prevailing so much among the children of crowded cities, and from scrophulous affections being very common in Norfolk. Still, however, the same difficulty presents itself, as occurs with regard to affections of the digestive organs; namely, why so minute a portion of scrophulous persons should be affected with this disease, and so large a number escape. I am disposed, indeed, to consider the idea of a probable connection between a scrophulous and calculous diathesis, as very much weakened by the consideration, that in many of those parts of Ireland, where calculous complaints are hardly at all known, scrophulous affections are stated to me to be exceedingly common.

I must still confess the difficulty of referring the disposition to calculous affections, to any known circumstances of air, water, soil, or habits of life. The eastern position of Norfolk and Suffolk, produces more than the average bleakness of other parts of the kingdom, particularly in the spring months; but this inclemency is exceeded, both in the northern parts of the island, and the more elevated parts of Ireland. The poorer part of the population have much of a farinaceous aliment, often not well fermented: but still the main article of food is fine wheaten bread, which has been, for a long period, in such common use over the whole of England. In Scotland, too, the latter is, as I am informed, fast superseding the bread made of the coarser grains.

In Ireland, the diet of the humbler classes of society is very much confined to potatoes, and butter-milk, or skim-milk; often, unfortunately however, without either of the latter; and it appears highly probable that this diet, and perhaps, to a certain extent, the use of ardent spirits, is unfavourable to the formation or deposition of lithic acid, with which the tendency to calculous diseases is much connected. The freedom from this formidable class of complaints seems, however, to be dearly purchased, by the want of many of the comforts which the labourers of England and Scotland possess; nor is it accompanied by a less than usual tendency to many of the most ordinary complaints of Great Britain, as pulmonary consumption and scrophula, which are mentioned by several of my intelligent correspondents, as being very frequent among the Irish commonalty. The effect of a town residence, in producing a tendency to calculous complaints, is very strongly exemplified in Dublin, where they are so much more prevalent than in other parts of Ireland. This circumstance will, I venture to hope, meet with the particular attention of the Dublin faculty, who unite, in so a high a degree, the requisites for prosecuting a difficult professional investigation.

The middle and active period of life, laborious and healthful occupations, together with a fair opportunity of obtaining nutritive aliment, and ordinary comforts, seem best adapted for producing a freedom from chronic ailments, and among them, from urinary calculus.

Mr. HUTCHISON has shown, how unfrequent the latter is among sailors; and there is every reason for supposing that this freedom is, in a considerable degree, participated by the military profession. On this point I have thought it desirable to institute some inquiries, the result of which I shall have the honour to lay before the Society.

In a valuable report published by Sir JAMES MACGRIGOR, on the diseases of the British army in the peninsula, under the command of the Duke of WELLINGTON, no case of calculus appears to have presented itself during the period of which he treats, viz. between December 1811 and June 1814, though above 330,000 cases were admitted into the general and regimental hospitals during that period*.

In the last 15 years, Sir JAMES informs me, that 4 cases only of calculus have occurred in the English army in Britain; and Mr. CRAMPTON, the surgeon-general of Ireland, states, that one example of lithotomy only, in which the operation was performed by himself, has occurred, within the same period, in the army in Ireland. I am, however, able to add to this, on the authority of Dr. PITCAIRNE of Cork, the case of an officer of the Scots Greys, whom I had occasion to visit at the barracks here, who was operated upon at Cork, by Dr. WOODROFFE, about two years since, in his way to join his regiment. Mr. CRAMPTON likewise informed me, on the authority of Sir JAMES WYLIE, physician to the late, and to the present Emperor of Russia, that calculous diseases are hardly known in the Russian army.

The Baron DELESSERT of Paris, has done me the favour to procure from the Baron LARREY, and M. GAMA, surgeons-in-chief to the great military hospitals of Gros Caillou, and Val de Grace, in the French metropolis, a report as to the prevalence of calculous complaints among the French soldiery. The Baron

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* Sketch of the Medical History of the British Armies of the Peninsula of Spain and Portugal during the late Campaigns. Medico-Chirurgical Transactions, vol. vi. p. 381.

LARREY states, that in the course of 30 years, only 5 operations of lithotomy have been performed at the Gros Caillou, (4 of which were on soldiers, and one on a soldier's child,) and one operation at the Val de Grace. M. GÁMA states, that during 6 years that he has been surgeon-in-chief of the military hospital of Val de Grace, and 8 previously, that he exercised the same functions in the military hospital at Strasbourgh, he has not once had occasion to perform the operation of lithotomy. He mentions likewise, that the disease is very rare in the army generally; and that no case of stone operation has occurred to him, during any part of his extensive military service.—It is very probable, however, that as the circumstances which concern health can be more particularly guarded in the naval service, than the military, according to the very judicious observations of Sir JAMES MACGRIGOR upon this subject, there may be, upon the whole, a less liability to disease in the former, than in the latter.

Before closing my observations, I cannot forbear expressing my regret, that since the introduction of lithotrity by M. CIVIALE, as a succedaneum for the operation of lithotomy, the beneficial effects of that practice do not seem to have been completely established in this country; though it has been recommended by the singular dexterity, and the conciliating deportment, of the Baron HEURTELOUP and Mr. COSTELLO. It is highly to the credit of our principal metropolitan surgeons, that all of them with whom I have conversed on the subject, are anxious that this plan of removing a calculus should have a fair trial; and I trust that in a matter in which the interests of humanity are so intimately concerned, such attention may be speedily given to the subject, by those who are qualified to direct public opinion, as may lead to a proper appreciation of the merits of the practice, the circumstances under which it may be best exercised, and the mode by which the manual dexterity which it requires, may be most readily obtained.

Carrow Abbey, near Norwich,
June 16, 1830.

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